

# Novel Estimation Algorithm For Bearings-Only Target Tracking

B.Omkar Lakshmi Jagan <sup>#1</sup>, S.Koteswara Rao <sup>#2</sup>, K.Lakshmi Prasanna <sup>#3</sup>, A.Jawahar <sup>#4</sup>,  
Sk.B.Karishma <sup>#5</sup>

<sup>#</sup> School of Electrical Sciences, KL University  
Greenfields, Vaddeswaram, Guntur District, Andhra Pradesh, India

<sup>1</sup>omkarjagan@gmail.com

<sup>2</sup>rao.sk9@gmail.com

<sup>3</sup>lakshmiprasannakancherla@gmail.com

<sup>4</sup>jawaharaee@outlook.com

<sup>5</sup>benazeerkarishma205@gmail.com

**Abstract**—The work focuses on deployment of novel underwater target estimation algorithm to determine the kinematic state of the target. The Target Motion Parameters (TMP) are found out using bearings-only measurements. If one of the target motion parameters is known, then the rest of them can be found out using the known parameter. The Pseudo Linear Estimator (PLE) algorithm is considered in this paper which is one of the simplest estimation algorithms is presented in this paper to estimate TMP. It is assumed that target speed is known by some means, then the target course and range is obtained using PLE. PLE incorporates the basic features of Kalman filter like sequentially processing, variance included in measurement. The Monte-Carlo simulation is carried out for the tactical geometries and various results demonstrate the superiority of PLE over its peers for underwater target tracking applications.

**Keywords** - sonar, target tracking, algorithm, estimation, simulation, sequential processing

## Nomenclature

|               |   |
|---------------|---|
| $X_s(k)$      | Target state vector                                 |
| $\phi(k+1 k)$ | State transient matrix                              |
| $b(k+1)$      | Deterministic vector                                |
| $t$           | Damping time  |
| $B_m(k)$      | Bearing measurement                                 |
| $R_x(k)$      | Range component in X-direction                      |
| $R_y(k)$      | Range component in Y-direction                      |
| $\eta(k)$     | Error in the bearing measurement                    |
| $R(k)$        | Range between observer and target at time index $k$ |
| $B_0$         | initial bearing measurement                         |
| $T_{cr}$      | target course                                       |
| $V_t$         | target speed  |

## I. INTRODUCTION

This Target Motion Analysis (TMA) especially in two coordinate approach is conventional utilized for undersea target tracking. The observer listens to the noise emitted from the radiating target (i.e., passive mode). The Line of Sight (LOS) or bearing measurements are only available in passive mode while range as well as bearing measurements are available in conventional active mode of surveillance. The process of target motion estimation in passive mode is posed as non-linear estimation problem due to non-linear relation between target state and LOS measurements. The LOS measurements from acoustic source are monitored by single observer and therefore the observer has to maneuver for range observability of the target. The absence of maneuver is compensated by using utilising Doppler measurements along with bearing measurements[1-3], altitude[4] and range as well as bearing measurements[5]. But without any observer movement, the paper introduces new method in passive target tracking. The target state vector consists of position and velocity in X and Y coordinates. The LOS measurements are processed to track the target.

In[5], batch mode is used which is not suitable for practical applications. This paper transforms the batch mode in recursive mode which less computational effort as well as time[9]. The covariance matrix is updated recursively in terms of increment of the new measurement with reduction in time as well as burden.

The mathematical modeling of PLE with recursive mode is demonstrated in section II. The simulation along with results is presented in the consecutive section and concluded in section IV.

## II. MATHEMATICAL MODELING

The modeling of target dynamics is presented in this section. The target state equation is given by [8]

$$X_s(k+1) = \phi(k+1, k) X_s(k) + b(k+1) \quad (1)$$

$$X_s(k) = \begin{bmatrix} \dot{x}(k) & \dot{y}(k) & R_x(k) & R_y(k) \end{bmatrix}^T \quad (2)$$

$$\phi(k+1|k) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ t & 0 & 1 & 0 \\ 0 & t & 0 & 1 \end{bmatrix} \quad (3)$$

$$b(k+1) = \begin{bmatrix} 0 & 0 & -\{x_o(k+1) - x_o(k)\} & -\{y_o(k+1) - y_o(k)\} \end{bmatrix} \quad (4)$$

$$B(k) = \tan^{-1}(R_x(k)/R_y(k)) \quad (5)$$

$$B_m(k) = B(k) + \eta(k) \quad (6)$$

where  $\eta(k)$  is zero mean Gaussian with variance  $\sigma^2$ . The TMP can be estimated in two methods, one the target is in non maneuvering mode and another in maneuvering mode. In non maneuvering state of the target,  $b(k+1)$  is equated to zero. The manipulation of these equations yields

$$R_x(k) \cos B_m(k) - R_y(k) \sin B_m(k) + \xi(k) = 0 \quad (7)$$

$$\xi(k) = \eta(k) R(k) \quad (8)$$

$$H(k) X_s(k) + \xi(k) = 0 \quad (9)$$

$$H(k) = \begin{bmatrix} 0 & 0 & \cos B_m(k) & -\sin B_m(k) \end{bmatrix} \quad (10)$$

The TMP estimation is carried out using the initial  $X_s(k)$ , then eqn. (9) is redefined in the following fashion

$$H(k) \phi(k,0) X_s(0) + \xi(k) = 0 \quad (11)$$

$$\phi(k,0) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ kt & 0 & 1 & 0 \\ 0 & kt & 0 & 1 \end{bmatrix} \quad (12)$$

In case of k measurements,

$$\begin{bmatrix} t \cos B_m(1) & -t \sin B_m(1) & \cos B_m(1) & -\sin B_m(1) \\ 2t \cos B_m(2) & -2t \sin B_m(2) & \cos B_m(2) & -\sin B_m(2) \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ kt \cos B_m(k) & -kt \sin B_m(k) & \cos B_m(k) & -\sin B_m(k) \end{bmatrix} \begin{bmatrix} \dot{x}_t \\ \dot{y}_t \\ R_x(0) \\ R_y(0) \end{bmatrix} + \begin{bmatrix} \xi(1) \\ \xi(2) \\ \cdot \\ \xi(k) \end{bmatrix} = 0 \quad (13)$$

$$\begin{bmatrix} t.\cos B_m(1) & -t.\sin B_m(1) \\ 2t.\cos B_m(2) & -2t.\sin B_m(2) \\ \vdots & \vdots \\ kt.\cos B_m(k) & -kt.\sin B_m(k) \end{bmatrix} \begin{bmatrix} \dot{x}_t \\ \dot{y}_t \end{bmatrix} + \begin{bmatrix} \cos B_m(1) & -\sin B_m(1) \\ \cos B_m(2) & -\sin B_m(2) \\ \vdots & \vdots \\ \cos B_m(k) & -\sin B_m(k) \end{bmatrix} \begin{bmatrix} R_x(0) \\ R_y(0) \end{bmatrix} + N(k) = 0 \quad (14)$$

$$N(k) = \begin{bmatrix} \xi(1) \\ \xi(2) \\ \vdots \\ \xi(k) \end{bmatrix}$$

$$A' X_v + DX_p(0) + N(k) = 0 \quad (15)$$

$$A' = \begin{bmatrix} t.\cos B_m(1) & -t.\sin B_m(1) \\ 2t.\cos B_m(2) & -2t.\sin B_m(2) \\ \vdots & \vdots \\ kt.\cos B_m(k) & -kt.\sin B_m(k) \end{bmatrix} \quad (16)$$

$$D = \begin{bmatrix} \cos B_m(1) & -\sin B_m(1) \\ \cos B_m(2) & -\sin B_m(2) \\ \vdots & \vdots \\ \cos B_m(k) & -\sin B_m(k) \end{bmatrix} \quad (17)$$

$$X_v = \begin{bmatrix} \dot{x}_t \\ \dot{y}_t \end{bmatrix} \quad (18)$$

$$X_p(0) = \begin{bmatrix} R_x(0) \\ R_y(0) \end{bmatrix} \quad (19)$$

Where we have  $A = -A'$ , then the least square of estimation of  $X_v$  is

$$\hat{X}_v = \left[ (A^T A)^{-1} A^T D \right] X_p(0) \quad (20)$$

$$\text{Let } S = \left[ (A^T A)^{-1} A^T D \right] \quad (21)$$

$$\dot{x}_t = S_{11} R_x(0) + S_{12} R_y(0)$$

$$\dot{y}_t = S_{21} R_x(0) + S_{22} R_y(0)$$

Replacing  $R_x(0)$  with  $\tan(B_0) R_y(0)$

$$\dot{x}_t = S_{11} \tan(B_0) R_y(0) + S_{12} R_y(0) \quad (22)$$

$$\dot{y}_t = S_{21} \tan(B_0) R_y(0) + S_{22} R_y(0)$$

$$\frac{\dot{x}_t}{\dot{y}_t} = \frac{S_{11} \tan(B_0) + S_{12}}{S_{21} \tan(B_0) + S_{22}} \quad (23)$$

$$\text{Tan}^{-1}\left(\frac{\dot{x}_t}{\dot{y}_t}\right) = \text{Tan}^{-1}\left(\frac{S_{11} \text{Tan}(B_0) + S_{12}}{S_{21} \text{Tan}(B_0) + S_{22}}\right) \tag{24}$$

The target course, tcr, can be calculated as

$$\text{tcr} = \text{Tan}^{-1}\left(\frac{S_{11} \text{Tan}(B_0) + S_{12}}{S_{21} \text{Tan}(B_0) + S_{22}}\right) \tag{25}$$

To calculate S matrix.

$$A^T A = \begin{bmatrix} -t \cos B_m(1) & -2t \cos B_m(2) & \dots & -kt \cos B_m(k) \\ t \sin B_m(1) & 2t \sin B_m(2) & \dots & kt \sin B_m(k) \end{bmatrix} \begin{bmatrix} -t \cos B_m(1) & t \sin B_m(1) \\ -2t \cos B_m(2) & 2t \sin B_m(2) \\ \vdots & \vdots \\ -kt \cos B_m(k) & kt \sin B_m(k) \end{bmatrix} \tag{26}$$

$$= \begin{bmatrix} \sum_{i=1}^k i^2 t^2 \cos^2 B_m(i) & \sum_{i=1}^k -i^2 t^2 \cos B_m(i) \sin B_m(i) \\ \sum_{i=1}^k -i^2 t^2 \cos B_m(i) \sin B_m(i) & \sum_{i=1}^k i^2 t^2 \sin^2 B_m(i) \end{bmatrix} \tag{27}$$

$$(A^T A) = \begin{bmatrix} \sum_{i=1}^k \frac{i^2 t^2 \cos^2 B_m(i)}{\sigma^2(i)} & \sum_{i=1}^k \frac{-i^2 t^2 \cos B_m(i) \sin B_m(i)}{\sigma^2(i)} \\ \sum_{i=1}^k \frac{-i^2 t^2 \cos B_m(i) \sin B_m(i)}{\sigma^2(i)} & \sum_{i=1}^k \frac{i^2 t^2 \sin^2 B_m(i)}{\sigma^2(i)} \end{bmatrix} \tag{28}$$

$$A^T D = \begin{bmatrix} -t \cos B_m(1) & -2t \cos B_m(2) & \dots & -kt \cos B_m(k) \\ t \sin B_m(1) & 2t \sin B_m(2) & \dots & kt \sin B_m(k) \end{bmatrix} \begin{bmatrix} \cos B_m(1) & -\sin B_m(k) \\ \cos B_m(2) & -\sin B_m(k) \\ \vdots & \vdots \\ \cos B_m(k) & -\sin B_m(k) \end{bmatrix} \tag{29}$$

$$= \begin{bmatrix} \sum_{i=1}^k -i t \cos^2 B_m(i) & \sum_{i=1}^k i t \cos B_m(i) \sin B_m(i) \\ \sum_{i=1}^k i t \cos B_m(i) \sin B_m(i) & \sum_{i=1}^k -i t \sin^2 B_m(i) \end{bmatrix} \tag{30}$$

$$A^T D = \begin{bmatrix} \sum_{i=1}^k \frac{-i t \cos^2 B_m(i)}{\sigma^2(i)} & \sum_{i=1}^k \frac{i t \cos B_m(i) \sin B_m(i)}{\sigma^2(i)} \\ \sum_{i=1}^k \frac{i t \cos B_m(i) \sin B_m(i)}{\sigma^2(i)} & \sum_{i=1}^k \frac{-i t \sin^2 B_m(i)}{\sigma^2(i)} \end{bmatrix} \tag{31}$$

In batch mode, computational effort is more due numerous samples. The following sub section describes the estimation problem in sequential mode.

### A. SEQUENTIAL PROCESSING:

After the arrival of the first bearing measurement

$$\begin{aligned}
 \text{ksums}[1] &= \frac{\cos^2 B_m(1)}{\sigma^2(1)} \\
 \text{ksums}[1] &= \frac{\cos^2 B_m(k)}{\sigma^2(k)} + \text{previous ksums}[1] \\
 \text{ksums}[2] &= \frac{-\cos B_m(k) \sin B_m(k)}{\sigma^2(k)} + \text{previous ksums}[2] \\
 \text{ksums}[3] &= \frac{\sin^2 B_m(k)}{\sigma^2(k)} + \text{previous ksums}[3] \\
 \text{ksums}[4] &= \frac{-kt \cdot \cos B_m(k) \sin B_m(k)}{\sigma^2(k)} + \text{previous ksums}[4] \\
 \text{ksums}[5] &= \frac{kt \cdot \cos^2 B_m(k)}{\sigma^2(k)} + \text{previous ksums}[5] \\
 \text{ksums}[6] &= \frac{kt \cdot \sin^2 B_m(k)}{\sigma^2(k)} + \text{previous ksums}[6] \\
 \text{ksums}[7] &= \frac{k^2 t^2 \cdot \cos^2 B_m(k)}{\sigma^2(k)} + \text{previous ksums}[7] \\
 \text{ksums}[8] &= \frac{-k^2 t^2 \cdot \cos B_m(k) \sin B_m(k)}{\sigma^2(k)} + \text{previous ksums}[8]; \\
 \text{ksums}[9] &= \frac{k^2 t^2 \cdot \cos^2 B_m(k)}{\sigma^2(k)} + \text{previous ksums}[9]
 \end{aligned} \tag{32}$$

$$A^T A = \begin{bmatrix} \text{ksums}[7] & \text{ksums}[8] \\ \text{ksums}[8] & \text{ksums}[9] \end{bmatrix} \tag{33}$$

$$A^T D = \begin{bmatrix} \text{ksums}[4] & \text{ksums}[5] \\ \text{ksums}[5] & \text{ksums}[6] \end{bmatrix} \tag{34}$$

In this way the target course can be found out easily after 2<sup>nd</sup> measurement onwards.

### B.Target Motion parameters when initial range is known

Using sophisticated techniques , prior knowledge about range is known . The procedure to obtain the remaining target motion parameters is given here.

$$X_s(1) = \begin{bmatrix} \dot{x}_t(1) \\ \dot{y}_t(1) \\ R_x(1) \\ R_y(1) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ t & 0 & 1 & 0 \\ 0 & t & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{x}_t(0) \\ \dot{y}_t(0) \\ R_x(0) \\ R_y(0) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -(x_0(1) - x_0(0)) \\ -(y_0(1) - y_0(0)) \end{bmatrix} = \Phi(1,0) \cdot X_s(0) + b(1) \tag{35}$$

$$b(1) = [0 \quad 0 \quad -(x_0(1) - x_0(0)) \quad -(y_0(1) - y_0(0))]$$

Similarly for  $X_s(2), X_s(3)$  and so on. The eqn. (14) is transformed as

$$\begin{bmatrix} t.\cos B_m(1) & -t.\sin B_m(1) & \cos B_m(1) & -\sin B_m(1) \\ 2.t.\cos B_m(2) & -2.t.\sin B_m(2) & \cos B_m(2) & -\sin B_m(2) \\ \vdots & \vdots & \vdots & \vdots \\ k.t.\cos B_m(k) & -k.t.\sin B_m(k) & \cos B_m(k) & -\sin B_m(k) \end{bmatrix} \begin{bmatrix} \dot{x}_i \\ \dot{y}_i \\ R_x(0) \\ R_y(0) \end{bmatrix} = \begin{bmatrix} \cos B_m(1)(x_0(1)-x_0(0))-\sin B_m(1)(y_0(1)-y_0(0)) \\ \cos B_m(2)(x_0(2)-x_0(0))-\sin B_m(2)(y_0(2)-y_0(0)) \\ \vdots \\ \cos B_m(k)(x_0(k)-x_0(0))-\sin B_m(k)(y_0(k)-y_0(0)) \end{bmatrix} + N(k)=0 \tag{36}$$

$$\begin{bmatrix} t.\cos B_m(1) & -t.\sin B_m(1) \\ 2.t.\cos B_m(2) & -2.t.\sin B_m(2) \\ \vdots & \vdots \\ k.t.\cos B_m(k) & -k.t.\sin B_m(k) \end{bmatrix} \begin{bmatrix} \dot{x}_i \\ \dot{y}_i \end{bmatrix} + \begin{bmatrix} \cos B_m(1) & -\sin B_m(1) \\ \cos B_m(2) & -\sin B_m(2) \\ \vdots & \vdots \\ \cos B_m(k) & -\sin B_m(k) \end{bmatrix} \begin{bmatrix} R_x(0) \\ R_y(0) \end{bmatrix} = \begin{bmatrix} \cos B_m(1)(x_0(1)-x_0(0))-\sin B_m(1)(y_0(1)-y_0(0)) \\ \cos B_m(2)(x_0(2)-x_0(0))-\sin B_m(2)(y_0(2)-y_0(0)) \\ \vdots \\ \cos B_m(k)(x_0(k)-x_0(0))-\sin B_m(k)(y_0(k)-y_0(0)) \end{bmatrix} + N(k)=0 \tag{37}$$

$$A^T X_v + DX_p(0) - C + N(k) = 0 \tag{38}$$

$$C = \begin{bmatrix} \cos B_m(1).(x_0(1) - x_0(0)) - \sin B_m(1).(y_0(1) - y_0(0)) \\ \cos B_m(2).(x_0(2) - x_0(0)) - \sin B_m(2).(y_0(2) - y_0(0)) \\ \vdots \\ \cos B_m(k).(x_0(k) - x_0(0)) - \sin B_m(k).(y_0(k) - y_0(0)) \end{bmatrix} \tag{39}$$

The least square solution is given by

$$X_v = (A^T . A)^{-1} . A^T . (D . X_p - C) \tag{40}$$

$A^T A$  is already available, as in eqn.(28). Now let us calculate  $A^T (B.X_p - C)$

$$A^T . DX_p = \begin{bmatrix} \sum -i.t \cos^2 B_m(i) & \sum i.t \sin B_m(i). \cos B_m(i) \\ \sum i.t \sin B_m(i). \cos B_m(i) & \sum -i.t \sin^2 B_m(i) \end{bmatrix} \begin{bmatrix} R_x(0) \\ R_y(0) \end{bmatrix} \tag{41}$$

$$= \begin{bmatrix} \sum -i.t \cos^2 B_m(i).R_x(0) + \sum i.t. \sin B_m(i). \cos B_m(i).R_x(0) \\ \sum i.t \cos B_m(i). \sin B_m(i).R_x(0) + \sum -i.t \sin^2 B_m(i).R_y(0) \end{bmatrix}$$

$$A^T . D . X_p - A^T . C = \begin{bmatrix} \sum -i.t \cos^2 B_m(i).R_x(0) + \sum i.t. \sin B_m(i). \cos B_m(i).R_x(0) + \sum i.t. \cos B_m(i) [\text{term}(i)] \\ \sum i.t \cos B_m(i). \sin B_m(i).R_x(0) + \sum -i.t \sin^2 B_m(i).R_y(0) - \sum i.t. \sin B_m(i) [\text{term}(i)] \end{bmatrix} \tag{42}$$

$$A^T . DX_p - A^T . C = \begin{bmatrix} \sum -i.t \cos^2 B_m(i).R_x(0) + \sum i.t. \sin B_m(i). \cos B_m(i).R_x(0) + \sum i.t. \cos B_m(i) [\text{term}(i)] \\ \sum i.t \cos B_m(i). \sin B_m(i).R_x(0) + \sum -i.t \sin^2 B_m(i).R_y(0) - \sum i.t. \sin B_m(i) [\text{term}(i)] \end{bmatrix} \tag{43}$$

where  $\text{term}(i) = [\cos B_m(i).(x_0(i) - x_0(0)) - \sin B_m(i).(y_0(i) - y_0(0))]$  (44)

$$\begin{bmatrix} \dot{x} \\ R_{y0} \end{bmatrix} = (A^T A)^{-1} \begin{bmatrix} \sum -i.t.\cos^2 B_m(i).\tan(B_0) + \sum i.t.\sin B_m(i).\cos B_m(i) + \left( \frac{\sum i.t.\cos B_m(i).term(i)}{R_y(0)} \right) \\ \sum i.t.\sin B_m(i).\cos B_m(i).\tan(B_0) + \sum -i.t.\sin^2 B_m(i) - \left( \frac{\sum i.t.\sin B_m(i).term(i)}{R_y(0)} \right) \end{bmatrix} \quad (45)$$

As we have done earlier, let us convert eqn. (45) for sequential processing.

$$\begin{aligned} ksums4\_mod &= -k.t.\cos^2 B_m(k)\tan(B_0) + previous7\_mod \\ ksums5\_mod &= k.t.\cos B_m(k).\sin B_m(k) + previous5\_mod \\ ksums6\_mod &= -k.t.\sin^2 B_m(k) + previous6\_mod \\ ksums5\_mod &= k.t.\cos B_m(k).\sin B_m(k)\tan(B_0) + previous5\_mod \\ term(k) &= \cos B_m(k)[x_0(k) - x_0(0)] - \sin B_m(k)[y_0(k) - y_0(0)] \\ lsums1 &= k.t.\cos B_m(k) * term(k) + previous lsums[1] \\ lsums2 &= -k.t.\sin B_m(k) * term(k) + previous lsums[2] \\ lsums1\_mod &= lsums1/R_y(0) \\ lsums2\_mod &= lsums2/R_y(0) \end{aligned} \quad (46)$$

$$R_y(0) = \text{initial range} * \cos B_m(0)$$

Using the above ,

$$A^T DX_p - A^T C = \begin{bmatrix} ksums4\_mod + ksums5 + Lsums1\_mod \\ ksums5\_mod + ksums6 + Lsums2\_mod \end{bmatrix} \quad (47)$$

and

$$\begin{aligned} (A^T A)^{-1} * (A^T DX_p - A^T C) &= \begin{bmatrix} ksums7 & ksums8 \\ ksums8 & ksums9 \end{bmatrix}^{-1} \begin{bmatrix} ksums4\_mod + ksums5 + Lsums1\_mod \\ ksums5\_mod + ksums6 + Lsums2\_mod \end{bmatrix} \\ &= [Q] \end{aligned} \quad (48)$$

tcr is

$$tcr = \tan^{-1} \left( \frac{Q(1,1)}{Q(2,1)} \right) \quad (49)$$

vt is

$$vt = \sqrt{Q(1,1)^2 + Q(2,1)^2} * R_y(0) \quad (50)$$

### C.Target Motion parameters when target course/speed is known

Initially , calculate tcr for assumed ranges. The value of range estimate for which estimated tcr is numerically equal to actual course is the accurate range estimate . If course and range is known , then the speed of the target is found out easily. The procedure can also be implemented if target speed is available initially. This procedure is easier compared to range estimation using eqn. (45)

## III. SIMULATION & RESULTS

The TMP estimation without observer maneuver which is very complicated. The target course can be determined using non maneuvering observer. The simulation study is carried for various tactical geometries, out of which one is presented for reference. Initial range and bearing are 5000 m and 40 degrees and target speed is 30 knots while ownship speed is 20 knots and with 90 degree course.

The ownship is assumed to be moving at a speed of 20 knots (10.3 m/sec) (at zero speed while estimating the target course) and at 90 deg course. (All angles are considered with respect to Y axis). Number of scenarios is tested by changing the course of the target in steps of 1 degree in such a way that the angle between the target course and line of sight is always less than 55 degrees, as only closing targets are of interest to the observer. In

general, the error allowed in the estimated target motion parameters in underwater are eight percent in range estimate, 0.2 degrees in bearing estimate, five degrees in course estimate and three meters/sec. in velocity. For the purpose of analysis, this scenario with target course equal to 165 degrees is considered. The results of this scenario after several Monte Carlo runs are shown in figures 1. From the results, it is observed that course estimate with required accuracy is obtained from around 100 seconds onwards. For the same scenario, the target course is fed from 100 seconds onwards to estimate the range and speed. The results are shown in Fig 1 (b) and Fig. 1.(c).

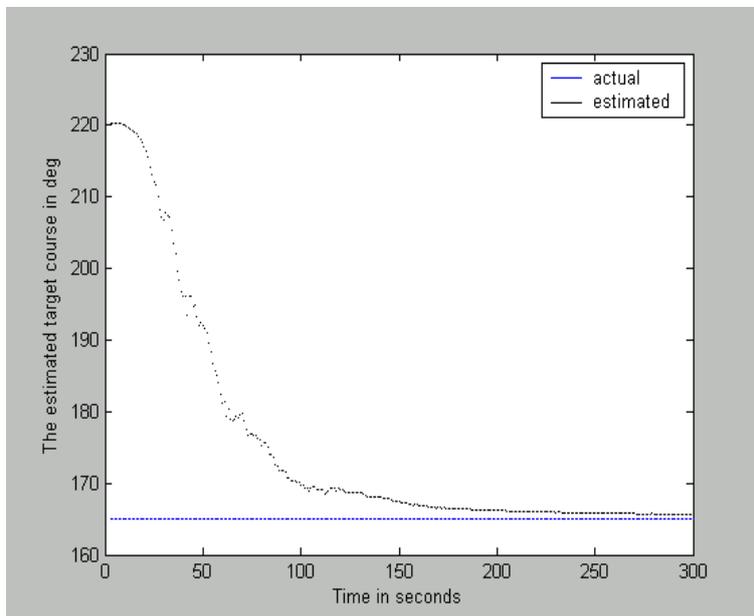


Fig 1. (a) Error in course estimate

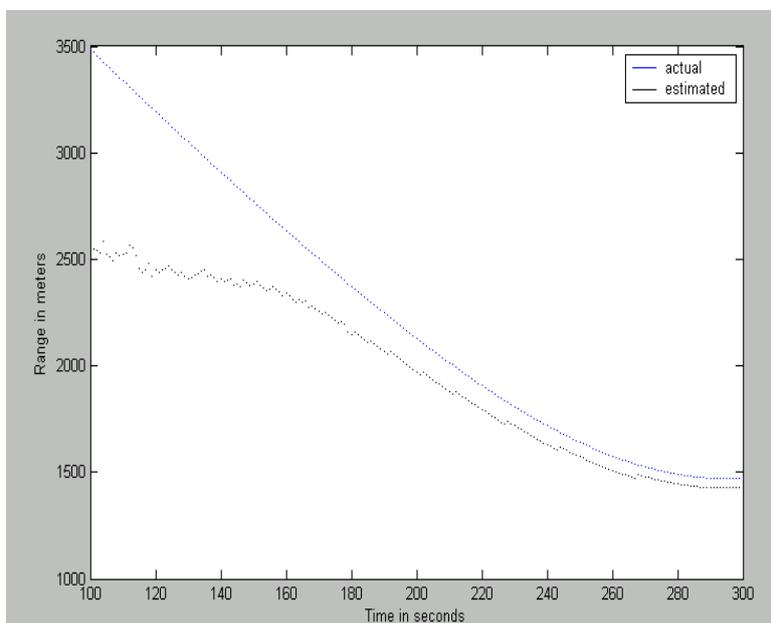


Fig 1. (b) Error in range estimate

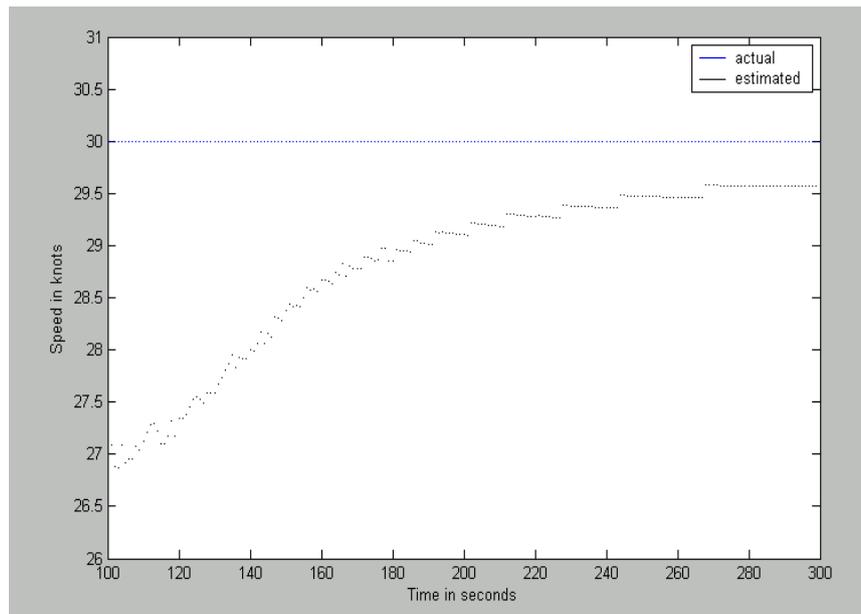


Fig 1. (c) Error in speed estimate

#### IV. SUMMARY & CONCLUSION

The paper discusses the sequential mode of PLE estimation algorithm against the batch type. The incremental values of the measurements are only computed at every time and updated to the variables, which are taken here as SUMS. The observer is to be in non-maneuvering state while course estimation. If one of the TMP is known by any means, then the rest of them can be determined with prior knowledge about this single parameter. The results inferred the sequential PLE is suitable for target tracking in undersea applications.

#### REFERENCES:

- [1] Jaffret, C., and Bar-Shalom, Y. (1990), "Track formation with bearing and frequency measurements," IEEE Transactions on Aerospace and Electronic Systems, 26, (Nov. 1990), 999-1009.
- [2] Y.T.Chan and S.W.Rudnicki (1992), "Bearings-Only and Doppler-Bearing Tracking using Instrumental Variables," IEEE Transactions on Aerospace and Electronic Systems, 28, (Oct. 1992), 1076-1082.
- [3] Xiuo-Jiao Tao Cai-Rong Zou and Zhen-Ya He (1996), "Passive Target Tracking using Maximum Likelihood Estimation", IEEE Transactions on Aerospace and Electronic Systems, 32, (Oct. 1996), 1348-1353.
- [4] T.L. Song and J.L. Speyer (1985), "A Stochastic Analysis of a modified gain extended kalman filter with applications to estimation with bearing only measurements," IEEE Transactions on Automatic Control, AC-30, No. 10, (Oct. 1985), 940-949.
- [5] Y.T.Chan and T.A.Rea, "Passive Tracking Scheme for a single stationary Observer", IEEE Trans. Aerosp. Electron.Syst. vol. 38, No.3, July 2002, 1046-1053.
- [6] A.G.Lindgren, K.F.Gong, "Position and Velocity Estimation via Bearing Observations", IEEE Trans. Aerosp. Electron.Syst. vol. AES-14, July 1978, PP 564-577.
- [7] A.G.Lindgren, S.C. Nardone, "Biased Estimation Properties of the pseudo linear estimator ", IEEE Trans. Aerosp. Electron.Syst. vol. AES-18, No.4, July 1982, PP 432-441.
- [8] S.C Nardone, A.G. Lindgren and K.F.Gong, "Fundamental properties and performance of conventional bearings only target motion analysis ", IEEE Trans. Automatic Control Vol. Ac -29, No.9, September 1984, PP 775-787.
- [9] S. Koteswara Rao, "Pseudo-linear estimator for bearings-only passive target tracking", IEE Proc., Radar, Sonar, Navigation, Vol.148, Feb '01, pp16-22.