

# Calculation of Noisy Level for CAD using Markov Chain Process

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**Abstract - This paper is a novel methodology for indentifying blocks in the heart using CMR (Cardiac Magnetic Resonance) for Coronary Artery Disease(Cardiomypopathy). This is a type of Image De-noising, taking image from video-graphic way from CMR and by applying fast Fourier transform(FFT) resultant value is compared Markov Model from this a physician can identify whether a person is in need of CABG (Coronary Artery Bye Pass Grafting). This novel method makes new technology era to go forward from previous one.**

**Key Words:** Cardiomypopathy, Cardiac Magnetic Resonance, Coronary Artery Bye Pass Grafting(CABG), FFT, Markov Model.

## 1. INTRODUCTION

Coronary Artery Disease (or) cardiomypopathy is one of the main issues in all over the world. Still there is awareness but many people lost their life due to this disease, this is because not maintaining their health properly. After getting Heart attack peoples know about Cardiomypopathy. Indians are affected by this cardiac disease. It is a disease about the heart muscle. The heart was disturbed and unable to pump blood; heart rhythm is disturbed which leads to irregular heart beat called as arrhythmias. The exact cause of the muscle damage is yet to be found. It affects middle-aged and older persons. Different types of this disease made into two categories a. Ischemic b. Non-Ischemic Ischemic refers to heart muscle damage and Non-Ischemic refers to Dilated, Hypertrophic and Restrictive.

## 2. Objectives of the Study

- To determine the percentage of block in the heart
- To determine flow of blood
- To determine need of CABG or not.

## 3. Literature Review

<sup>[1]</sup> Jole Shackelford discussed about Sir William Harvey famous and a Royal Physician made his work on the operation of the heart and moment of blood into and out of the heart. Harveys discoveries was the heart forcefully contracts and presses blood into the arteries in the phase called "SYSTOLE". When the contraction of two small chambers that sit atop the ventricles called as "DIASTOLE" which causes the ventricles to fill with blood and expand. <sup>[4]</sup> According to VU Reddy Fast Fourier Transform is an efficient way for computing DFT. It is a method of Spectrum estimation FFT is used to change time domain signals into Frequency domain <sup>[2]</sup> Suyash P.Awate et al discussed about the re-construction problem in an Inverse Fourier Transform <sup>[5]</sup> Matthias W.Seeger et al in his work of Bayesian Experimental Design of Magnetic Resonance Imaging Sequence used the Bayesian inference of MR images and re-construction can be through FFT. <sup>[3]</sup> Thomas sangild Sorensen et al used non equi-spaced Fast Fourier Transform used in the medical imaging. <sup>[6]</sup> Seth J. Worley et al in his work by comparing Time Domain and Frequency Domain calculated through Single Averaged Electro-Cardiogram to determine whether signal processing beyond signal averaging improves identification of patients with ventricular tachycardia. To compare time domain analysis using methods similar to those of frequency domain analysis of the signal averaged ECG in the same group of patients to determine using multivariable analysis the best signal averaged ECG variable for identifying patients with ventricular tachycardia. According to Dale L.Bailey *et al* <sup>[10]</sup>, Albert C.Lardo *et al* <sup>[11]</sup> in their work namely Positron Emission Tomography worked about sinogram and laminogram which is used in the Radon Transform.

## 4. Methodology

ECG is one of the most common tests for the heart problems to identify. It is simple and often very valuable. Fig 1 shows the basic representation of ECG. It contains PQRST Segments and tells the presence of Ischemia, and ECG cannot confirm the exact anatomical distribution or the severity of damage in the coronary artery involved.



Fig 1: Basic Representation of ECG

Table : 1 Represents PQRST Segments.

P WAVE	Atrial Depolarization
QRS WAVE	Ventricular Depolarization
T WAVE	Repolarization of the Ventricles

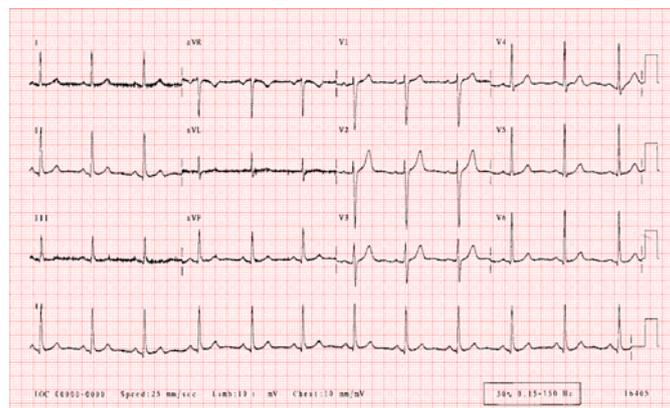


Fig 2 : Normal ECG

Rate of Rhythm is calculated , The number of R waves on the strip and it is multiplied by 6 which equals 72 beat/min .Planar ST Elevation can be represented as ischemic disease .In the above Normal ECG strip in fig 2 all the P Wave an QRS wave and ST segment, R waves are Normal. If any of the waves are elevated or depressed hearty rhythm would be disturbed. ECG is an integral part of the diagnostic work. Dynamic Changes in the waveform shows about the disease. Proposed work ECG Waveform calculated through FFT which is extremely useful for analyzing un-steady measurements. According to Thomas Sangild<sup>[3]</sup> and Reddy<sup>[4]</sup> complete Fourier Series Consists of  $F(x) = a_0 + \sum a_n \cos(n\omega_0 t) + \sum b_n \sin(n\omega_0 t) \dots (1)$ . Any Patients having irregular waves represented in fig 3 in the ECG have to go for one more process i.e. CARDIAC MAGNETIC RESONANCE.



Fig 3: Ir-regular Waves in the ECG.

Here we are presenting a data for a single patient and how the process is done. Taking single patient data, and going to calculate the prognosis for proceed to further step or not. A P wave represented in the Table 1 is used for depolarization in the arteries and it should consist of 2.5mm and PR interval is in between 0.12 and 0.2 sec. If there is a long PR interval first degree heart block is present. If the PR interval widens every beat and then a QRS complex is missing there is 2<sup>nd</sup> degree heart block. QRS wave is used for ventricles for polarizing and de-

polarizing. T wave is used to do the action of polarize and depolarize is said to Repolarization of ventricles. If there is no discernable relationship between P waves QRS complexes then 3<sup>rd</sup> degree heart block is present. QRS complex looks like 'w' 'v' and 'M' (represented in fig 1) in v6 segment there is left bundle branch block.

Table : 2 Values of P,QRS,RS,ST segments

Lead type	P-Segment (mm)	QRS – Segment (mm)	RS – Segment (mm)	ST-Segment (mm)
I,III,III	0.52	0.10	0.10	0.2
VR,VL,VF	5.4	-3.0	4.4	1.4
V1,V2,V3	4.5	2.3	2.4	2.2
V4,V5,V6	3.3	3.5	3.3	2.50

Unbormal ECG report of a patient Fig(3) taken and it is calculated,. According to Seth Worley<sup>[6]</sup> et al they calculated frequency and time domain using the signals of ElectroCardiogram. In this work we are calculating the frequency and magnitude and complex values using fourier domains data shown in table (3) and a graph is generated to find that the pvalue (fig 4).

Table :3 fft-freq,fft-mag,fft-complex for p waves

Time	Data	FFT-freq	FFT-mag	FFT-COMPLEX
0.4	0.52	0	0.02	13.72
0.8	5.4	1.25	0.04	-3.98
0.12	4.5	2.5	0.06	-3.68
0.16	3.3	3.75	0.08	-3.98

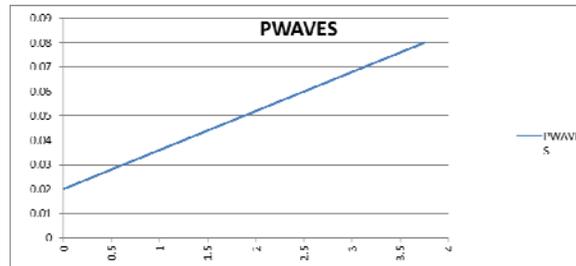


Fig 4: p wave graph

QRS Segment was calculated using fig (3) with fast Fouier transform. Frequency, magnituded with complex values were calculated show in table (4). From fig (5) there is a long interval in betweenQRS Segment and it is compared with fig(2) of normal ECG report. It infers that the flow of blood is slow due to block in the left atrium. Fig(10) shows the inflow of flood that is slow due to block.

Table 4: fft-freq,fft-mag,fft-complex for QRS segment

Time	Data	FFT-freq	FFT-mag	FFT-COMPLEX
0.4	0.1	0	0.725	2.9
0.8	-3	1.25	1.715554	-2.2+6.5i
0.12	2.3	2.5	0.475	1.9
0.16	3.5	3.75	1.715554	-2.2-6.5i

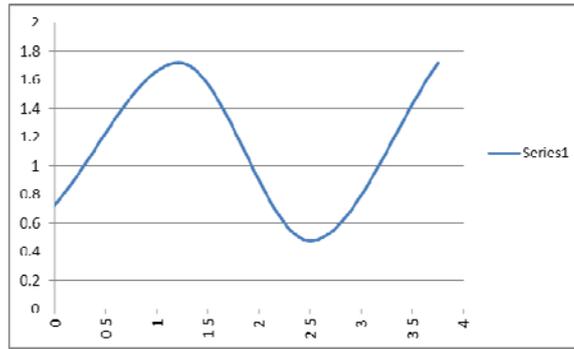


Fig 5: QRS Segment.

RS Segment was taken from the fig(3) it is calculated resultant value shown in the table(5) The RS Segment in graph fig(6) infers that long interval between RS Segment this confers the CAD

Table 5: FFT-freq,fft-mag,fft-complex for RS segment

Time	Data	FFT-freq	FFT-mag	FFT-COMPLEX
0.4	0.1	0	2.55	10.2
0.8	4.4	1.25	0.637377	-2.3-1.1i
0.12	2.4	2.5	1.3	-5.2
0.16	3.3	3.75	0.637377	-2.3+1.1i

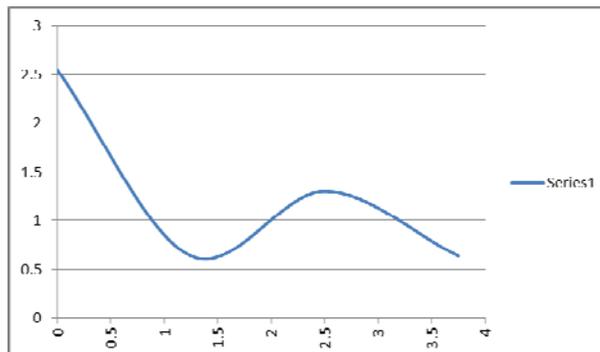


Fig 6: RS segment.

ST Segment calculated with fig(3) values shown in the table 6 and the graph in fig(7) shows a normal blood flow in the right atrium.

Table 6: fft-freq,fft-mag,fft-complex for ST segment

Time	Data	FFT-freq	FFT-mag	FFT-COMPLEX
0.4	0.2	0	1.575	6.3
8	1.4	1.25	0.570636	-2+1.1i
0.12	2.2	2.5	0.375	-1.5
0.16	2.5	3.75	0.570636	-2-1.1i

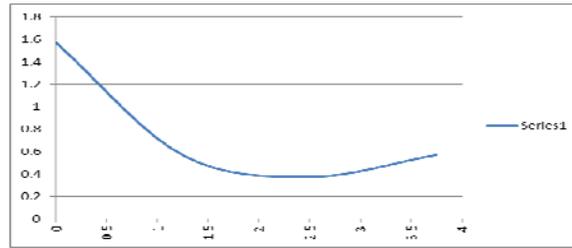


Fig 7: ST segment.

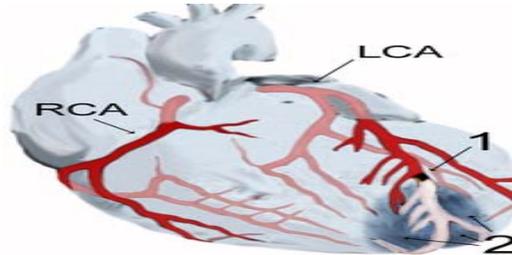


Fig 8: Diagrammatic representation of myocardial infarction

In the above fig(8) coronary diseased heart severe blockage had occurred in the Right Coronary Artery and Left Coronary Artery. After thorough clinical testing of ECG a physician can confirm that a person is suffering from coronary artery disease. For percentage of block in the heart a physician has to decide for cardiac (MRI).

### 5. Cardiac Magnetic Resonance Imaging

According to Mansfield &A.A.Maudsley<sup>[7]</sup>,Tel Geva<sup>[8]</sup>,Vadim Kupermann<sup>[9]</sup> Magnetic Resonance Imaging(MRI) is a diagnostic tool for coronary artery disease, It is used to monitor the disease by: Evaluating the function of the heart chambers, valves, size and blood flow through major vessels, and surrounding structures such as the pericardium. Diagnosing a variety of cardiovascular disorders. Evaluating the effects of coronary artery disease such as limited blood flow to the heart muscle and scarring within the heart muscle after a heart attack. 1.Planning a patient treatment for cardiovascular disorders. 2.Monitoring the progression of certain disorders over time.3.Evaluating the anatomy of the heart and blood vessels for a person suffering in this disease. 4.Evaluating the effects of surgical changes, especially in patients. A 0.15 tesla, resistive magnet was used for magnetic resonance imaging. The resonance frequency for hydrogen is 6.25 MHz with this device. An electrocardiographic signal was obtained from standard surface electrodes placed distally on both arms and on the right lower leg. Individual tomographic sections were performed in conjunction with an electrocardiographic telemetry gating device.

The position within the magnet was then adjusted by multiples of 0.5. cm in the longitudinal axis until separate transverse images. Through the aortic valve and at the tip of the papillary muscles were obtained. Without moving the patient, the transverse image at the level of the papillary muscle to was repeated, but gated to systole rather than diastole. Data acquisition for each transverse image required 0.5 to 0.11 minutes, depending on the heart rate below range values are used by P.Mansfield&A.A.Maudsley<sup>[7]</sup>.

Table 8 Range of Left and Right Ventricles using NMR

Range	NMR
Aorta(Cm)	3.1±0.4
Left Atrium	2.9±0.4
Right Ventricle (Diastole)(Cm)	4.2±0.6
Left Ventricle(Systole)(Cm)	3.0±0.6
Shortening Fraction	28±1.0
Septal Thickness	1.0±0.1
Posterior Wall Thickness	1.1±0.1

The Measurement conventions for magnetic resonance imaging were Left atrial and aortic magnetic resonance measurements were made from the transverse section through the aortic valve. Measurements of both the aorta and left atrium were from anterior to posterior wall. The left and right ventricular cavity and left ventricular wall thickness measurements were made. The left ventricular measurements were made along a line drawn

perpendicular to the septum and posterolateral left ventricular wall and positioned to bisect the left ventricular cavity. Shortening fraction was calculated as the End Diastolic Dimension minus End Systolic Dimension divided by the End Diastolic Dimension

$$SF = EDD - (ESD/EDD) \tag{2}$$

**5.1 Radon Transform**

From P.Mansfield&A.A.Maudsley<sup>[7]</sup> the foundation of analytical reconstruction is the Radon Transform, which relates a 2D function f(x,y) the collection of line integrals of that function.

Definition: Let L(r,ψ) denote the line in the Euclidean plane at angle ψ counter clockwise from the y axis and at a signed distance r from the origin

$$L(r, \psi) = \{(x,y) \in R^2 : x \cos \psi + y \sin \psi = r\} \\ = \{r \cos \psi - l \sin \psi, r \sin \psi + l \cos \psi : l \in R\} \tag{2.1}$$

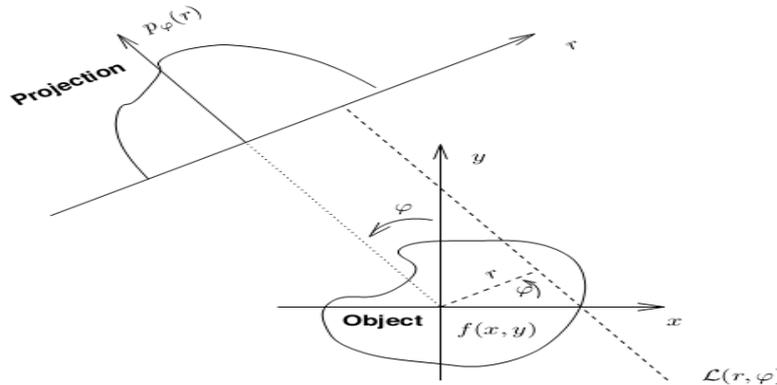


Fig 9 Geometry with line integral using Radon Transform

Taking the eqn(2.1) in the matrix form

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \cos \psi & -\sin \psi \\ \sin \psi & \cos \psi \end{pmatrix} \begin{pmatrix} r \\ l \end{pmatrix} \tag{3}$$



Fig 10 shows about the blood inflow through artery using radon transform for identification of noisy level.

**5.2 Sinogram**

According to Dale L.Bailey et al <sup>[10]</sup>,Albert C.Lardo et al<sup>[11]</sup> Sinogram has two co-ordinates (r and ψ) to reduce noise many linear and non-linear filters have applied to it. To identify the noisy level of an image we are using Jacobian Matrix. Jacobian is used to transform an image i.e. from (x',y) = f(x,y) now the image is converted into an jacobian matrix of 2D images. Using the above equation(1) to construct J<sub>F</sub>(r,θ,ψ)

$$J_F(r, \theta, \psi) = \begin{bmatrix} \cos \psi \sin \theta & \cos \psi \cos \theta & -\sin \psi \sin \theta \\ \sin \psi \sin \theta & \sin \psi \cos \theta & \cos \psi \sin \theta \\ \cos \theta \sin \psi & \sin \theta \cos \psi & 0 \end{bmatrix} \tag{4}$$

$f(x',y') = f(x,y)$   $f(x',y') = u^2+v^2$  (taking  $u = x$  and  $v = y$ ) taking the matrix of eqn(4) consider matrix 0's are said to noisy and 1's are not noisy creating a 3 \* 3 matrix  $\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$  taking  $1 = x$  and  $0=y$  and  $J_f = [2u,2v]$  therefore  $J_g$

.  $J_f =$   
 $J_g = \frac{\partial u}{\partial x} * \frac{\partial u}{\partial y} = \begin{bmatrix} (2x+1)(2+y) \\ (2x+1)(2+y) \\ (2+x)(2y+1) \end{bmatrix} * [2x, 2y] = 4.$  (5)

The equation (5) shows that the four ventricles are in the noisy level. The above matrix indicates that the image in its original is transformed into noisy level.

**5.3 Laminogram**

The Radon transform maps a 2D object  $f(x,y)$  into a sinogram  $p_\psi(r)$  consisting of line integrals through the object. To recover the object from  $p_\psi(r)$  would be to take each sonogram value and smear it back into object space along the corresponding ray. This type of operation is called backprojection it is the basic for the tomographic image reconstruction. To Recover the object  $f(x,y)$  instead yields a blurred version of the project is called laminogram represented by  $f_b(x,y)$

**5.4 Back Project-Filter Method**

The noisy level between the laminogram and the original object is back-project filter (BPF) method 2d Fourier Transform Yields

$F(U,V) = \sqrt{U^2+ V^2} / W(U,V) * F_B(U,V)$  (6)

**Identification of Noisy Level**

**Step 1:** Choose a non zero angular weighting function  $w(\psi)$ . Perform angularly weighted backprojection of the sinogram  $p_\psi(r)$  to form a laminogram  $f_b(x,y)$

**Step 2:** Take 2D FT of  $f_b(x,y)$  to get  $F_b(u,v)$

**Step 3:** Apply the angularly modulated cone filter to the Fourier Domain

**Step 4:** Take Inverse 2D FT of  $F(u,v)$  to get  $f(x,y) + e$  where  $e$  is noisy level

**Step 5:** Construct noisy level ( $e$ ) to a transient matrix and calculate the probability for block or severely damaged block in the arteries.

**6. Stochastic Process**

According to Sheldon Ross<sup>[13]</sup> A stochastic process  $X = \{ x(t),t \in T\}$  is a collection of random variables. For each  $t$  in the index set  $T$ ,  $X(t)$  is a random variable. We interpret  $t$  as time and call  $X(t)$  the state of the process at time  $t$ . Consider a Stochastic process  $\{X_n, n=0,1,2,\dots\}$  that takes on a finite or countable number of possible values. If  $X_n = i$ , then the process is said to be in state  $i$  at time  $n$ . Whenever the process in the state  $i$  then there is a fixed probability  $P_{ij}$  if it is in the next state  $j$ .if the probability in the next state is also  $j$  then it is Markov-Chain. Markov chain concerns about a sequence of random variables, which correspond to the states of a certain system, in a such a way that the state at one time depends only on the in the previous time. Consider now a finite state of markov chain and suppose that the states are numbered so that  $T = \{1,2,\dots,t\}$  denotes the set of transient states. Let

$Q = \begin{bmatrix} p_{11} & p_{12} & p_{1t} \\ p_{i1} & p_{i2} & p_{it} \\ p_{t1} & p_{t2} & p_{tt} \end{bmatrix}$

$Q$  specifies only the transition probabilities from transient states into transient states. For transient states  $I$  and  $j$ . let  $m_{ij}$  denote the expected total number of time periods spent in state in  $j$  given that the chain starts in state  $i$ .

Initial Transition yields  $m_{ij} = \delta(I,J) + \sum P_{IK}M_{KJ}$  (7)

$M_{KJ} = 0$  where  $k$  is a recurrent state.

Let  $M$  denote the matrix of values  $m_{ij}$  where  $I$  and  $j = 1,2,\dots,t$  let  $M = \begin{pmatrix} m_{11} & m_{12} & m_{1t} \\ m_{i1} & m_{i2} & m_{it} \\ m_{t1} & m_{t2} & m_{tt} \end{pmatrix}$

In matrix notation  $M = I + QM$  (8)

Where  $I$  is identity matrix of size  $t$

$(I - Q) M = I$  (9)

Multiplying both side by  $(I-Q)^{-1}$  then

$M = (I-Q)^{-1}$  (10)

For  $i \in T$  and  $j \in T$  the quantify  $f_{i,j}$  = equal to the probability of ever making a transition into state  $j$  given that the chain starts in state  $i$  easily determined from  $M$

$$M_{i,j} = E [\text{number of transition into state } j/\text{start in } i] \\ = m_{i,j} * f_{i,j} \tag{11}$$

Where  $m_{i,j}$  is the expected number of time periods spent in state  $j$ . We see that

$$f_{i,j} = m_{i,j} / m_{j,j} \tag{12}$$

**6.1 Problem**

Let  $x^{(n)}$  be the heart of  $n$  arteries which can be defined as  $M = \{ \text{block, non block} \}$   $x^{(0)} = \text{non block}$   $x^{(1)} = \text{block}$  with probability =0.4 and  $n=3$  Starting from state 1 to find out the expected time spent in each state. Expected number of visits to each state and the probability of ever visiting state. Before calculating markov process we have to find the joint density function. Taking Two random variables  $X,Y...$  that are defined on a probability space, the joint density probability distribution for  $X,Y...$  is a probability distribution that gives the probability that each of  $X,Y...$  falls in any particular range (or) discrete set of values specified for that variable.

By taking a first row of values from the transition probability matrix and dividing into two groups then by making integration we get result which indicates the septal thickness of wall

$$P = [0.2, 0.5, 0.3, 0.0, 0.4, 0.0] \\ \iint (10x + 4y) dy dx = 1.4 \text{ ---- (13)}$$

Eqn(13) shows that the valves are septal thickness and the density of blood inside arteries is slow. After Examining the heart through MRI . We are constructing a transition matrix, arteries which are having block are taken into account Markov chain process is taken by taking three arteries  $\{0,1,2\}$  with the transition probability matrix

$$P = \begin{bmatrix} 0.2 & 0.5 & 0.3 & 0.0 & 0.4 & 0.0 \\ 0.3 & 0.1 & 0.3 & 0.6 & 0.0 & 0.4 \\ 0.5 & 0.4 & 0.4 & 0.0 & 0.6 & 0.0 \end{bmatrix}$$

$P =$  three types of arteries having a block to be replaced using this markov process . Inverting  $(I - Q)$  gives that  $M = (I - Q)^{-1} =$

$$\begin{bmatrix} 1.5865 & 0.9774 & 0.5714 \\ 1.4662 & 2.4436 & 1.4286 \\ 1.2857 & 2.1429 & 2.7143 \end{bmatrix}$$

$$M_{3,3} = 2.7143 \quad M_{3,2} = 2.1429$$

$$F_{3,4} = M_{3,4} / M_{4,4} = 1.4286 / 2.4436 = 0.5846$$

$$F_{3,4} = 1 - (0.6/0.4)^3 / (1 - (0.6/0.4)^4) = 38/65 = 0.5846$$

From the result we infer that the heart is septal thickness arteries.

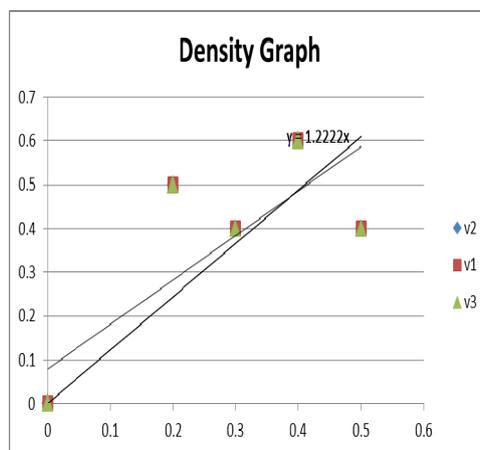


Fig 11 Joint Probability Density Function Graph.

The above graph fig 11 indicates the blood flow of an artery. The two values that had taken as sample intercepts at a point with a value  $y=1.2222$  which represents that artery is thickened.

## 7. Conclusion

we emphasize the specificity of electro magnetic resonance anatomic correlations in patients. Human Sedation is reduced and angiogram is mostly avoided. Hence patients with severe CAD DISEASE can adopt this method.

## 8. Future Work

In Future we can implement it to the cardiology and avoid Angiogram using CRI and Stochastic process results a Cardiologist, that he/she can decide for further process.

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