Multi-response Optimization of Spring Parameters of Two Wheeler Rear Suspension System Using Grey Relational Analysis in Taguchi Method

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Abstract—In this paper, a suitable approach for the optimization of spring parameters, namely spring wire diameter, spring index and number of active coils of two wheeler rear suspension systems with multiple responses such as strain energy and weight based on orthogonal arrays with grey relational analysis is used. A grey relational grade is obtained from the grey analysis. Based on the grey relational grade, optimum levels of parameters have been identified and significant contribution of parameters is determined by ANOVA. Confirmation test is conducted to validate the test result. Experimental results have shown that the responses of spring can be improved effectively through this novel approach.

Keyword- Rear suspension, Coil spring, Strain energy, Taguchi method, ANOVA, Grey relational analysis.

I. INTRODUCTION

The Suspension system is an assembly of springs, shock absorbers and linkages that connects a vehicle to its wheels [1]. The primary functions of a suspension system are to:

- Provide vertical compliance so the wheels can follow the uneven road, isolating the chassis from the roughness in the road.
- Maintain the wheels in the proper steer and camber attitudes to the road surface.
- React to the control forces produced by the tires.
- Keep the tires in contact with the road with minimal load variations [2].

A motorcycle's suspension system consists of a spring coupled to a viscous damping element, a piston, in a cylinder filled with oil. It serves a dual purpose: contributing to the vehicle's handling and braking, and providing safety and comfort by keeping the vehicle's passengers comfortably isolated from road noise, bumps and vibrations. The typical motorcycle has a pair of fork tubes for the front suspension, and a swing arm with one or two shock absorbers for the rear suspension [3].

A. Spring

The springs are a very important component of the suspension system that provides ride comfort. The springs support the weight of the vehicle, maintain ride height, and absorb road shock. In spring design, it is important to understand sprung and unsprung weight. Sprung weight is the weight supported by the springs such as the vehicle's body, transmission and frame. Unsprung weight is the weight that is not carried by springs such as the tires, wheels and brake assemblies. The most commonly used springs is the coil spring. The coil spring is a length of round spring steel rod that is wound into a coil. The force of a coil spring depends on wire diameter, coil diameter, number of coils and materials [4]-[5].

II. EXPERIMENTAL DETAILS

A. Experimental work

Taguchi's L9 orthogonal array is used to design the experiments with three factors and three levels. Experiments were conducted based on the Taguchi's method which is a powerful tool used in the design of experiments [6]. Taguchi advocates use of orthogonal array designs to assign the factors chosen for the experiment. The advantage of Taguchi method is that it uses a special design of orthogonal arrays to study the entire parameter space with only a small number of experiments. Compared to the conventional approach of experimentation, this method reduces drastically the number of experiments that are required to model the response functions [6]-[8].

In this study, the 3 factors which affect majorly on quality characteristic such as Spring Wire Diameter (d), Spring Index (C) and Number of Active Coils (N). The design of experiments is carried out by Taguchi methodology using Minitab 16 software. In this technique the main objective is to optimize the helical spring of rear suspension that is influenced by various parameters [9]. The spring nomenclature is as shown in Table I.

TABLE I
Spring Nomenclature

S.No	Description	Symbol	Unit
1	Spring Wire Diameter	d	mm
2	Spring Index	C	-
3	Number of Active Coils	N	Number of turns
4	Strain Energy	E	N/mm^2
5	Weight	W	Kg

B. Experimental Set Up

The test is carried out on spring testing Machine for strain energy. The deflection of a helical spring is measured by using a spring testing machine. The experiment is conducted by keeping all other parameters constant. The constant parameters were damper components, oil level and other assembly of the rear suspension system [10]-[13].

III. EXPERIMENTAL CONDITIONS

There are three inputs controlling factors selected having three levels. The assignment of the levels of the factors and the various parameters used are given in Table II. A Series of experiment is conducted to evaluate the influence of rear suspension parameters on strain energy and weight. The experimental results for L9 orthogonal array are given in Table III.

TABLE II
Assignment of the Levels to the Factors

Sl.No	Factors	Unit	Symbols	Level 1	Level 2	Level 3
1	Spring Wire Diameter (mm)	mm	d	5	5.3	5.6
2	Spring Index	-	C	6	6.25	6.5
3	Number of Active Coils	Number of turns	N	13	14	15

TABLE III
Experimental Results for L9 Orthogonal Array

Trial No.	Actual value			Strain Energy E (N/mm^2)	Weight W (Kg)
	d (mm)	C	N (Number of turns)		
1	5	6	13	11898.00	0.17
2	5	6.25	14	14542.00	0.20
3	5	6.5	15	17516.50	0.22
4	5.3	6	14	11898.00	0.23
5	5.3	6.25	15	14542.00	0.25
6	5.3	6.5	13	14211.50	0.23
7	5.6	6	15	12228.50	0.29
8	5.6	6.25	13	11898.00	0.26
9	5.6	6.5	14	14542.00	0.31

A. Multi-objective optimization using grey relational analysis

The Grey relational grade is used to convert optimization problem from a multi-objective to a single-objective. The aim of this study is to determine the optimal combination of spring parameters that simultaneously maximize strain energy (E) and minimize weight (W). To do so, grey relational analysis is used in this study. Steps of grey relational analysis are as follows: [14]-[16].

Step 1: Grey relational generation

The first step of the grey relational analysis is to normalize (in the range between 0 and 1) the experimental data according to the type of performance response. If the target value of the original sequence is infinite, such as strain energy, it has "the-larger-the-better" characteristic and the original sequence can be normalized as follows:

$$x_i^*(k) = \frac{x_i^k(k) - min\,(x_i^0(k))}{max(x_i^0(k)) - min(x_i^0(k))} \quad (1)$$

If a defined target value, OB, exists, the original sequence can be normalized as follows:

$$x_i^*(k) = 1 - \frac{|x_i^0(k) - OB|}{max\,[max\left(x_i^0(k)\right) - OB,\ OB - min(x_i^0(k))]} \quad (2)$$

In the present study, as weight is to be minimized ("the smaller-the-better" is a characteristic of the original sequence), the original sequence should be normalized as follows:

$$x_i^*(k) = \frac{\max\left(x_i^0(k)\right) - x_i^0(k)}{\max(x_i^0(k)) - \min(x_i^0(k))} \quad (3)$$

Where $x_i^*(k)$ is the value after grey relational generation (normalized value), $\max\left(x_i^0(k)\right)$ and $\min(x_i^0(k))$are the largest and smallest values of $x_i^0(k)$ for the kth response, respectively, k being 1 for strain energy, 2 for weight.

The processed data after grey relational generation is given in Table IV. The normalized values are ranged between zero and one. The Larger normalized result means to better performance and the best normalized result should be equal to 1.

Step 2: Grey relational coefficient

Grey relational coefficients denote the relationship between the ideal and the actual experimental results. Grey relational coefficient (ξi(k)) can be calculated as:

$$\xi i(k) = \frac{\Delta min + \zeta\,\Delta max}{\Delta 0i(k) + \zeta\,\Delta max} \quad (4)$$

$$0 < \xi i(k) \leq 1 \quad (5)$$

Where Δ0i(k) is the deviation sequence of reference sequence $x_0^*(k)$ and comparability sequence $x_i^*(k)$,

$$\Delta 0i(k) = \|\, x_0^*(k) - x_i^*(k)\| \quad (6)$$

$$\Delta min = \underset{\forall j\epsilon i\ \ \forall k}{\min\min} \|x_0^*(k) - x_j^*(k)\| \quad (7)$$

$$\Delta max = \underset{\forall j\epsilon i\ \ \forall k}{\max\max} \|x_0^*(k) - x_j^*(k)\| \quad (8)$$

ζ is the distinguishing coefficient (ζ ϵ [0,1]) and is used to adjust the difference of the relational coefficient. In this study ζ is taken as 0.5 and the grey relational coefficient calculated using Eq.(4) is given in Table IV.

Step 3: Grey relational grade

Grey relational grade shows the relationship between the series and is calculated as follows:

$$\alpha i = \frac{1}{n}\sum_{k=1}^{n} \xi i(k) \quad (9)$$

Where n is the number of performance characteristics. Higher relational grade corresponds to the closer experimental value to the ideal normalized value. Thus, higher grey relational grade shows that the corresponding parameter combination is closer to the optimum.

Step 4: Grey relational ordering

The highest grey relational grade is assigned an order of 1. Grey relational grade computed using Eq. (9) and the grey relational order is given in Table IV. According to Table IV, the control parameter's setting of 3(experiment 3) had the highest grey relational grade and this meant that experiment 3 was the optimal spring factors setting for maximum strain energy and minimum weight simultaneously among the other experiments.

Since higher multiple performance characteristics were desirable, the larger better S/N quality characteristics were adopted for grey relational grade. A quality characteristic of the larger-the-better is calculated in the following equation:

$$S/N_{LB} = -10\,\log_{10}\left(\frac{1}{n}\sum_{i=1}^{n}\frac{1}{y_{i^2}}\right) \quad (10)$$

yi is the ith measured experimental results in a run/row and n explains the number of measurements in each test trial.

The level of a parameter with the highest S/N ratio gives the optimal level. So the optimal process parameter setting for the multiple performance characteristic was d1C3N3 (Fig. 1). Main effects plot by means for grey relational grade as shown in Fig.2.Thus, the best combined values for maximizing the multiple performance characteristics or Grey relational grade (GRG) where spring wire diameter of 5 mm, spring index of 6.5 and the number of active coils of 15 number of turns. The response table for S/N Ratios of grey relational grade as shown in Table V. The response table for the means of grey relational grade as shown in Table VI.From the Table V and VI, it is clear that, most significant factor is spring wire diameter (d) followed by spring index (C) and number of active coils (N).

ANOVA result of the multiple performance characteristics was given in Table VII. The analysis is made for the level of confidence 95% (the level of significance is 5%). Spring wire diameter, spring index and number of active coils influenced the multiple performance characteristics by 85.82%, 7.88% and 3.56% respectively. From the analysis of this table, it could be concluded that, spring wire diameter is one of the most dominant parameters that affect the grey relational grade.

TABLE IV
Normalized values, grey relational coefficients and grey relational grades of responses

Trail No	**Normalized values of responses**		**Grey relational coefficients**		**GRG**	**ORDER**
	E	**W**	**E**	**W**		
1	0	1	0.333333333	1	0.666667	2
2	0.470588235	0.785714286	0.485714286	0.7	0.592857	3
3	1	0.642857143	1	0.583333333	0.791667	1
4	0	0.571428571	0.333333333	0.538461538	0.435897	6
5	0.470588235	0.428571429	0.485714286	0.466666667	0.47619	5
6	0.411764706	0.571428571	0.459459459	0.538461538	0.49896	4
7	0.058823529	0.142857143	0.346938776	0.368421053	0.35768	9
8	0	0.357142857	0.333333333	0.4375	0.385417	8
9	0.470588235	0	0.485714286	0.333333333	0.409524	7

TABLE V
Response Table for S/N Ratios

Level	**d**	**C**	**N**
1	-3.364	-6.555	-5.947
2	-6.565	-6.422	-6.503
3	-8.322	-5.274	-5.801
Delta	4.958	1.281	0.701
Rank	1	2	3

TABLE VI
Response Table for Means

Level	**d**	**C**	**N**
1	0.6837	0.4867	0.5170
2	0.4703	0.4848	0.4794
3	0.3842	0.5667	0.5418
Delta	0.2995	0.0819	0.0624
Rank	1	2	3

TABLE VII
ANOVA for grey relational grade (multiple response characteristics)

Source	**DF**	**Seq SS**	**Adj SS**	**Adj MS**	**F**	**P**	**%Contribution**
d	2	0.142666	0.142666	0.071333	31.33	0.031	85.82%
C	2	0.013106	0.013106	0.006553	2.88	0.258	7.88%
N	2	0.005926	0.005926	0.002963	1.30	0.435	3.56%
Error	2	0.004554	0.004554	0.002277		2.74%	
Total	8	0.166251					
S = 0.0477168 R-Sq = 97.26% R-Sq(adj) = 89.04%							

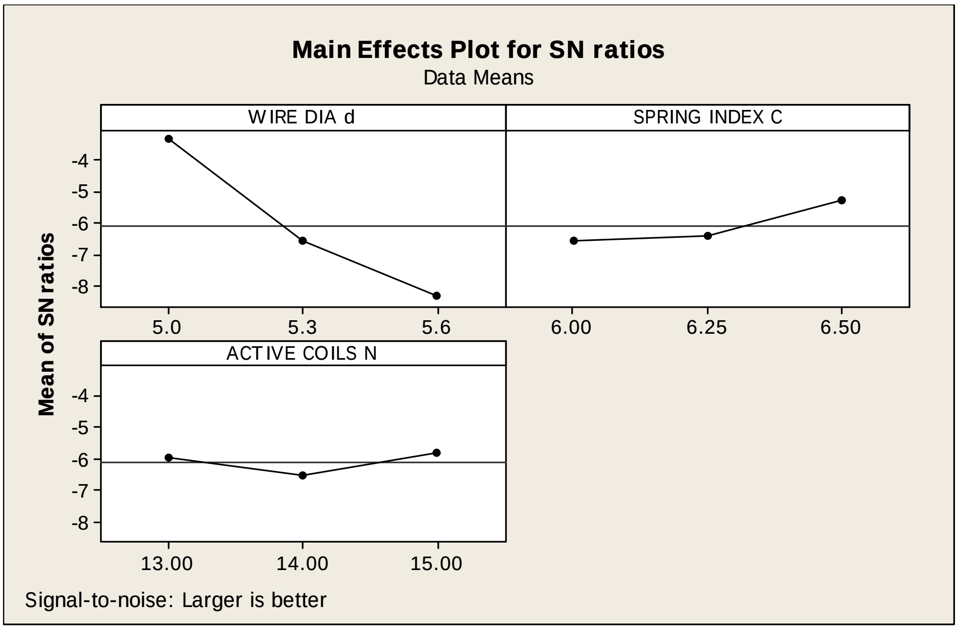

Fig. 1. Main effects plot of S/N ratios for grey relational grade (multiple performance characteristics)

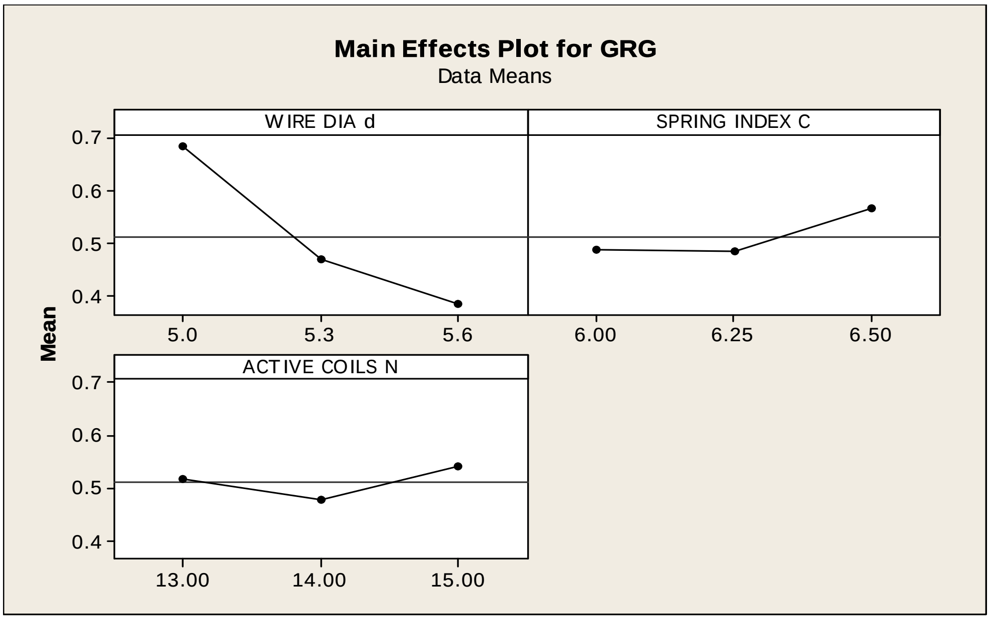

Fig. 2. Main effects plot of Means for the grey relational grade

B. Confirmation experiments:

After the optimal level has been selected, one could predict the optimum response using the following equation:

$$\gamma_{\text{predicted}} = \gamma_m + \sum_{i=1}^{n}(\gamma i - \gamma m) \quad (11)$$

where γ_m is the total mean S/N ratio, γi is the mean S/N ratio at optimal level, n is the number of main design parameters that affect the quality characteristics. It is very essential to perform a confirmation experiment in the parameter design, particularly when the less number of data is utilized for optimization. The purpose of this confirmation experiment is to verify the improvement in the quality characteristics. Based on the Eq.(3), the grey relational grade (GRG) is predicted for the optimal combination of parameters (d1-C3-N3) and its value is 0.76673. Lastly confirmation test is conducted using the optimum combination of parameters (d1-C3-N3). Table VIII shows the comparison of predicted multiple performance characteristics (GRG) with the actual one. The grey relational grade for the confirmation experiment is found to be 0.79167. This result is within the 95% confidence interval of the predicted optimum condition. Hence the grey relational analysis based on the Taguchi method for the optimization of the multi response problems is a very useful tool for predicting the strain energy and weight of the spring with multiple responses. Good agreement between the predicted and actual multiple performance characteristics are being observed. The increase of the S/N ratio of the initial spring parameters to optimal spring parameters is 1.49268 dB.

TABLE VIII
Results of Confirmation Experiment

	Initial spring parameters	**Optimal spring parameters**	
		Prediction	**Experiment**
Level	d1C1N1	d1C3N3	d1C3N3
GRG	0.66667	0.76673	0.79167
SN Ratio	-3.52183	-2.30711	-2.02915

Improvement of SN Ratio 1.49268 dB

IV. CONCLUSION

Experiments were conducted on a spring testing machine and strain energy and weight have been determined. The following conclusions were drawn.

- Grey relational analysis of the Taguchi method for the optimization of the multi response problems is a very useful tool for predicting strain energy and weight.
- It does not involve complicated mathematical theory or computation and thus can be employed by the engineers without a strong statistical background.
- From this analysis, it is revealed that spring wire diameter is the prominent factor which affect the spring design.The spring wire diameter (P = 85.82%) is the most influencing factor in determining the multiple performance characteristics or grey relational grade (GRG) followed by spring index (P =7.88%) and number of active coils (P =3.56%).
- The best multiple performance characteristics are obtained with the lower spring wire diameter of 5mm, higher spring index of 6.5 and higher number of active coils of 15 number of turns with the estimated multiple performance characteristics (GRG) of 0.76673. The experimental value of GRG for this combination of parameters is 0.79167.
- The increase in the S/N ratio of the initial spring parameters to optimal spring parameters is 1.49268 dB.
- The optimal levels of process parameters were found to be d1C3N3. i.e.

Spring Wire Diameter (mm)	5
Spring Index	6.5
Number of Active Coils (Number of turns)	15

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