Theoretical, Numerical (FEM) and Experimental Analysis of composite cracked beams of different boundary conditions using vibration mode shape curvatures

Pankaj Charan Jena [1], Dayal R. Parhi [2], G. Pohit [3]

[1]Department of Mechanical Engineering, Orissa Engineering College, Bhubaneswar, Odisha, India, 752050, Tel: ++91 8895100075, Email: pankajcharanjena@yahoo.com.

[2]Department of Mechanical Engineering, N.I.T, Rourkela, Odisha, India, 769008, Tel: ++91 661-2464509, Fax: ++91 6612462999, Email: dayalparhi@yahoo.com.

[3]Department of Mechanical Engineering, Jadavpur University, Jadavpur, Kolkata, India, 700032 Tel: ++91 033-2457-2660 (O), gpohit@vsnl.net, gpohit@gmail.com.

Abstract: This manuscript presents theoretical, experimental and numerical (finite element methods) analysis of fibers composite beams with transverse crack subjected to free vibration. Epoxy- glass fibers (unidirectional) composite has been taken as the material under study. Vibration parameters have been evaluated using various boundary conditions of beam. The beam subjected to different boundary conditions has a principal effect on dynamic characteristics of composite beam. The current work presents the evaluation of changes in natural frequencies and corresponding mode shapes curvature for different boundary conditions by varying crack positions and crack depth. The numerical results were found in good agreement with experimental and analytical results. The study concludes that structure with crack can be diagnosed by using vibration signatures and it help to monitoring the health of beam type structures.

Keywords: crack, composite, beams, natural frequency, mode shapes; finite element.

Nomenclature:

A	= cross-sectional area of the beam
C_{ij}	= flexibility influence coefficient of flexible matrix
E	= young’s modulus of elasticity of the beam material
F_{i} ,(i = 1, 2)	= experimentally determined function
H	= thickness of the beam
h_1	= depth of crack
I	= moment of inertia
i, j	= variables
J_C	= strain-energy release rate
K_{Ii} ,(i = 1, 2)	= stress intensity factors for P_i loads
K_{ij}	= local flexibility matrix elements
L	= length of the beam
L_1	= location (length) of the crack from one end
M_i,(i=1,4)	= compliance constant
P_i,(i=1,2)	=P_i= axial force (i=1) and P_j = bending load (i=2)
Ω	= stiffness matrix
u_i(i=1,2)	= additional displacement functions
U_c	= strain energy due to crack
W	= Breadth of the composite beam
X, Y, and Z	= co-ordinate of the composite beam
y	= co-ordinate of the composite beam
Y_0	= amplitude of the exciting vibration

y_i(i=1,2) = normal functions (transverse) $y_i(x)$

$\omega_{n(}$ = natural frequency of uncracked composite beam

ω_c = natural frequency of cracked composite beam

$\psi = h/H$ =Relative crack Depth (RCD)

$\chi = L_1/L$= relative crack position (RCP)

ρ = mass-density of the composite beam

1. Introduction

Higher strain to weight ratio is a desirable characteristics in its applications to light weight structures, aircraft, ships, automobiles, Composite structures have seen an increased applications in automobiles, ship building, aerospace, mechanical, and civil structure etc in the recent past. It has been established that the presence of crack, changes the natural frequency of the structure and its mode shape. In the present manuscript efforts have been made to detect the crack locality and depth using theoretical, experimental and numerical analysis.

Vision et al. [1] have examined that the fibers composite orthotropic material properties exhibit to exemplify its suitability in high speed purposes of civil structures and mechanical engineering components. Erdelyi et al. [2] have described mostly composites have different characteristics, such as high strength to weight ratio, good buckling resistance, and high stiffness. Irwin [3] has explained that existence of cracks in beam kind structures proves variation of local stiffness that considerably affects on natural frequency and mode shape of the beam. Gupta [4] has given an approach to study the stress value around the crack tip interface. Nikpur et al. [5] have evaluated compliance matrix of composite cracked bodies. Sreekanth et al. [6] have studied on composite structure taking into account various forms of transverse cracks like, fibers fracture, matrix cracking and surface-breaking cracks with by spectral finite element methods. They established FEM-2D model is more suitable for complex cracked beam to detect crack location and crack depth. Pandey et al. [7] have studied to identify the crack in beam using Eigen parameters and concluded that mode shape curvature is suitable to find out damage position. Marek [8] has used finite element approach on non propagating middle cracked beam to find out its dynamic and static behaviour. Amara et al. [9] have analyzed mechanical properties of angle-ply composites laminates and studied the effect of transverse cracks on the stiffness of laminated composite beam with different fibers orientations. Ramanamurthy et al.[10] have presented the detection of damage in a composite structure using finite element approach. Nag et al. [11] have modeled a laminate cracked composite using node based 2D finite element approach based on spectral wave scattering. Lakshmi et al.[12] have worked on detection of crack location by the first four torsional modes and bending modes and calculated corresponding changes in natural frequencies of a cracked beam. Murat [13] has investigated the effects of cracks on the vibration characteristics of beam made of graphite fiber-reinforced polyamide using finite element methods. Shu et al. [14] have done analysis work on delaminated non overlapping composite beam using free vibration techniques by theoretical and experimental procedure to find the locations of delaminations. They have concluded that the mode shape and frequency will change significantly in presence of delaminations. Singh et al. [15] have used free vibration mode shape procedure on non uniform composite to evaluate its physical parameter.

The objective of this paper is to establish the mode shape curvatures behaviour of a composite beam with a transverse open crack subjected to free vibration. In the present study, a composite beam made up epoxy glass fibres is considered for analysis. The effort is made to study the changes in ω_n i.e. natural frequency and mode shapes in presence and absence of cracks in composite structures, and the effort is also made for recognition of the cracks through non destructive and inexpensive ways.

2. Analytical Approaches

In the present article, composite beam with various end conditions has been considered for analysis with a single transverse crack. The mathematical analysis in the present study will lead to the derivation of stiffness matrix of the cracked beam. Referring to Irwin theory [3], **t**he strain energy release rate at the cracked section can be expressed as,

$$J_C = \frac{1}{E'}(K_{I1} + K_{I2})^2 \tag{1}$$

Where, the stress intensity factors are K_{I1}, K_{I2} of mode I (opening of the crack) under load P_1 and P_2 respectively.

$$\frac{1}{E'} = \frac{1-\nu_1^2}{E} \quad \text{(For plane strain condition)} \quad \text{and} \quad \frac{1}{E'} = \frac{1}{E} \quad \text{(for plane stress condition)}$$

The expression for stress intensity factors from earlier studies Irwin et al. [3], are

$$K_{I1}=\frac{P_1}{WH}\sqrt{\pi h}(F_1(\frac{h}{H})),K_{I2}=\frac{6P_2}{WH^2}\sqrt{\pi h}\,(F_2(\frac{h}{H})) \tag{2}$$

Where parameters F_1 and F_2 can be presented as bellow,

$$F_1(\frac{h}{H}),=\sqrt{(\frac{2H}{\pi h}\tan(\frac{\pi h}{2H}))}\left\{\frac{0.752+2.02\,(\mathrm{h/\,H})+0.37\,(1-\sin(\pi h/2H))^3}{\cos(\pi h/2H)}\right\} \tag{3}$$

$$F_2(\frac{h}{H}),=\sqrt{(\frac{2H}{\pi h}\tan(\frac{\pi h}{2H})}\left\{\frac{0.923+0.199\,(1-\sin(\pi h/2H))^4}{\cos(\pi h/2H)}\right\} \tag{4}$$

Let Uc be the strain energy due to the crack, then from Castiglione's theorem, the additional displacement along the direction of force P_i is $u_i=\frac{\partial U_c}{\partial P_i}$ (5)

The strain energy (Uc) can be related to strain energy release rate (J_c) as**,**

$$U_c=\int_0^{h_1}\frac{\partial U_c}{\partial h}dh=\int_0^{h_1}J_c(h)\,dh \quad \text{Where } J_C=\frac{\partial U_c}{\partial h} \tag{6}$$

From equations (5) and (6), thus we have

$$\boldsymbol{u_i=\frac{\partial}{\partial P_i}\left[\int_0^{h_1}J_c(\mathrm{h})\,dh\right]} \tag{7}$$

Defining the flexibility influence co-efficient C_{ij} per unit depth,

$$\boldsymbol{C_{ij}=\frac{\partial u_i}{\partial P_j}=\frac{\partial^2}{\partial P_i\partial P_j}\int_0^{h_1}J_c(\mathrm{h})\,dh} \tag{8}$$

Integrating over the breadth 'W', the final flexibility matrix element can be obtained as,

$$C_{ij}=\frac{\partial u_i}{\partial P_j}=\frac{\partial^2}{\partial P_i\partial P_j}\int_{-W/2}^{+W/2}\int_0^{h_1}J_c(\mathrm{h})\,dh\,dz \tag{9}$$

Using the value of strain energy release rate from equation (1) and putting in equation (8),

$$\boldsymbol{C_{ij}=\frac{B}{E'}\frac{\partial^2}{\partial P_i\partial P_j}\int_0^{h_1}(K_{I1}+K_{I2})^2\,dh} \tag{10}$$

The local stiffness matrix can be obtained by taking the inversion of compliance matrix. i. e.

$$\mathrm{K}=\begin{bmatrix}\mathrm{K}_{11} & \mathrm{K}_{12}\\ \mathrm{K}_{21} & \mathrm{K}_{22}\end{bmatrix}=\begin{bmatrix}\mathrm{C}_{11} & \mathrm{C}_{12}\\ \mathrm{C}_{21} & \mathrm{C}_{22}\end{bmatrix}^{-1} \tag{11}$$

Figure 1 shows the relative crack depth to that of variation of dimensionless compliances.

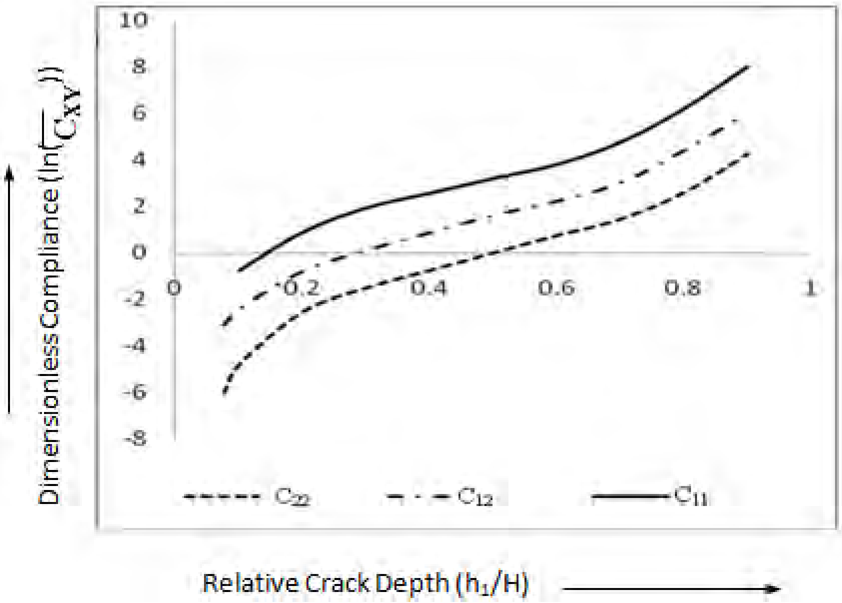

Figure 1. Relative crack depth (h_1/H) vs. dimensionless compliance (ln ($\overline{C}_{xy}$))

2.2. Analysis of Vibration Characteristics of the Cracked Composite Beam

A composite beam of length 'L' width 'W' and depth 'H', with a crack of depth 'h_1' at a distance 'L_1' from the one end is considered as shown in figure 2.

Taking $u_1(x, t)$ and $u_2(x, t)$ as the amplitudes of longitudinal vibration for the sections before and after the crack and $y_1(x, t)$, $y_2(x, t)$ are the amplitudes of bending vibration for the same sections as shown in fig. (2).

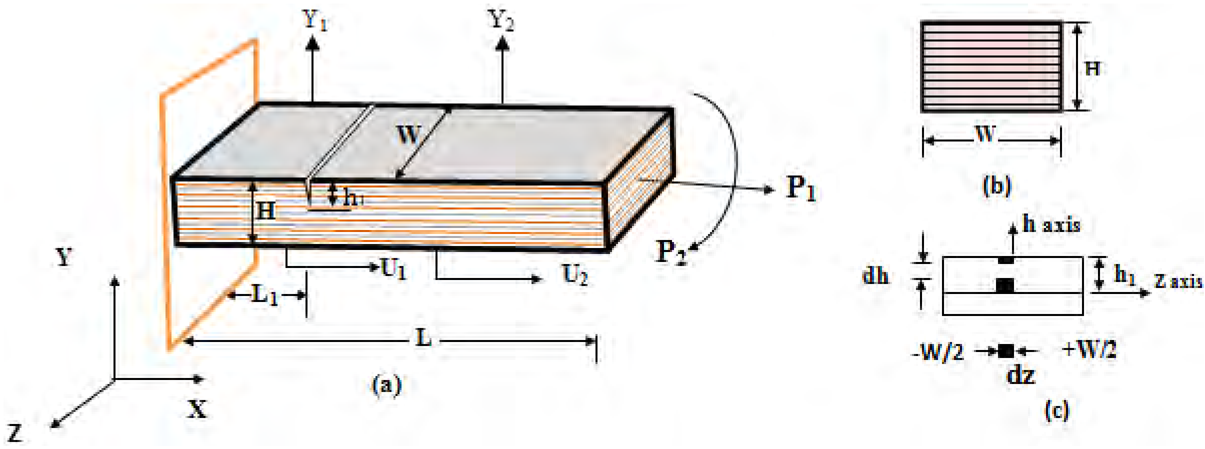

Figure 2: Geometry of composite beam: (a) Composite beam (b) cross-sectional view of the beam (c) crack depth and crack location.

The normal function for the system can be defined as

$$\bar{u}_1(\bar{x})=A_1\cos(\bar{K}_u\bar{x})+A_2\sin(\bar{K}_u\bar{x}) \quad (12)$$

$$\bar{u}_2(\bar{x})=A_3\cos(\bar{K}_u\bar{x})+A_4\sin(\bar{K}_u\bar{x}) \quad (13)$$

$$\bar{y}_1(\bar{x})=A_5\cosh(\bar{K}_y\bar{x})+A_6\sinh(\bar{K}_y\bar{x})+A_7\cos(\bar{K}_y\bar{x})+A_8\sin(\bar{K}_y\bar{x}) \quad (14)$$

$$\bar{y}_2(\bar{x})=A_9\cosh(\bar{K}_y\bar{x})+A_{10}\sinh(\bar{K}_y\bar{x})+A_{11}\cos(\bar{K}_y\bar{x})+A_{12}\sin(\bar{K}_y\bar{x}) \quad (15)$$

Where, $\bar{u}=\frac{u}{L}$, $\bar{x}=\frac{x}{L}$, $\bar{y}=\frac{y}{L}$, $\chi=\frac{L_1}{L}$

$$\overline{K}_u = \frac{\omega L}{C_u}, \overline{K}_y = \left(\frac{\omega L^2}{C_y}\right)^{1/2}, C_u = \left(\frac{E}{\rho}\right)^{1/2}, \ C_y = \left(\frac{EI}{\mu}\right)^{1/2}, \quad \mu = A\rho$$

A_i, (i=1, 12) Constants are to be determined, from boundary conditions. The boundary conditions of the cantilever beam in consideration are:

$$\overline{u}_1(0)=0; \quad \overline{y}_1(0)=0; \quad \overline{y}_1'(0)=0; \quad \overline{u}_2'(1)=0; \quad \overline{y}''_2(1)=0; \quad \overline{y}_2'''(1)=0$$

At the cracked section: $\overline{u}_1(\chi)=\overline{u}_2(\chi)$; $\overline{y}_1(\chi)=\overline{y}_2(\chi)$; $\overline{y}_1''(\chi)=\overline{y}_2''(\chi)$; $\overline{y}_1'''(\chi)=\overline{y}_2'''(\chi)$

Also at the cracked section at distance L_1 from fixed end of cantilever beam, we have:

$$AE\frac{du_1(L_1)}{dx}=K_{11}(u_2(L_1)-u_1(L_1))+K_{12}\left(\frac{dy_2(L_1)}{dx}-\frac{dy_1(L_1)}{dx}\right) \tag{16}$$

Multiplying both sides of the above equation by $\frac{AE}{LK_{11}K_{12}}$ we get;

$$M_1M_2\overline{u}'(\chi)=M_2(\overline{u}_2(\chi)-\overline{u}_1(\chi))+M_1(\overline{y}_2'(\chi)-\overline{y}_1'(\chi)) \tag{17}$$

Similarly, $$EI\frac{d^2y_1(L_1)}{dx^2}=K_{21}(u_2(L_1)-u_1(L_1))+K_{22}\left(\frac{dy_2(L_1)}{dx}-\frac{dy_1(L_1)}{dx}\right) \tag{18}$$

Multiplying both sides of the above equation by $\frac{EI}{L^2K_{22}K_{21}}$ we get,

$$M_3M_4\overline{y}_1''(\chi)=M_3(\overline{u}_2(\chi)-\overline{u}_1(\chi))+M_4(\overline{y}_2'(\chi)-\overline{y}_1'(\chi)) \tag{19}$$

Where, $M_1=\frac{AE}{LK_{11}}$, $M_2=\frac{AE}{K_{12}}$, $M_3=\frac{EI}{LK_{22}}$, $M_4=\frac{EI}{L^2K_{21}}$

The normal functions, equation (11) along with the boundary conditions as mentioned above, yield the characteristic equation of the system as: $|\Omega|=0$ (20)

Where, Ω is a 12×12 Matrix. This determinant is a function of natural circular frequency (ω), the relative location of the crack (χ) and the local stiffness matrix (K) which in turn is a function of the relative crack depth (h_1/H).

3. Numerical analysis of cracked composite beam using ANSYS

Finite element approach (FEA) is widely used numerical methods for vibration analysis of structures. ANSYS is a software tool based on FEA is used here for free vibration analysis of a composite beam under consideration. A Composite beam of dimensions 650mm×60mm×6mm with eleven layers of epoxy glass fibers for determinations of natural frequency and mode shapes under various end conditions with and without crack were analyzed using ANSYS 12. The crack depth and crack locations were varied in a systematic manner for analysis. Following procedures for a cracked composite beam were followed for the analysis:

Step-1: geometry modeling of composite beam.

Step-2: selection of element type (here solid 186 layered element for 3D modeling) for composite beam. 20 nodes Solid 186 layered element is used for solid modeling.

Step-3: material modeling of test composite beam (properties of orthotropic material).

Step-4: section layup of model (here 0^0 fibers orientation). With a shell section or layered composite specified with layer thickness, orientation, number of integration points through the thickness of layer.

Step-5: model meshing

Step-6: boundary conditions and modal solution

Step-7: result and plotting

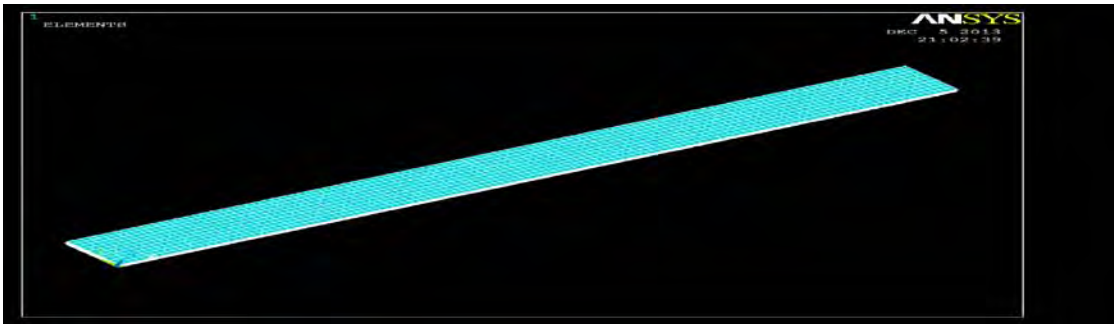

Figure 4: Epoxy E-glass fibre composite cracked beam meshing

Figure 5: Epoxy E-glass fibre composite cracked beam meshing

Table: 1. Mechanical properties of prepared Epoxy E-glass with 0^0 fiber orientation composite beam:

(E_{XX}) in KN/mm^2	(E_{YY}) in KN/mm^2	(E_{ZZ}) in KN/mm^2	(G_{XY}) in KN/mm^2	(G_{YZ}) in KN/mm^2	(G_{ZX}) in KN/mm^2	γ_{XY}	γ_{YZ}	γ_{ZX}	ρ in Kg/m^3
39.36	7	7	1.2	1.2	1.2	0.23	0.23	0.23	1740

4. Experimental Procedure

Composite beam of epoxy glass fibres with dimensions 650mm×60mm×6mm were prepared using hand layout techniques. Crack depths with respect to fixed different locations are taken separately for experimental analysis. A total of eleven layers (unidirectional fibers) of uniform thickness with matrix put alternately were prepared using hand layout techniques. The mechanical properties of the composite beam recorded using Instron setup machine are given in Table 1.

The transverse cracks were created at different locations of the beam individually by inserting sharp thin smooth plate at time of preparation of composite beams. The transverse cracks were located at different locations from one end of the beam to study the changes in dynamic characteristics. The results were compared with a beam without crack. The composite beam with different boundary fixations i.e. fix-free, fix-fix, were considered. For each experiment the crack location were changed for different boundary fixations keeping the crack depth ratio constant. Pre fixed distance of ranging from 50mm to 500mm with 50mm steps along the length were considered for crack location for each experiment with a specified boundary fixation. The experiments were repeated for various boundary fixations and investigation. The experimental setup is shown in figure (6) the beams were put into the holder for test with different boundary fixations. External excitation was created in the beam with a impact hammer initially at mid position of beam.

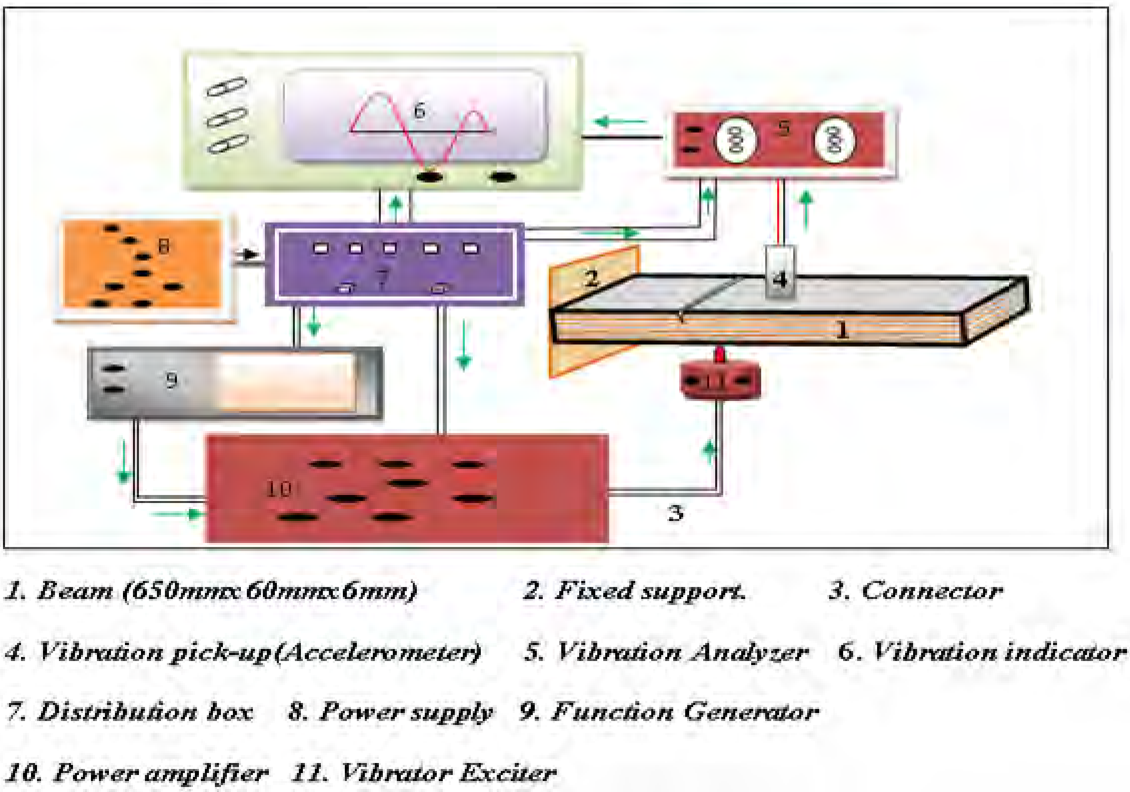

Fig.6: Schematic diagram of experimental setup

The excitation created by hammer is transmitted in the form of a signal to the FFT (model pulse lite 3560-L Device) through amplifier. The readings of the amplitude of transverse vibration and resonant frequencies at different crack locations along the length are recorded by using vibration set up. The experimental set up with beam under test is shown in the fig. (6).

5. Results and Discussions Analysis:

Theoretical, Finite element (numerical) and Experimental analysis of a cracked composite beam were carried out. Relative natural frequency was calculated varying the crack depth and crack locations. Finite element analysis (ANSYS) was used to extract natural frequency and mode shapes of various end supports and results were compared with experimental and theoretical results. The relative crack depth to that of variation of dimensionless compliances is given in figure (1) and in figure (2) is present the Geometry of composite cracked beam. 20 nodes Solid 186 layered element is used for solid modelling in numerical analysis. In figure (4-5) are shown the mesh form of (epoxy E-glass fiber) composite beam. The schematic diagram of experimental setup of beam under test is given in figure (6). In figures (7-9) are presented the first three mode shape of composite cracked beam. It is observed from 1st mode of vibration that relative frequency decreases with increase in crack depth for a given crack position. The slop of the curve is more significant between 0.4 to 0.6 of crack depth ratio and indicating a steep fall in relative frequency. Similar trend is observed for the corresponding relative frequency of second mode of vibration. However the drop in relative frequency is higher in 3rd mode figure (10) as compared to 1st and 2nd mode of vibration in respect to crack depth. Similar trend is observed for variation in relative frequency of C-C end support composite beam figure (11). Variation in relative frequency are recorded by varying the crack positions for a given crack depth ratio. It is observed, 1st mode of vibration that the relative frequency increases with the shifting of crack positions from one end support with respect to 1st mode of vibration. However similar trend is not observed for 2nd and 3rd mode of vibration for given crack depth ratio. For Clamp-Clamp beam figure (13), a drop in relative frequency is significant as compared to Clamp-Free beam. The relative frequency variation in 2nd mode of vibration is more pronounced as compared to its corresponding 3rd mode of vibration.In general, the natural frequency decreases with increase in crack depth due to beam losses its stiffness.

(a). Mode shape of epoxy glass fiber Composite cantilever(C-F) Beam using ANSYS at Relative crack position $(\chi = \frac{L_1}{L})$ 0.08 and Relative crack depth(h_1/H)=0.58:

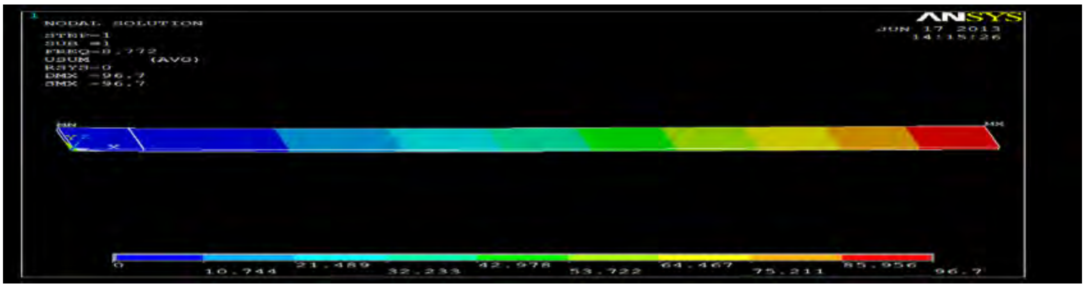

Figure 7. First mode shape of C-F cracked (epoxy glass fiber) composite beam

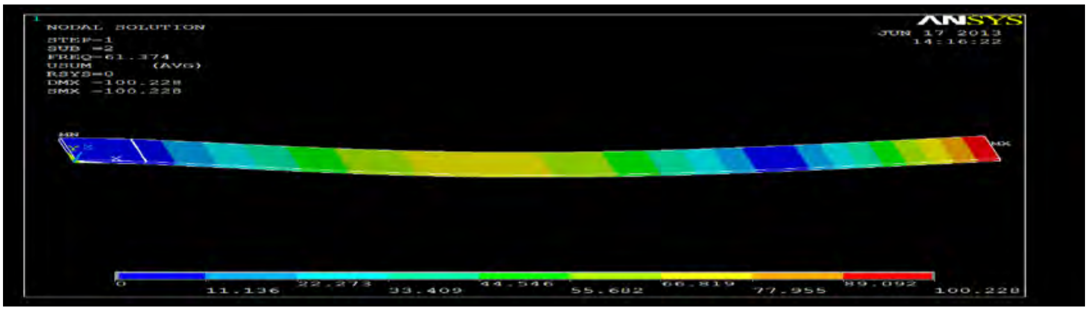

Figure 8. 2nd mode shape of C-F cracked (epoxy glass fiber) composite beam

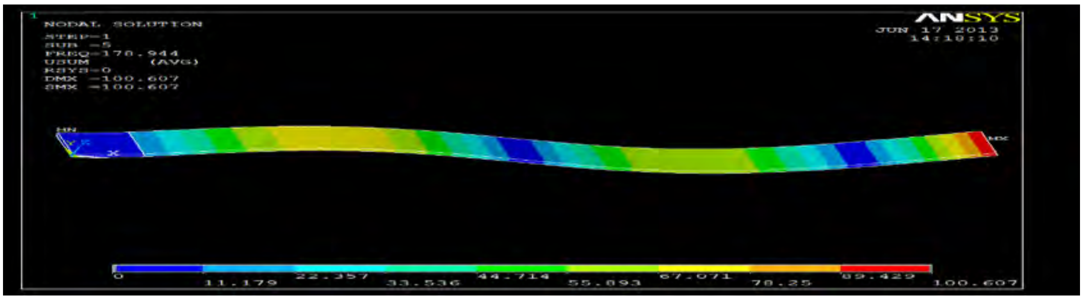

Figure 9. Third mode shape of C-F cracked (epoxy glass fiber) composite beam

(b). (h_1/H) vs. (ω_c/ω_n) at fixed ($\chi = \frac{L_1}{L}$) of C-F Composite beam:

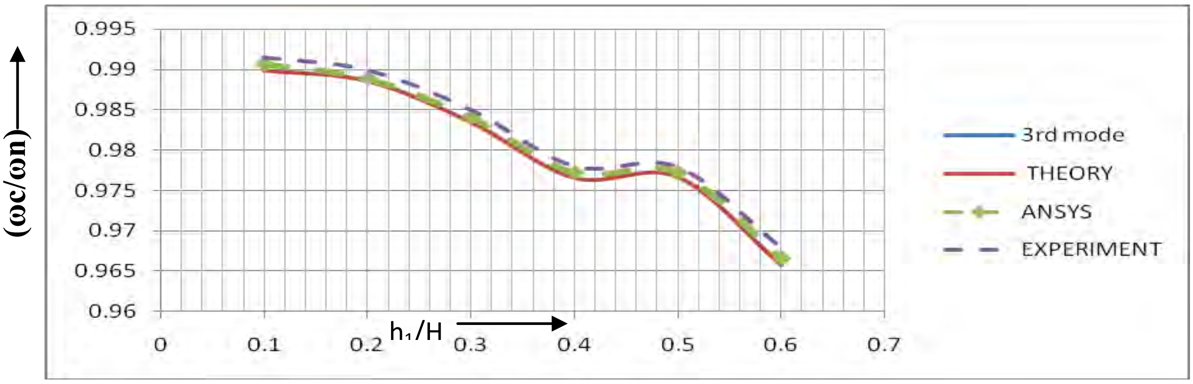

Figure 10. (h_1/H) vs. (ωc/ωn) at χ = 0.08 of C-F Composite beam in 3rd mode

(c). (h_1/H) vs. (ω_c/ω_n) at $(\chi = \frac{L_1}{L})$ = 0.08 of C-C Composite beam

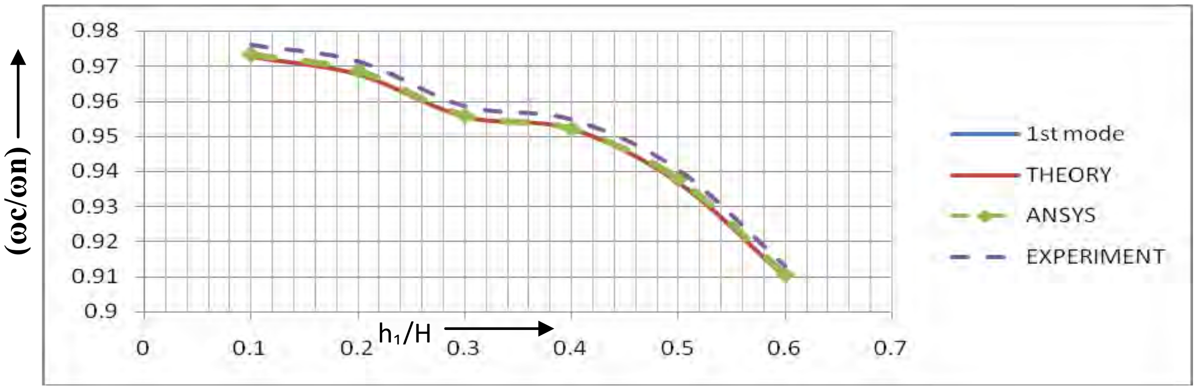

Figure 11. (h_1/H) vs. (ωc/ωn) at $\chi =$ 0.08 of C-C Composite beam in 1st mode

(d). ($\chi = \frac{L_1}{L}$) vs. (ω_c/ω_n) of C-F at fixed (h_1/H) :

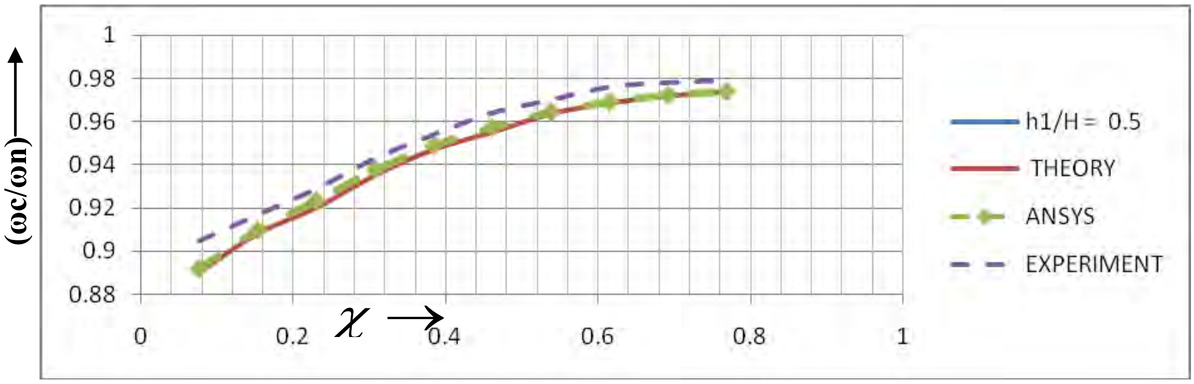

Figure12. (χ) vs. (ω_c/ω_n) of C-F at crack depth Ratio h_1/H= 0.48 in 1st Mode

(e). ($\chi = \frac{L_1}{L}$) vs. (ω_c/ω_n) of C-C at fixed (h_1/H):

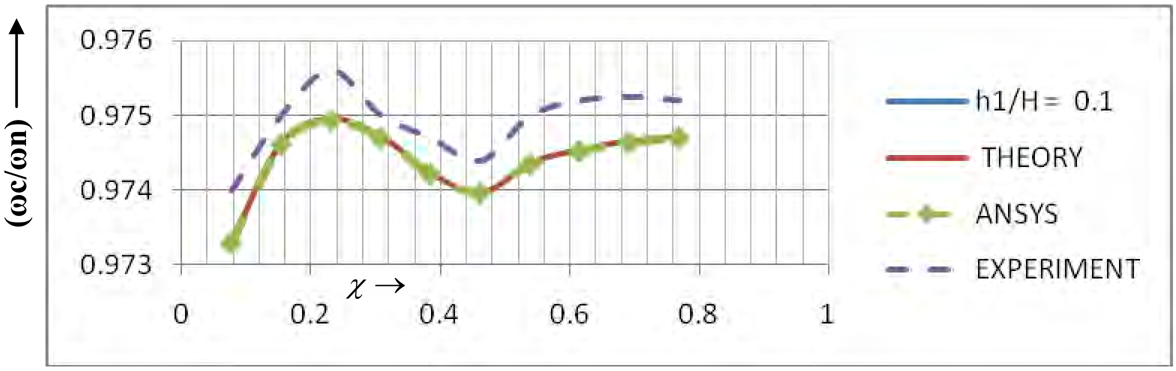

Figure 13. (χ) vs. (ω_c/ω_n) of C-C at crack depth Ratio (h_1/H)= 0.12 in 1st Mode.

6. Conclusions:

The results of experimental, analytical and finite element analysis investigation are given and the observed that with increase in crack depth, the relative frequencies reduce in order due to drop in stiffness of the composite beam. Further it is observed that the relative crack position affects the relative frequency of the composite beam. The relative frequency increases with higher relative crack location. Further **a**ttempts will be made to investigate the effect of frequency for composite beams with variation in fibre angles and volume fractions.

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