Tuning and Analysis of Multiple Interactive Loop Process by Model Predictive Control

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Abstract -General process consists of a number of loops that may be of interactive or non interactive types. A Single Input Single Output includes less complex interactive loops when compared with Multi Input Multi Output processes. In this work a multiloop interactive process known to be an Octal Tank Process (OTP) with 8 tanks is considered that are placed one above another. The input voltages are fed through two motors however the two outputs are water levels of the lower tanks. Similarly four inputs are fed through motors $V_1, V_2, V_1^{'}, V_2^{'}$ and four outputs are water levels of h1, h2 and h5, h6. Optimized tuning methodology is applied to Multiple Interactive Loop Process through a Model Predictive Control (MPC) technique meant for calculating present and future values. Different tuning methods are applied to the process and response water levels of the tanks are observed. MPC can stabilize all linear processes effectively and efficiently, it tunes output and inputs simultaneously to provide more stable (optimized) output within the permissible limits of tolerance / error. It can also work with nonlinear processes under extreme conditions. It offers an optimized result under Predictive control "P" and Horizon control "M", by tuning P and M and it is applied to OTP as a Nonlinear Model Predictive Control.

Keyword- Nonlinear Model Predictive Control1, optimized2, OTP3, Predictive control 4, Horizon control 5

I. INTRODUCTION

MPC technique is more applicable for obtaining optimized performance and is attractive as it offers feedback strategy for linear processes. This same method can be applied to nonlinear systems to obtain equally good response or result. This is referred to as moving horizon or receding horizon control. MPC methods use linear or nonlinear models, to calculate present and future values of dynamic systems. Linear MPC theory is quite mature and has wide ranging applications from chemicals to aerospace industries. Most of the physical systems are inherently nonlinear in existence because of the economical constraints and product quality in process industry. This requires maintaining and operating the system within the admissible operating region which is a part of the boundary.

This work mainly focuses on the application of MPC techniques to nonlinear model of OTP [1], [2]. The basic principle of MPC is as shown in fig 1. It depicts the dynamic behavior of system over predicted horizon P and control horizon M and determines the predefined open loop performance objective function which needs to be optimized. Performance measure is obtained at time t, the controller predicts the future values by tuning for a value of P greater than l. MPC has two methods of approach, for non linear and linear models of OTP [4]. Linear MPC approaches the operating points by tuning objective function however the Nonlinear MPC approaches the problem without consideration to operating point by tuning the objective function for nominal values.

MPC method has two ways of tuning. The first method is based on simulation of process model by adjusting the parameters based on process dynamics, which is approximately adjustable [5], [6]. Similarly second one is based on explicit derivation of formulas by considering various parameters of the process model with respect to dynamics. The controller design sets the prediction horizon P, control horizon M, weights on the output Q, weights on the rate of change of input λ, the reference trajectory parameter α and some constant parameters.

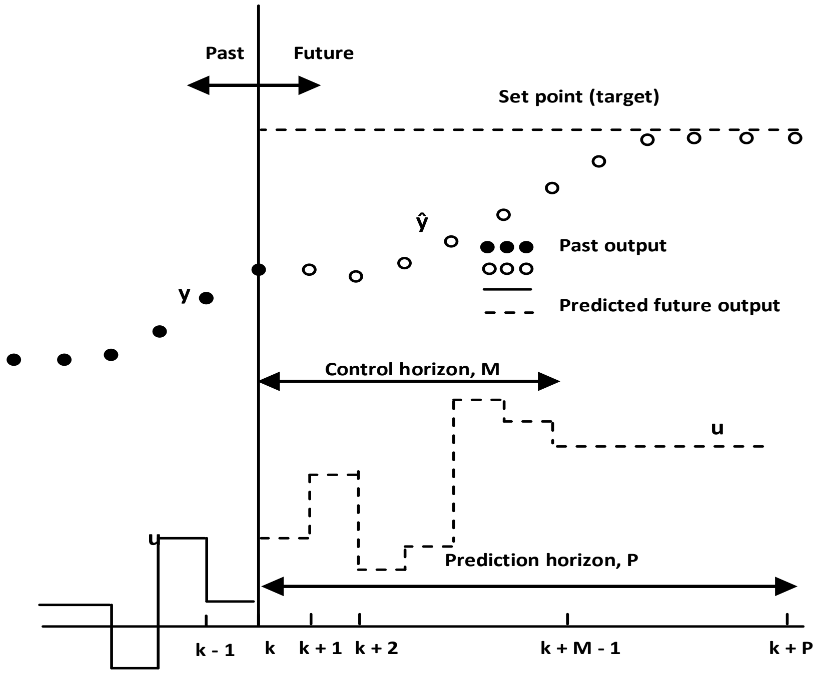

Fig 1 Basic principle of Model Predictive Control

II. MODEL DESCRIPTION

A. Nonlinear model

Octal tank process consists of eight interconnected water tanks and two pumps as shown in Figure 2. The main aim is to control the water levels in the two lower tanks using two pumps. The process inputs are v_1 and v_2 (input voltages to the pumps) and the outputs are $y_1 = k_c h_1$ and $y_2 = k_c h_2$ (voltages from level measurement devices).

where, γ_i is the flow distribution to lower and diagonal upper tank, A_i is the cross-section area, a_i is the outlet hole cross section and h_i is the water level, in tank i respectively. The voltage applied to pump i is v_i and the corresponding flow is $k_i v_i$.

The parameters $\gamma_1, \gamma_3, \gamma_5, \gamma_7, \in (0,1)$ $\gamma_2, \gamma_4, \gamma_6, \gamma_8, \in (0,1)$ are determined based on the proper setting of the valves prior to the experiment. The flow to tank 1 is $\gamma_1 k_1 v_1$, the flow to tank 4 is $\gamma_3 k_1 v_1$, the flow to tank 5 is $\gamma_5 k_1 v_1$ and the flow to tank 8 is $\gamma_7 k_1 v_1$. Similarly the flow to tank 2 is $\gamma_2 k_2 v_2$, the flow to tank 3 is $\gamma_4 k_2 v_2$, the flow to tank 6 is $\gamma_6 k_2 v_2$ and the flow to tank 7 is $\gamma_8 k_2 v_2$. The acceleration of gravity is denoted ‘g’. The parameter values for the process are given in Table 1. Mass balances and Bernoulli’s law yield the following model [3], [1], [2]:

$$\frac{dh_1}{dt}=-\frac{a_1}{A_1}\sqrt{2gh_1}+\frac{a_3}{A_1}\sqrt{2gh_3}+\frac{\gamma_1 k_1}{A_1}v_1$$
$$\frac{dh_2}{dt}=-\frac{a_2}{A_2}\sqrt{2gh_2}+\frac{a_4}{A_2}\sqrt{2gh_4}+\frac{\gamma_2 k_2}{A_2}v_2$$
$$\frac{dh_3}{dt}=-\frac{a_3}{A_3}\sqrt{2gh_3}+\frac{a_5}{A_3}\sqrt{2gh_5}+\frac{\gamma_4 k_2}{A_3}v_2$$
$$\frac{dh_4}{dt}=-\frac{a_4}{A_4}\sqrt{2gh_4}+\frac{a_6}{A_4}\sqrt{2gh_6}+\frac{\gamma_3 k_1}{A_4}v_1$$
$$\frac{dh_5}{dt}=-\frac{a_5}{A_5}\sqrt{2gh_5}+\frac{a_7}{A_5}\sqrt{2gh_7}+\frac{\gamma_5 k_1}{A_5}v_1$$
$$\frac{dh_6}{dt}=-\frac{a_6}{A_6}\sqrt{2gh_6}+\frac{a_8}{A_6}\sqrt{2gh_8}+\frac{\gamma_6 k_2}{A_6}v_2$$
$$\frac{dh_7}{dt}=-\frac{a_7}{A_7}\sqrt{2gh_7}+\frac{\gamma_8 k_2}{A_7}v_2$$
$$\frac{dh_8}{dt}=-\frac{a_8}{A_8}\sqrt{2gh_8}+\frac{\gamma_7 k_1}{A_8}v_1 \tag{1}$$

TABLE I
Parameters process values

parameters	Units	Values
A_1, A_3, A_5, A_7	$\left[cm^2\right]$	28
A_2, A_4, A_6, A_8	$\left[cm^2\right]$	32
a_1, a_3, a_5, a_7	$\left[cm^2\right]$	0.071
a_2, a_4, a_6, a_8	$\left[cm^2\right]$	0.057
k_c	$\left[V/cm\right]$	0.5
g	$\left[cm/s^2\right]$	981

This typical system has two finite zeros for $\gamma_1,\gamma_3,\gamma_5,\gamma_7 \in (0,1)$ & $\gamma_2,\gamma_4,\gamma_6,\gamma_8 \in (0,1)$ one always lies in the left half-plane and the other can be placed either in the left or the right half-plane depending on the valve setting of $\gamma_1,\gamma_2,\gamma_3,\gamma_4,\gamma_5,\gamma_6,\gamma_7,\gamma_8$.

B. Linear model

Octal tank process called the 2×1 matrix Quadruple Tank Process (QTP) consists of eight interconnected water tanks and four pumps as shown in Figure 3. The model consists of 8 water tanks arranged in a 4x2 matrix format numbered in the order 1, 2 starting from the bottom row and so on till the top row. This model focuses upon controlling the water level of the tanks 1, 2 and 5, 6. The process inputs are v_1, v_2 and v_1^1, v_2^1 (input voltages to the pumps) and the outputs are $y_1 = k_c h_1$, $y_2 = k_c h_2$ and $y_5 = k_c h_5$, $y_6 = k_c h_6$ (voltages from level measurement devices).

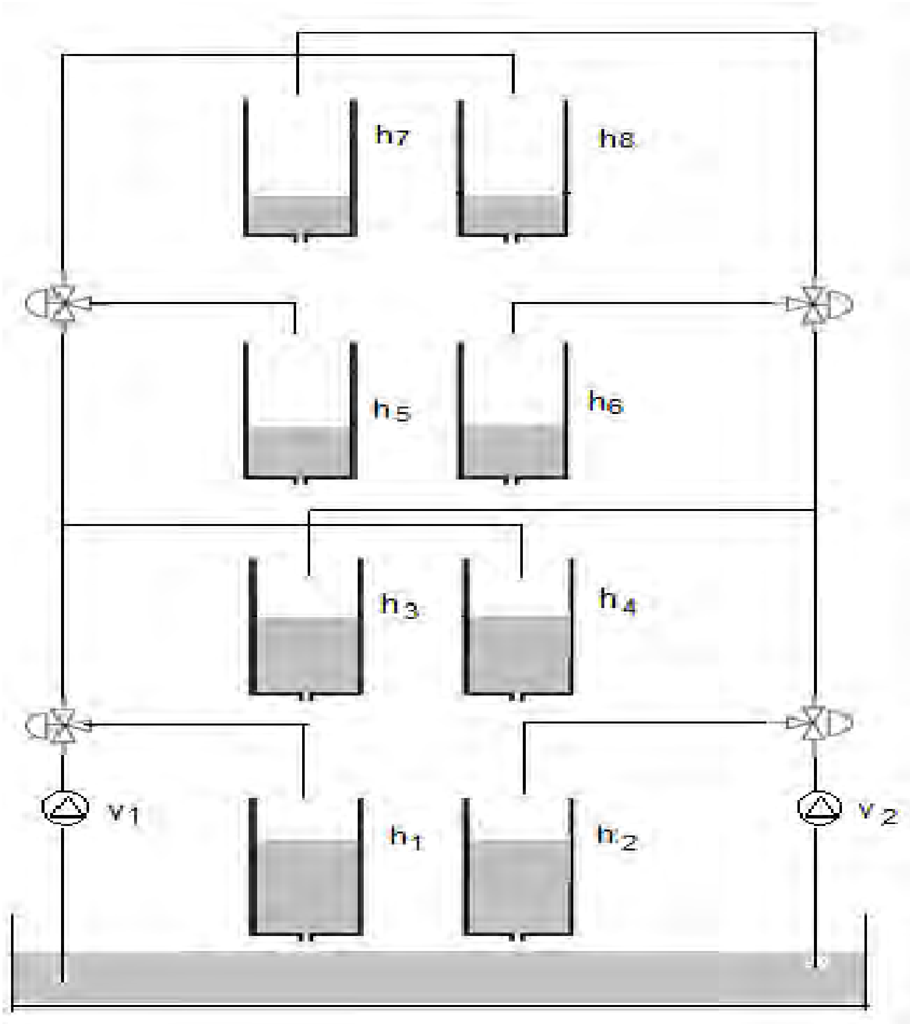

Fig 2. Daigram of non linear Octal Tank Process

The parameters $\gamma_1, \gamma_3, \gamma_5, \gamma_7, \in (0,1)$ $\gamma_2, \gamma_4, \gamma_6, \gamma_8, \in (0,1)$ are determined based on the proper setting of the valves prior to the experiment. The flow to tank 1 is $\gamma_1 k_1 v_1$, the flow to tank 4 is $(1-\gamma_1)k_1 v_1$, the flow to tank 5 is $\gamma_1^{'} k_1^{'} v_1^{'}$ and the flow to tank 8 is $(1-\gamma_1^{'})k_1^{'} v_1^{'}$. Similarly the flow to tank 2 is $\gamma_2 k_2 v_2$, the flow to tank 3 is $(1-\gamma_2)k_2 v_2$, the flow to tank 6 is $\gamma_2^{'} k_2^{'} v_2^{'}$ and the flow to tank 7 is $(1-\gamma_2^{'})k_2^{'} v_2^{'}$. The acceleration of gravity is denoted ‘g’. The parameter values for the process are given in Table 1.

It is derived at two operating points with linearized model and control of the Octal tank process studied at two operating points $p\_$ at which the system is shown to have minimum phase characteristic and p_+ at which the system is shown to have non-minimum phase characteristic. The chosen operating points correspond to parameter value in table 2. Mass balances and Bernoulli’s law yield the following model [1], [2], [3]: Linearised the model has two sets of operating points with state space equation at operating points $x_i = h_i - h_i^0$ and $u_i = v_i - v_i^0$.

$$\frac{dh_1}{dt} = -\frac{a_1}{A_1}\sqrt{2gh_1} + \frac{a_3}{A_1}\sqrt{2gh_3} + \frac{\gamma_1 k_1}{A_1} v_1$$
$$\frac{dh_2}{dt} = -\frac{a_2}{A_2}\sqrt{2gh_2} + \frac{a_4}{A_2}\sqrt{2gh_4} + \frac{\gamma_2 k_2}{A_2} v_2$$
$$\frac{dh_3}{dt} = -\frac{a_3}{A_3}\sqrt{2gh_3} + \frac{a_5}{A_3}\sqrt{2gh_5} + \frac{(1-\gamma_2) k_2}{A_3} v_2$$
$$\frac{dh_4}{dt} = -\frac{a_4}{A_4}\sqrt{2gh_4} + \frac{a_6}{A_4}\sqrt{2gh_6} + \frac{(1-\gamma_1) k_1}{A_4} v_1$$
$$\frac{dh_5}{dt} = -\frac{a_5}{A_5}\sqrt{2gh_5} + \frac{a_7}{A_5}\sqrt{2gh_7} + \frac{\gamma_1^{'} k_1^{'}}{A_5} v_1^{'}$$
$$\frac{dh_6}{dt} = -\frac{a_6}{A_6}\sqrt{2gh_6} + \frac{a_8}{A_6}\sqrt{2gh_8} + \frac{\gamma_2^{'} k_2^{'}}{A_6} v_2^{'}$$
$$\frac{dh_7}{dt} = -\frac{a_7}{A_7}\sqrt{2gh_7} + \frac{(1-\gamma_2^{'}) k_2^{'}}{A_7} v_2^{'}$$
$$\frac{dh_8}{dt} = -\frac{a_8}{A_8}\sqrt{2gh_8} + \frac{(1-\gamma_1^{'}) k_1^{'}}{A_8} v_1^{'} \qquad (2)$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \\ \dot{x}_7 \\ \dot{x}_8 \end{bmatrix} = \begin{bmatrix} -\frac{1}{T_1} & 0 & \frac{A_3}{A_1 T_3} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{T_2} & 0 & \frac{A_4}{A_2 T_4} & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{T_3} & 0 & \frac{A_5}{A_1 T_5} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{T_4} & 0 & \frac{A_6}{A_2 T_6} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{T_5} & 0 & \frac{A_7}{A_1 T_7} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{T_6} & 0 & \frac{A_8}{A_2 T_8} \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{T_7} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{T_8} \end{bmatrix} x + \begin{bmatrix} \frac{\gamma_1 k_1}{A_1} & 0 & 0 & 0 \\ 0 & \frac{\gamma_2 k_2}{A_2} & 0 & 0 \\ 0 & \frac{(1-\gamma_2) k_2}{A_3} & 0 & 0 \\ \frac{(1-\gamma_1) K_1}{A_4} & 0 & 0 & 0 \\ 0 & 0 & \frac{\gamma_1^{'} K_1^{'}}{A_5} & 0 \\ 0 & 0 & 0 & \frac{\gamma_2^{'} K_2^{'}}{A_6} \\ 0 & 0 & 0 & \frac{(1-\gamma_2^{'}) K_2^{'}}{A_7} \\ 0 & 0 & \frac{(1-\gamma_1^{'}) K_1^{'}}{A_8} & 0 \end{bmatrix} u$$

(3)
$$\mathrm{y} = \begin{bmatrix} \mathrm{k_c} & 0 & 0 & 0 \\ 0 & \mathrm{k_c} & 0 & 0 \\ 0 & 0 & \mathrm{k_c} & 0 \\ 0 & 0 & 0 & \mathrm{k_c} \end{bmatrix} h_i \qquad (4)$$

Here $i = 1, 2, 5, 6$

Here the time constant is

$$T_i = \frac{A_i}{a_i}\sqrt{\frac{2h_i^0}{g}} \qquad (5)$$

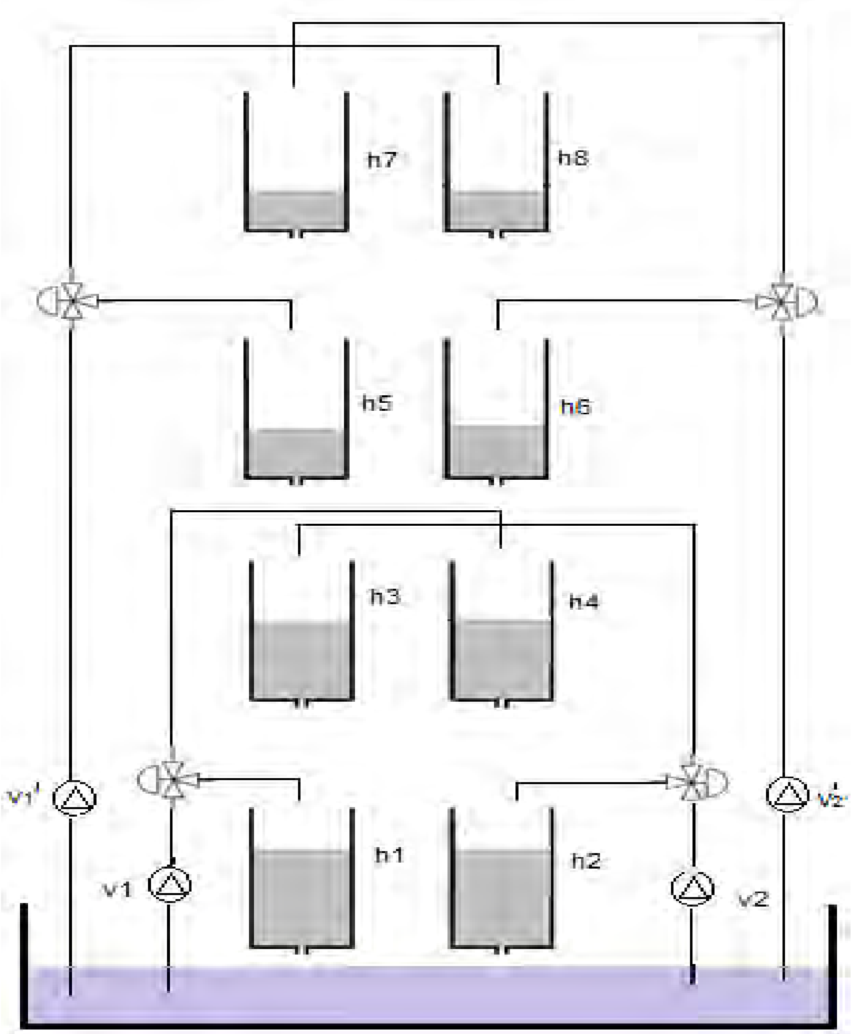

Fig 3. Daigram of linear Octal Tank Process(2×1 matrix of QTP)

TABLE II
Operating Points

Operating points	Units	P_-	P_+
$\left(h_1^0, h_2^0\right)$	$[cm]$	$(12.4, 12.7)$	$(12.6, 13.0)$
$\left(h_3^0, h_4^0\right)$	$[cm]$	$(1.8, 1.4)$	$(4.8, 4.9)$
$\left(h_5^0, h_6^0\right)$	$[cm]$	$(12.4, 12.7)$	$(12.6, 13.0)$
$\left(h_7^0, h_8^0\right)$	$[cm]$	$(1.8, 1.4)$	$(4.8, 4.9)$
(v_1, v_2)	$[V]$	$(1,1)$	$(1,1)$
(v_1^1, v_2^1)	$[V]$	$(1,1)$	$(1,1)$
(k_1, k_2)	$\left[cm^3/Vs\right]$	$(3.33, 3.35)$	$(3.14, 3.29)$
$(k_1^{'}, k_2^{'})$	$\left[cm^3/Vs\right]$	$(4.35, 4.37)$	$(4.78, 5.21)$
(γ_1, γ_2)		$(0.7, 0.6)$	$(0.43, 0.34)$
$(\gamma^{'}_1, \gamma^{'}_2)$		$(0.64, 0.73)$	$(0.35, 0.32)$

III. FORMULATION OF NMPC

Consider the non linear differential equation for stabilizing the problem [9].

$$\dot{x}\left(t\right) = f\left(x\left(t\right), u(t)\right), \quad x(t_0) = x\left(0\right) \tag{6}$$

$$y\left(t\right) = g\left(x\left(t\right), u\left(t\right)\right) \tag{7}$$

$$u\left(t\right) \in u, \forall t \geq 0 \text{ , } x\left(t\right) \in x, \forall t \geq 0 \text{ } y\left(t\right) \in y, \forall t \geq 0 \tag{8}$$

Where $x\left(t\right) \subseteq R^n$ and $u\left(t\right) \subseteq R^m$ determine the vector of states and inputs. Denotes x and u are feasible set of inputs and states and y is estimated or measured output.

We assume a set of feasible assumptions i.e., x and y as follows:

Assume 1: In its simplest form, u and x are given by constraints of the form

$$u_{\min} \leq u \leq u_{\max} \tag{9a}$$

$$x_{\min} \leq x \leq x_{\max} \tag{9b}$$

Assumption 2: The vector field $f\left(x\left(t\right), u(t)\right)$ is continuous and satisfies $f\left(0,0\right) = 0$ at initial condition.

Assumption 3: Equation (6) has a unique continuous solution for any initial condition in the region of interest and continuous manipulated input function $u\left(t\right) : \left[0, M\right] \rightarrow u$ and continuous predicted state function $x\left(t\right) : \left[0, P\right] \rightarrow x$

Real systems and models are mainly used for predicting the future values within the limits selected by the controller. The finite horizontal open loop described above is mathematically formulated as follows. $\overline{u}\left(t\right)$ is represented as internal controller.

$$\min_{\overline{u}(t)} J\left(x\left(t\right), \overline{u}\left(t\right); M, P\right)$$

$$J\left(x\left(t\right), \overline{u}\left(t\right); T_P, T_C\right) := \int_t^{t+P} f\left(x\left(t\right), \overline{u}\left(t\right)\right) d + \int_t^{t+M} g\left(\overline{u}\left(t\right)\right) d \tag{10}$$

Subject to:

$$\overline{x}(t) = f\left(\overline{x}\left(t\right), \overline{u}\left(\tau\right)\right), \quad \overline{x}(t) = x\left(t\right) \tag{11a}$$

$$\overline{x}\left(t\right) \in x, \quad \forall t \in \left[t, t+P\right] \tag{11b}$$

$$\overline{u}\left(t\right) \in u, \quad \forall t \in \left[t, t+M\right] \tag{11c}$$

$$\overline{u}\left(\tau\right) = \overline{u}\left(\tau + P\right), \quad \forall t \in \left[t+M, t+P\right] \tag{11d}$$

IV. STABILITY

Comparing the predicted result of open and closed loop behavior is always different. An NMPC strategy that achieves stability independent of the choice of performance measure, cost function and constraints of model is desirable. We assume that the prediction horizon and control horizon if set such that $P \neq M$, and $P < M$ will result in instability[8]. The one way to achieve stability is the use of an infinite horizon cost function, i.e., T_p in equation (10) is set at ∞ . Practically as well as theoretically this may not determine the response. More appropriate and feasible condition is $P > M$. Similarly the model is examined under non feasible conditions, where $P = M$, $P < M$. Whereas $P = M$ exhibits somewhat admissible response, $P < M$ exhibits a more aggressive response.

The input computed as the solution of NMPC optimization problem is equal to the closed loop trajectory of non-linear system at any given instance of time. Basic steps for infinite horizon proof are based on use of value functions [7], [8]. Feasibility at one sampling instance does impel for next sampling instance for the normal case.

V. OPTIMIZED THEORY FOR MPC

The designer needs to optimize control algorithm to minimize cost and maximize performance measure. These depend on the system variables, which are states x, output y, tracking error e and control u.

Describe the process state equation of nonlinear time invariant [12], [8] and [9]

$$\dot{x}(t) = f(x(t), u(t)) \quad (12)$$

$$y(t) = g(x(t), u(t)) \quad (13)$$

Performance function:

$$J(x(t), u(t), y(t)) = \int_{t_0}^{t_f} w_{xyu}(x, y, u)dt \quad (14)$$

Is minimized to the dynamic system which is represented as maximized performance [12] measure for determining control law with penalty term $h\left(x\left(t_f\right)\right)$

$$J = h(x(t)) + \int_{t_0}^{t_f} f(x(t), u(t))d \quad (15)$$

t_f = final time, t_0 = initial time; $t_0 \le t \le t_f$

Optimal solution to optimized problem is denoted $u^*(t)$ and repeatedly solved at sampling instants t=kδ K=0, 1, 2... for open loop control problem. Admissible optimal control law is defined for closed loop control for equation (6) at sampling instants

$$u^*(t) = \bar{u}^*(\tau, x(t), P, M), \ \tau \in [t, \delta] \quad (16)$$

The optimal value of NMPC open loop optimal control as a function of the state will be denoted in the following as value function

$$V(x, P, M) = J\left(x(t), \bar{u}^*(t), P, M\right) \quad (17)$$

In a similar method, we obtain performance measure form, from equations (15) to (17)

$$J(x(t), u(t), P, M) = h(x(P)) + \int_{t_0}^{t_0+P} f(x(t), u(t))d + \int_{t_0}^{t_0+M} g(u(t))d \quad (18)$$

The admissible controls are constrained to lie in a set U; i.e. $u \in U$. We first approximate the continuous operation of equation (8) by a discrete system

$$\frac{x(t+\Delta t) - x(t)}{\Delta t} \approx f(x(t), u(t)) \quad (19)$$

$$x(t+\Delta t) = x(t) + \Delta t f(x(t), u(t)) \quad (20)$$

Shortening the above notion

$$x(k+1) = x(k) + \hat{f}(x(k), u(k)) \quad (21)$$

$$x(k+1) \approx \hat{f}(x(k), u(k)) \quad (22)$$

In a similar manner, we get performance measure form as

$$J = h(x(k)) + \sum_{k=0}^{N-1} (f(x(k), u(k)) + g(u(k))) \quad (23)$$

To minimize the deviation of the final state of system from its desired values, there are more analytical squared terms much more analytically solvable than other types. Because positive & negative deviations are equally undesirable, so absolute value could be used in quadratic form.

Using matrix notation:

$$J = x^T(k)Hx(k) + \sum_{k=0}^{N-1}\left(x^T(k)Qx(k) + u^T(k)Rx(k)\right) \quad (24)$$

$$J(x(k), u(k), P, M) = \min\left(\sum_0^P (x^T(k)Qx(k)) + \sum_0^M (u^T(k)Ru(k)) + x^T(k)Hx(k)\right) \quad (25)$$

Optimized solution for equation (25) with number of intervals

$$J\left(x\left(k/k-1\right),u\left(k/k-1\right),P,M\right)=\min\sum_{0}^{P}(x^{T}\left(k/k-1\right)Qx\left(k/k-1\right))+\sum_{0}^{M}(u^{T}\left(k/k-1\right)Ru\left(k/k-1\right))+x^{T}\left(k/k-1\right)Hx\left(k/k-1\right)) \quad (26)$$

Here if, k is present, k-1 is past, if k-1 is present then k is future. Here k is described as discrete or continuous function Q, H; R is real symmetric positive semi-definite $n \times n$ matrix. Q is output weighted matrix and R input weighted matrix. H is solution of Ricatti equation from linear standard state space equation (27)

$$\begin{aligned} x(k+1) &= Ax(k)+Bu(k) \\ y(k) &= Cx(k)+Du(k) \end{aligned} \quad (27)$$

$$H = A^{T}HA - A^{T}HB\left(B^{T}HB+R\right)^{-1}BHA+Q \quad (28)$$

VI. TUNING METHODOLOGY OF NMPC

A. Prediction Horizon P

Different outputs will be obtained because of the input values of P, as the settling time and rise time are quite different. Increasing the value of P minimizes controller aggressiveness [6]. The final horizon is set to be finite or infinite to ensure stability. In this case, the final horizon is described based on tuning result for closed loop stability of control system or process.

The Proposed new tuning methods for MPC are as specified under [5], [6]:

$$P = k\eta + t_r/\eta T_S \quad (29)$$

$$P \geq \text{int}\left(M+C+\eta k\right) \pm 1 \quad (30)$$

$$t_r < P < T_p/k(or)\eta \quad (31)$$

By default P = 10 is probable value of objective function, as per stability criterion P is tuned from the various parameters , like, settling time t_s, rise time t_r , no of outputs k , higher order of process η , no of controllers C, process response time t_p , sampling time T_s, delay time t_d and response of rise time 60,80,90,95 w.r.t t_p. P value is calculated as average of number of outputs.

B. Control Horizon M

Evaluating the value of M, if it increases in value, it tends to become more aggressive over the prediction horizon (M>P). This is to monitor and control the response of data from output by adjusting the manipulated variable. This leads to a trade-off between increasing performance and robustness of formulation of control law, as a default control horizon is equal to 1. Formulate control horizon without more aggressiveness and existing robustness of permissible computation load [5], [6]. The proposed new tuning methods are designed mainly based on parameter settling time t_s, rise time t_r, number of outputs k, higher order of process η, sampling time T_S .

$$M = \min\left(\text{int}(t_S/2), \text{int}(P/4)\right) \pm 1 \quad (32)$$

$$M = \text{int}\left(kn/t_S\right) \quad (33)$$

$$M \propto k/t_r \quad (34)$$

$$M \propto \eta/t_s \quad (35)$$

C. Output weighted matrix is represented by Q

The output variables are relatively weighted according to their significance in the process model. It provides individual significance relative to output variable, with the most important variable having a larger weight compared to others. Increasing linearly the weight on the upper limit of output to achieve a smooth response till the desired output is obtained. The elements of Q that correspond to corrected error have nonzero weight to help in relative prediction. Derived expression for the output weight for minimum phase also works for non minimum phase for the closed loop [5].

Smoothness is totally based upon the output weights, expression for both non minimum and minimum phase will be

$$Q < 1 \tag{36}$$

$$Q \leq \det\left|C^T C\right| \tag{37}$$

Here C is output matrix of linear state space equation.

D. Weights on the magnitude of the inputs R

In similar fashion, R allows to be weighted for input variable according to their relative importance. R is normally considered as diagonal matrix with diagonal elements of $rM \times rM$ matrix. It is referred as input weighting matrix or move suppression matrix. It is more convenient for tuning parameters based on parameter of r_{ij} as suppression factor [5], [6] and [10].

E. Weights on the rate of change of inputs λ

This section discusses existing and new approaches for tuning the weights on the rate of change of inputs. Penalizing the rate of change produces a more robust controller but at the cost of the controller becoming more sluggish. Small value adjustments yield a more aggressive controller. Even a small change in the input affects the rise time and settling time [5], [6]. This is compensated by output weights

$$\lambda < 1/\eta P \tag{38}$$

F. Reference Trajectory parameters

In MPC application, reference trajectory provides the necessary path to reach final desired set point [10]. It can be specified in several different ways. It is designed between initial value and final value between $0 \leq \beta_j < 1$, j=1…P.

$$\beta_j = closedloopt_s / openloopt_s \tag{39}$$

$$0 < \beta_j < 1 \tag{40}$$

VII. SIMULATION ANALYSIS

We discussed extensively the application of non linear processes to OTP for the tanks response by giving appropriate weights to the tuning system. We generated conditions that are applicable for representation with non linear differential equations. Responses are calculated by means of the tuning equations for optimized solutions of NMPC for both the linear and nonlinear models. Responses for different tuning conditions are plotted for step input.

For non linear models, the responses are plotted for step input for two types of valve settings. Parameters of valves are as provided in table no 3, based on minimum and non minimum phase conditions. The simulation results of non linear model for the minimum phase and non minimum phase are as shown in figs 4, 5 and 6, 7.

TABLE III
Valve Settings

$(\gamma_1, \gamma_3, \gamma_5, \gamma_7)$	$(0.6, 0.1, 0.1, 0.2)$	Minimum phase
$(\gamma_2, \gamma_4, \gamma_6, \gamma_8)$	$(0.7, 0.1, 0.1, 0.1)$	Minimum phase
$(\gamma_1, \gamma_3, \gamma_5, \gamma_7)$	$(0.4, 0.3, 0.1, 0.2)$	Non minimum phase
$(\gamma_2, \gamma_4, \gamma_6, \gamma_8)$	$(0.45, 0.25, 0.1, 0.2)$	Non minimum phase

Response 1 and Response 2 parameters P, M, Q, λ for the non linear model of minimum phase are calculated from equations 29,34,35,36, 38 and 29, 32, 36, 38.However the response 1 is defined as P= 18, M=4, Q=1, λ=0.0138 while response2 is defined as P= 17, M=2, Q=1, λ=0.0147. For h1 reference value is 0.00075 and h2 reference value is 1.

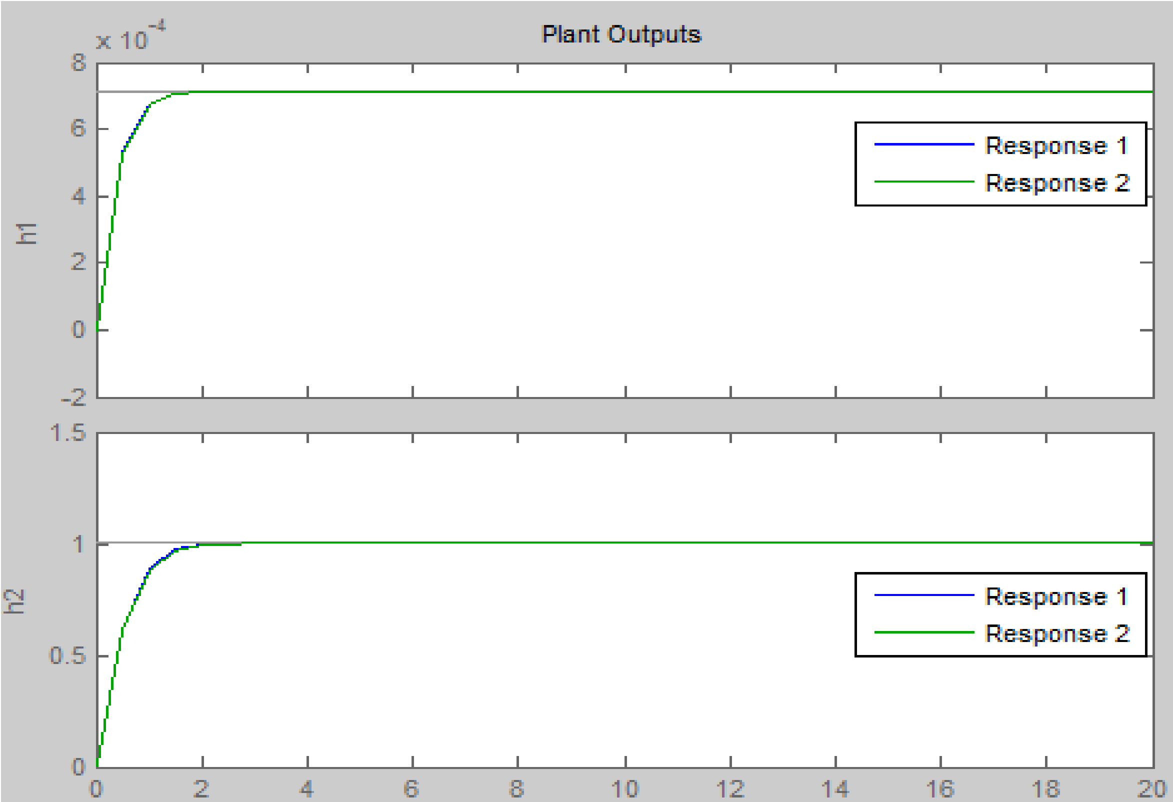

Fig. 4. Response of lower two tanks h1, h2 of step input

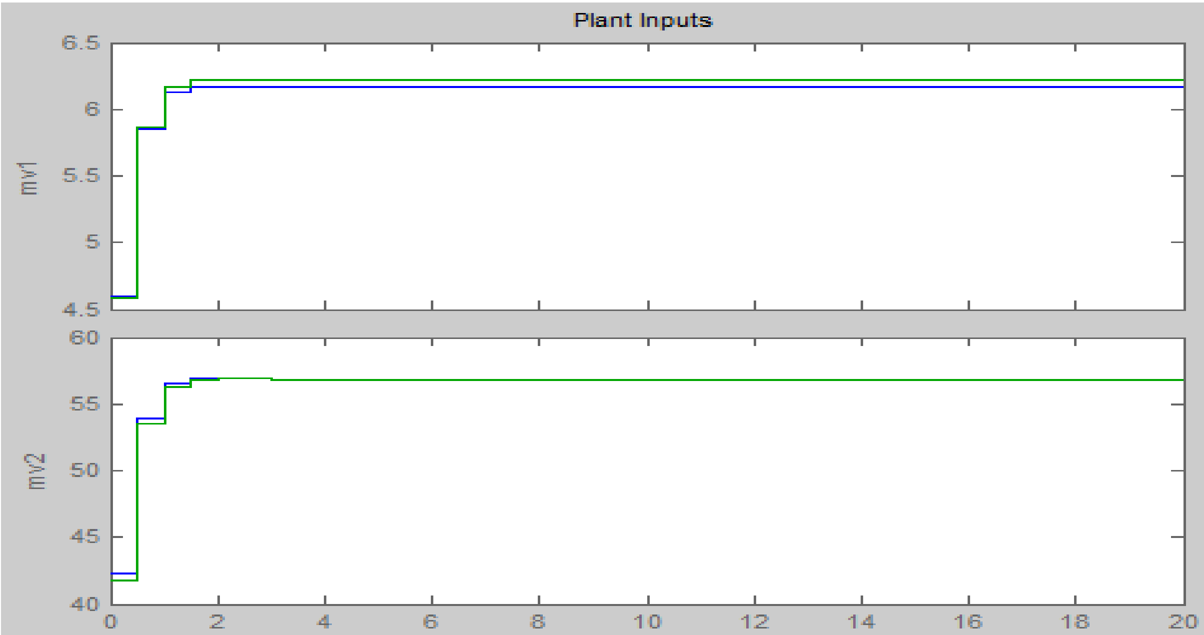

Fig. 5. Tuning of manipulated input

Response 1 and Response 2 parameters P, M, Q, λ for the non linear model of non minimum phase are calculated from equations 29,34,35,36, 38 and 29, 32, 36, 38.However the response 1 is defined as P= 18, M=4, Q=1, λ=0.0138 while response2 is defined as P= 17, M=2, Q=1, λ=0.0147. For h1 reference value is 0.002 and h2 reference value is 1.

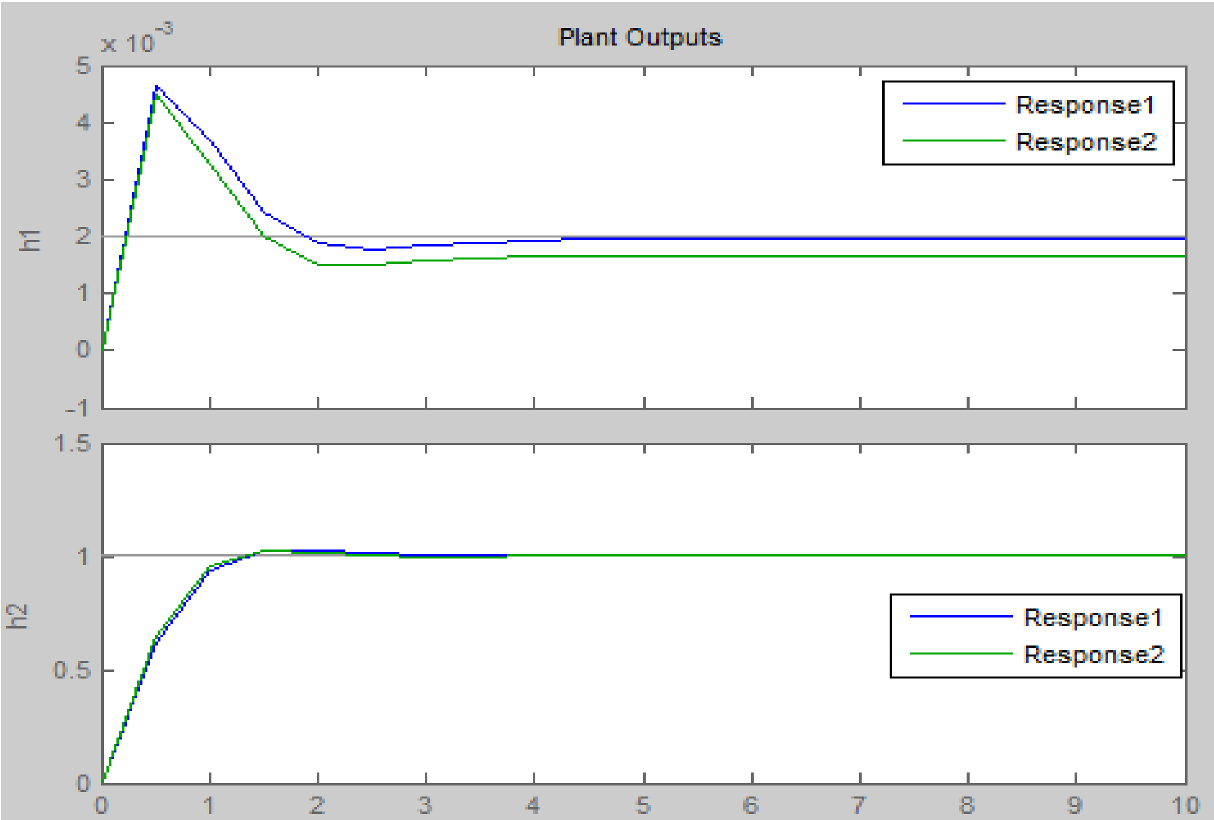

Fig. 6. Response of lower two tanks h1, h2 of step input

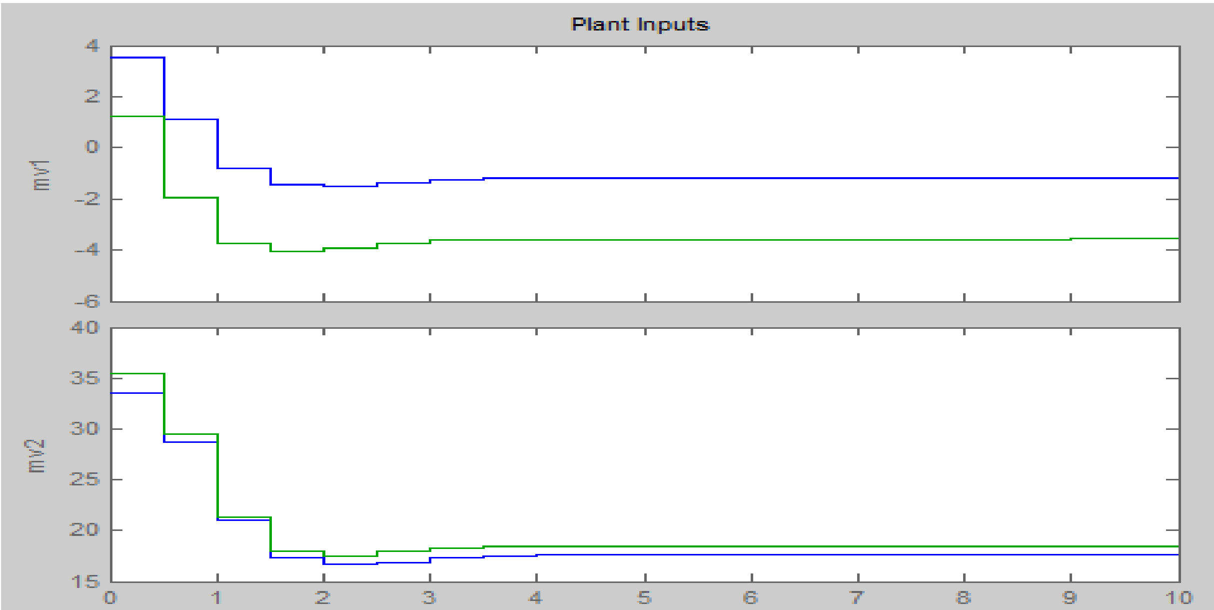

Fig.7. Tuning of manipulated input

Similarly for linear model of 2×1 matrix Quadruple Tank Process (QTP) responses are plotted for step input for two types of valve settings. Parameters of valves are as provided in table no 2, based on minimum and non minimum phase conditions. The simulation results of non linear model for the minimum phase and non minimum phase are as shown in figs 8, 9 and 10, 11.

Response 1 and Response 2 parameters P, M, Q, λ for the non linear model of minimum phase are calculated from equations 29,34,35,36, 38 and 29, 33, 36, 38.However the response 1 is defined as P= 32, M=4, Q=1, λ=0.0078125 while response2 is defined as P= 31, M=15, Q=1, λ=0.008064.

Similarly Response 1 and Response 2 parameters P, M, Q, λ for the non linear model of non minimum phase are calculated from equations 29,34,35,36, 38 and 29, 33, 36, 38.However the response 1 is defined as P= 32, M=4, Q=1, λ=0.0078125 while response2 is defined as P= 31, M=15, Q=1, λ=0.008064.

All the responses are plotted based on different tuning valves for both the linear & non linear models

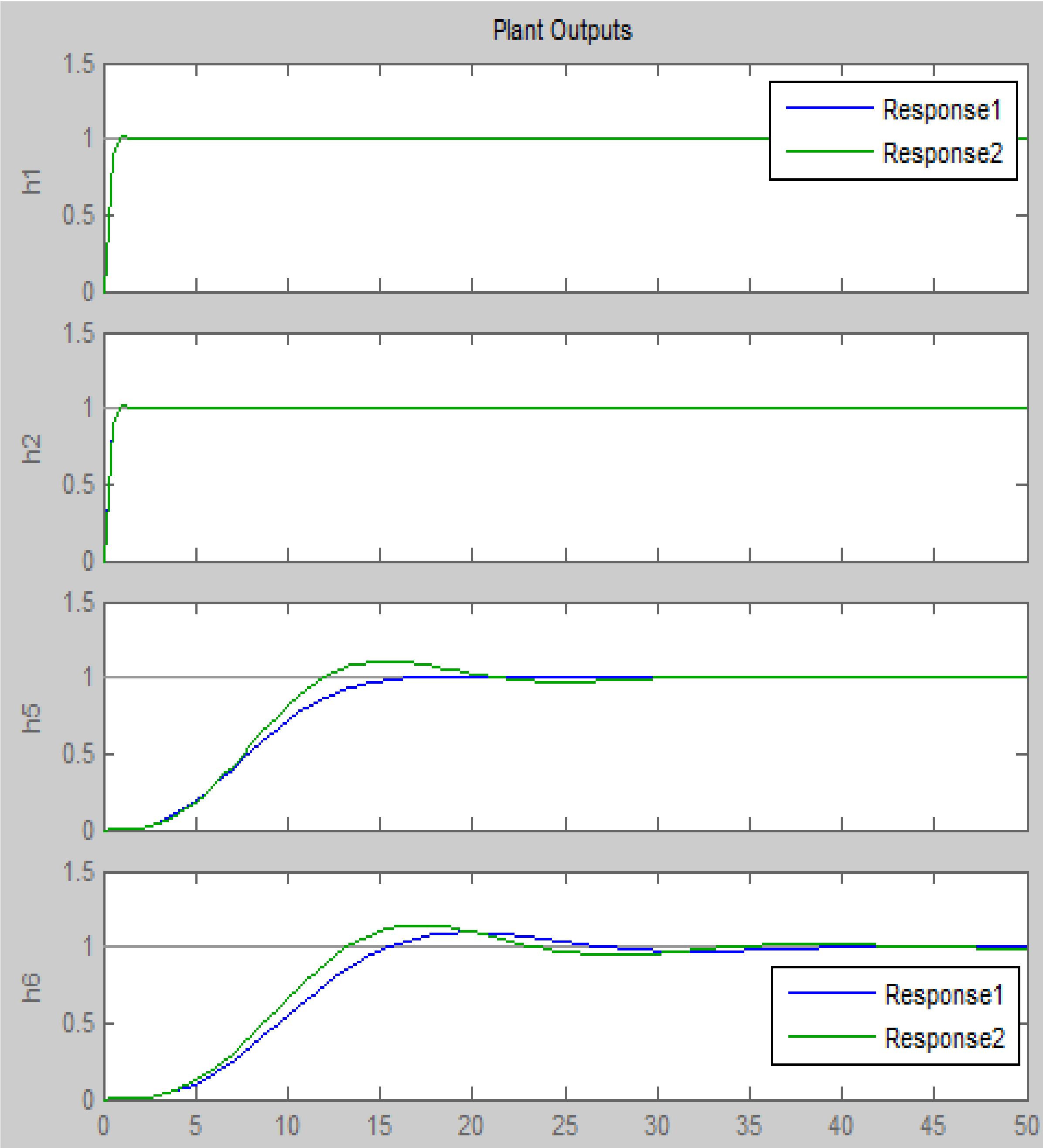

Fig.8. response of lower two tanks h1, h2, h5 and h6 of step input for minimum phase

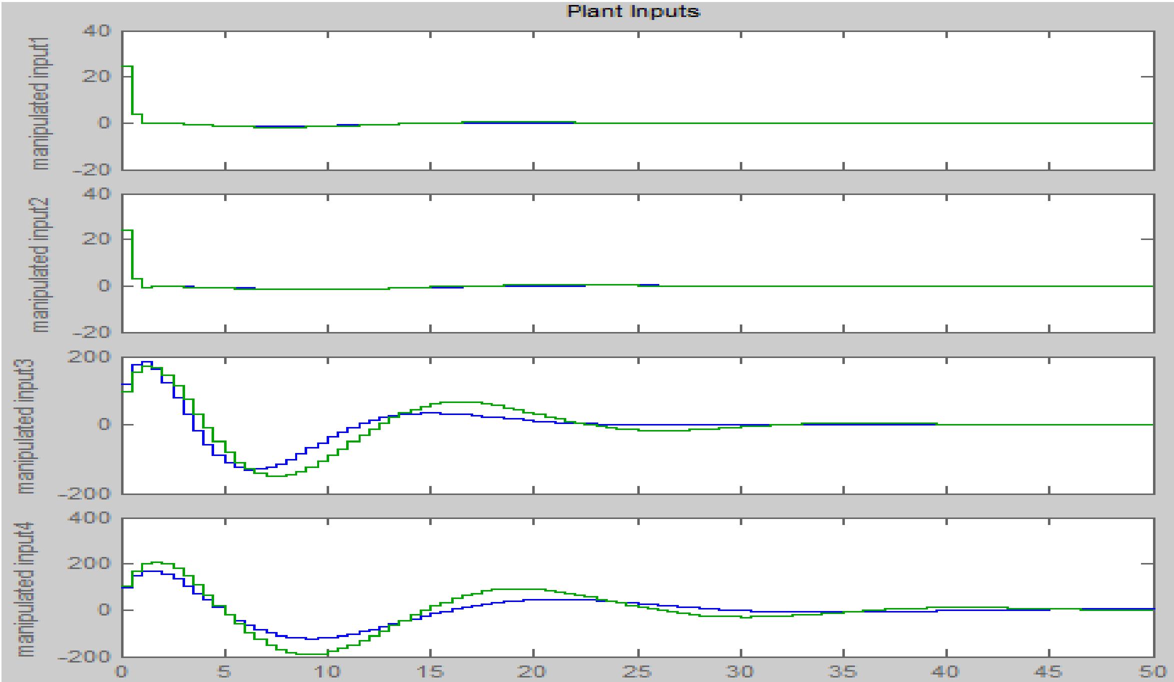

Fig. 9. Tuning of manipulated input

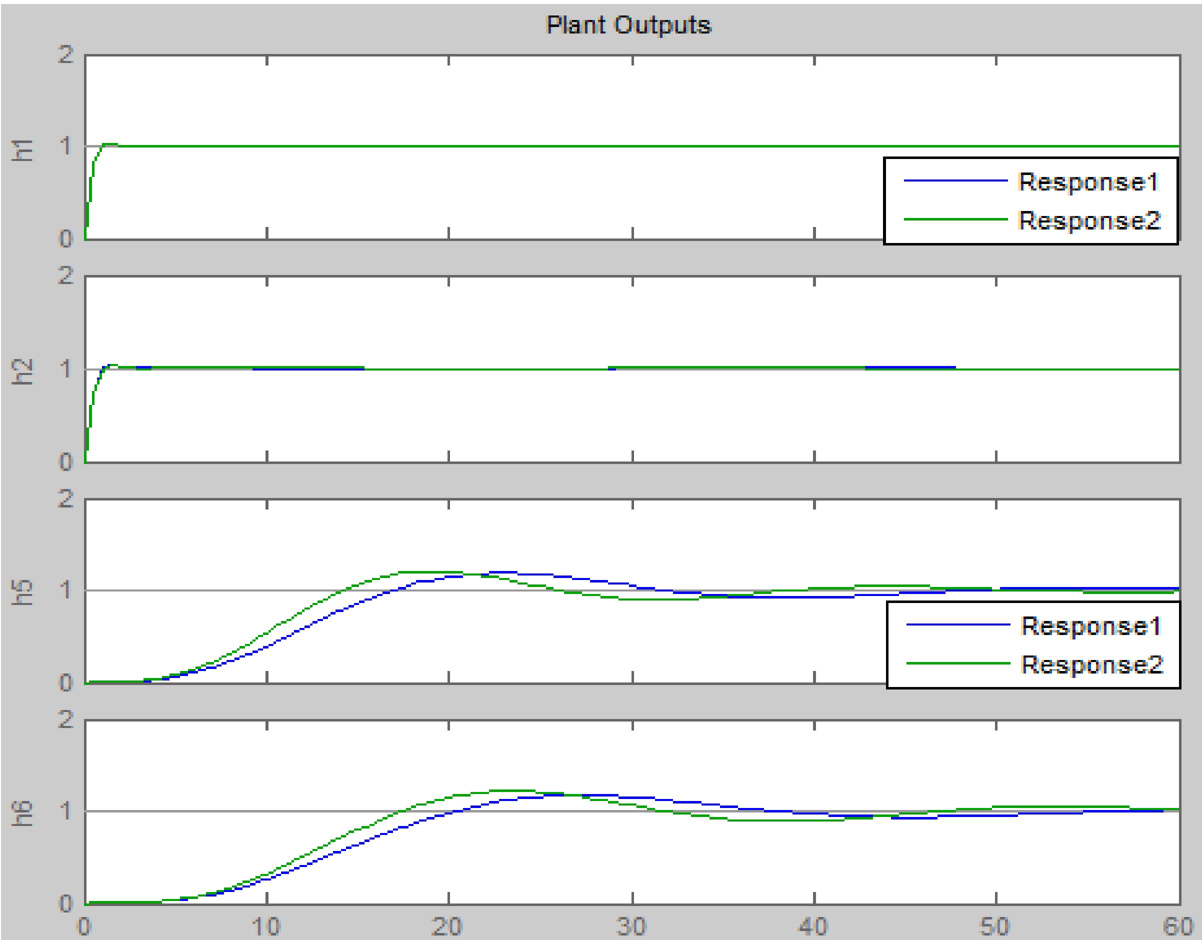

Fig.10. Response of lower two tanks h1, h2, h5 and h6 of step input for non minimum phase

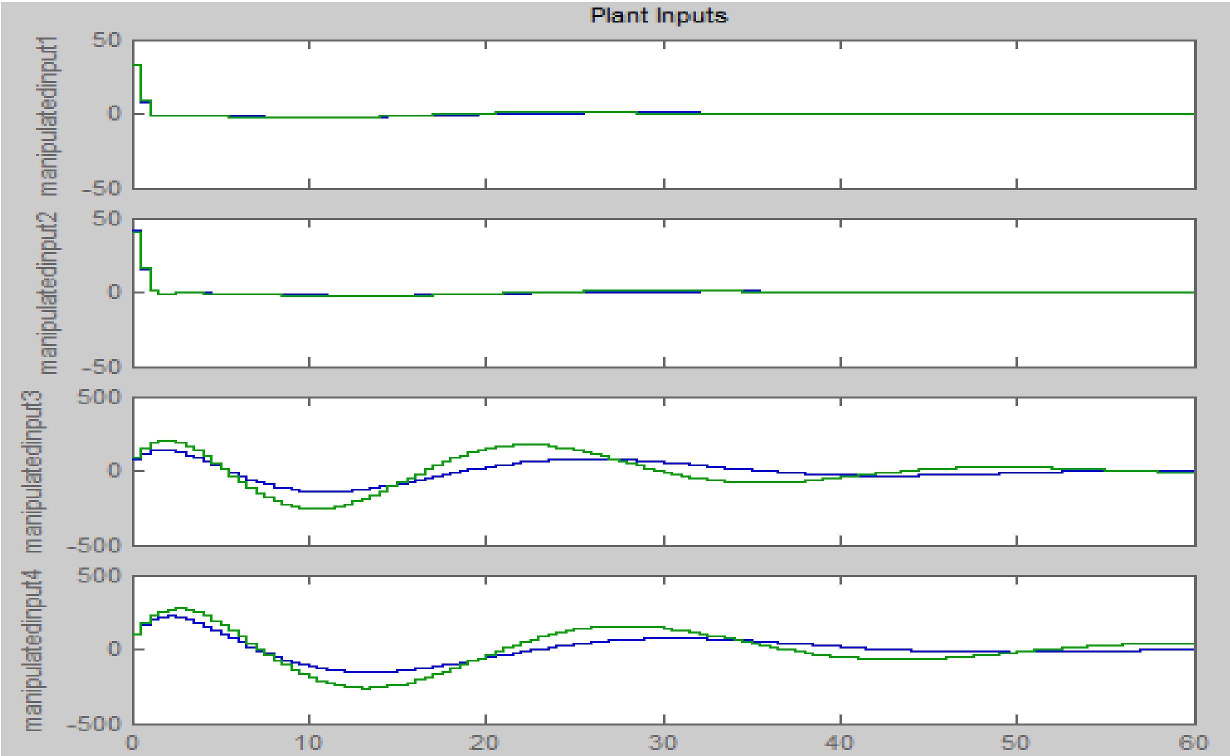

Fig.11. Tuning of manipulated input

VIII. CONCLUSION

This work provides elaborate tuning methods for octal Tank Process through Nonlinear Model Predictive Control with several conditions and constraints. We generated different responses for linear and nonlinear models with minute deviations for obtaining the steady state conditions; also stable output for nonlinear model of OTP is obtained.

The 2 x 1 matrix Quadruple Tank Process (QTP) also obtained stable response for four water tanks, at the same time tank h5, h6 has taken more time to reach the steady state. All the possible stability conditions are verified based on control and predicted horizon. Proposed new tuning methods for NMPC, provides stable response.

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