

Modulo- p Addition Based Constant Weight Variable Length Prime Codes for SAC-OCDMA Systems

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Abstract - In this paper, a novel prime code family called constant weight and variable length prime code (CWVLP) is presented. The CWVLP prime code is constructed from the basic prime codes which are generated using modulo- p addition. Since the proposed code family possesses very good correlation property (cross correlation equal to zero), the system eliminates the phase induced intensity noise (PIIN) and multi user interference (MUI) completely. In addition, for any given prime number p , we can generate p number of code families with different length. Further, the weight of the proposed code is kept always equal to one independent of the prime number p and the p number of code families; thus it can simplify the design of encoder/decoder structures of the proposed SAC-OCDMA system. Also, bit error rate performance of the developed code is much better than the commonly used MQC code. To illustrate, for $p=17$ and an acceptable bit error rate of 10^{-9} , the proposed coding system supports 225 simultaneous users with code length of 289 at an effective source power of -5 dBm.

Keyword - optical code-division multiple-access, spectral amplitude-coding (SAC), constant weight variable length (CWVLP) prime codes, modified quadratic congruence (MQC) codes.

I. INTRODUCTION

The performance of any communication system depends basically on the available band width, signal to noise ratio of the received signal and the codes used to relate the original information to the transmitted signal. Numbers of multiple access techniques are available for effective utilization of bandwidth and to increase the throughput of the communication system. Optical code-division multiple-access (OCDMA) is one robust multiple access technique that takes advantage of abundant bandwidth, low attenuation and no need for frequency and time management as all users transmit at the same time [1][2].

Optical CDMA systems can be broadly classified into coherent and incoherent systems, depending on the modulation and detection techniques deployed. Incoherent systems use the simple and the standard intensity modulation with direct detection technique, while coherent systems are based on the modulation and detection of optical phase component.

The most commonly used spreading techniques for the proposed SAC-OCDMA systems are based on the different types of optical sources (coherent / incoherent, narrowband / broadband), detection schemes (coherent / incoherent) and coding techniques (time, wavelength, amplitude and phase). From the choices made, coding schemes can be generally classified into six types, viz, pulse-amplitude-coding, pulse-phase-coding, spectral-amplitude-coding, spectral-phase-coding, spatial coding and wavelength-hopping time spreading coding [3]. In the first two techniques, coding is performed in the temporal domain. Pulse-amplitude-coding technique is based on incoherent processing with incoherent optical sources. Though this scheme is easier to implement, it requires the use of unipolar codes. Pulse-phase-coding technique uses phase modulators for introducing phase shift to pulses in a code sequence. This scheme allows the use of bipolar codes with close to zero cross correlation functions and thus reduces the MUI. Nevertheless, the requirement of ultrashort pulses limits the performance of the time domain techniques.

In spectral-amplitude-coding and spectral-phase-coding, coding is performed in the wavelength domain. In these techniques, an ultrashort pulse is first dispersed into multiple wavelengths. Spreading of spectrum is achieved by a grating. Spectral coding is then performed by passing spectral components through a phase or amplitude mask. Finally, the coded spectral components are recombined by another grating to form a code sequence. Spectral spreading is different from the temporal nature of the data, so that code length is independent of data rate. Spectral spreading based systems are advantageous over temporal spreading systems by avoiding the limitations of unipolar codes. Spatial coding and wavelength coding require the use of multiple fibers with 2D optical codes in the time and space domain or time and wavelength domain respectively. In this work, we consider incoherent synchronous one dimensional spectral amplitude coding optical CDMA system

for our analysis.

Since optical CDMA system is a multiple access spread spectrum technique, it allows multiple users to share a single bandwidth. Thus, for effective utilization of bandwidth and for increasing the number of simultaneous users, the OCDMA system requires the spreading codes with excellent correlation properties. Lower the cross correlation value better will be system performance. In the literature, different types of codes have been proposed for OCDMA systems with cross correlation value equal to one (MQC, MFH, MDW, EMPSC) [4-7], less than one (PMP) [8] or zero (ZCC) [9]. The above said codes suffer from various limitations in one way or another. The limitations of zero cross-correlation code shown in [9] are longer code length and requirement of higher number of mapping steps to achieve the particular code size for the given weight. Therefore, in order to simplify the code construction procedure and to improve the overall bit error rate performance, in this work a novel prime code family with zero cross correlation property is proposed using a different approach. The important feature of the proposed code is that the weight of the code is always equal to unity irrespective of the code size and hence assuring the simplified encoder and decoder structures. Further, these codes are derived from the basic prime codes which are generated from the Galois field using modulo- p addition while modulo- p multiplication is generally used in most of the other code families.

The rest of this paper is organized as follows. Code construction procedure is discussed in section II. In section III, we describe the basics of SAC-OCDMA system with block diagram. The performance analysis of the proposed code is carried out in section IV. Section V deals with the numerical results obtained from the analysis of the proposed system. Finally, concluding remarks are given in section VI.

II. CODE CONSTRUCTION

Prime codes are normally constructed based on the finite field arithmetic. Finite fields are also called Galois fields, in honor of their discoverer, Evariste Galois, a French mathematical prodigy [10]. For any prime number p , there exists a finite field of p elements. The finite field is denoted by $GF(p) = \{0, 1, \dots, p-1\}$. In this proposed work, Galois field with modulo- p addition is used to generate the basic prime code sequences. Then, the constant weight variable length prime code (CWVLPC) family is derived from the basic prime code sequences where p is the prime number.

The steps employed to construct the CWVLPC family are described below.

Step 1:

Consider the code construction procedure for $p=5$

Since the proposed codes are the families of prime code sequences, first generate the basic prime sequences using modulo- p addition. Elements of a basic prime sequence is generated by adding each element in the Galois field $GF(p) = \{0, 1, \dots, p-1\}$ by a preset number chosen from $GF(p)$. Thus, there are p prime sequences. The prime code sequences for $p = 5$ is shown in Table I.

TABLE I
Basic Prime Code Sequences

\oplus	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

The basic prime code sequences from the table are represented by 01234, 12340, 23401, 34012 and 40123. But, in this paper in order to generate codes with variable length, the diagonal elements are considered as the prime sequences. Therefore, the prime sequences are represented by 4, 30, 241, 1302, 02413. These five sequences are considered as the source sequences for the proposed code families. Therefore, the CWVL basic prime code sequences are given by

$$VLPC_{M=1, W=1} = [4]$$

$$VLPC_{M=2, W=1} = [30]$$

$$VLPC_{M=3, W=1} = [241]$$

$$VLPC_{M=4, W=1} = [1302]$$

$$VLPC_{M=5, W=1} = [02413]$$

where $VLPC_{M=1,W=1}$, $VLPC_{M=2,W=1}$, \dots , represent the different code families with weight (W) one and length p, 2p, and so on.

Step 2:

The second step is the generation of constant weight variable length prime code sequences. The CWVL prime code sequences can be generated by time shifting the basic source sequences. By time shifting the prime sequence, we can increase the number of prime sequences present in each and every family except the first code family.

$$VLPC_{M=1, W=1} = [4]$$

$$VLPC_{M=2, W=1} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$VLPC_{M=3, W=1} = \begin{bmatrix} 2 & 4 & 1 \\ 1 & 2 & 4 \\ 4 & 1 & 2 \end{bmatrix}$$

$$VLPC_{M=4, W=1} = \begin{bmatrix} 1 & 3 & 0 & 2 \\ 2 & 1 & 3 & 0 \\ 0 & 2 & 1 & 3 \\ 3 & 0 & 2 & 1 \end{bmatrix}$$

$$VLPC_{M=5, W=1} = \begin{bmatrix} 0 & 2 & 4 & 1 & 3 \\ 3 & 0 & 2 & 4 & 1 \\ 1 & 3 & 0 & 2 & 4 \\ 4 & 1 & 3 & 0 & 2 \\ 2 & 4 & 1 & 3 & 0 \end{bmatrix}$$

After time shifting the prime code sequences, the first code family supports one number of sequence, second, third, fourth and fifth code group's supports two, three, four and five numbers of sequences respectively.

Step 3:

In order to increase the number of simultaneous users supported by the system, a spreading technique is employed in this work. For that first we generate 1 number of size 1x1 matrix (it is actually a constant) with X (don't care) for the first code family. Similarly, we have to generate don't care matrices for the remaining code families.

For $VLPC_{M=1, W=1}$ code family,

$$VLPC_{M=1, W=1} = [X]$$

For $VLPC_{M=2, W=1}$ code family,

$$VLPC_{M=2, W=1} = \begin{bmatrix} X & X \\ X & X \\ X & X \\ X & X \end{bmatrix}$$

For $VLPC_{M=3, W=1}$ code family,

$$VLPC_{M=3, W=1} = \begin{bmatrix} X & X & X \\ X & X & X \end{bmatrix}$$

For $VLPC_{M=4, W=1}$ code family,

$$VLPC_{M=3,W=1} = \begin{bmatrix} 2 X X \\ X 4 X \\ X X 1 \\ 1 X X \\ X 2 X \\ X X 4 \\ 4 X X \\ X 1 X \\ X X 2 \end{bmatrix}$$

$$VLPC_{M=4,W=1} = \begin{bmatrix} 1 X X X \\ X 3 X X \\ X X 0 X \\ X X X 2 \\ 2 X X X \\ X 1 X X \\ X X 3 X \\ X X X 0 \\ 0 X X X \\ X 2 X X \\ X X 1 X \\ X X X 3 \\ 3 X X X \\ X 0 X X \\ X X 2 X \\ X X X 1 \end{bmatrix}$$

$$VLPC_{M=5,W=1} = \begin{bmatrix} 0XXXX \\ X2XXX \\ XX4XX \\ XXX1X \\ XXXX3 \\ 3XXXX \\ X0XXX \\ XX2XX \\ XXX4X \\ XXXX1 \\ 1XXXX \\ X3XXX \\ XX0XX \\ XXX2X \\ XXXX4 \\ 4XXXX \\ X1XXX \\ XX3XX \\ XXX0X \\ XXXX2 \\ 2XXXX \\ X4XXX \\ XX1XX \\ XXX3X \\ XXXX0 \end{bmatrix}$$

Step 5:

Now, the CWVL prime codes can be generated by placing '1' at the pth position and with 'p-1' zeros and replacing don't care terms (X) with 'p' zeros in each row.

$$VLPC_{M=1,W=1} = [00001]$$

$$VLPC_{M=2,W=1} = \begin{bmatrix} 00010 & 00000 \\ 00000 & 10000 \\ 10000 & 00000 \\ 00000 & 00010 \end{bmatrix}$$

$$VLPC_{M=3,W=1} = \begin{bmatrix} 00100 & 00000 & 00000 \\ 00000 & 00001 & 00000 \\ 00000 & 00000 & 01000 \\ 01000 & 00000 & 00000 \\ 00000 & 00100 & 00000 \\ 00000 & 00000 & 00001 \\ 00001 & 00000 & 00000 \\ 00000 & 01000 & 00000 \\ 00000 & 00000 & 00100 \end{bmatrix}$$

$$VLPC_{M=4,W=1} = \begin{bmatrix} 01000 & 00000 & 00000 & 00000 \\ 00000 & 00010 & 00000 & 00000 \\ 00000 & 00000 & 10000 & 00000 \\ 00000 & 00000 & 00000 & 00100 \\ 00100 & 00000 & 00000 & 00000 \\ 00000 & 01000 & 00000 & 00000 \\ 00000 & 00000 & 00010 & 00000 \\ 00000 & 00000 & 00000 & 10000 \\ 10000 & 00000 & 00000 & 00000 \\ 00000 & 00100 & 00000 & 00000 \\ 00000 & 00000 & 01000 & 00000 \\ 00000 & 00000 & 00000 & 00010 \\ 00010 & 00000 & 00000 & 00000 \\ 00000 & 10000 & 00000 & 00000 \\ 00000 & 00000 & 00100 & 00000 \\ 00000 & 00000 & 00000 & 01000 \end{bmatrix}$$

$$VLPC_{M=5,W=1} = \begin{bmatrix} 10000 & 00000 & 00000 & 00000 & 00000 \\ 00000 & 00100 & 00000 & 00000 & 00000 \\ 00000 & 00000 & 00001 & 00000 & 00000 \\ 00000 & 00000 & 00000 & 01000 & 00000 \\ 00000 & 00000 & 00000 & 00000 & 00010 \\ 00010 & 00000 & 00000 & 00000 & 00000 \\ 00000 & 10000 & 00000 & 00000 & 00000 \\ 00000 & 00000 & 00100 & 00000 & 00000 \\ 00000 & 00000 & 00000 & 00001 & 00000 \\ 00000 & 00000 & 00000 & 00000 & 01000 \\ 01000 & 00000 & 00000 & 00000 & 00000 \\ 00000 & 00010 & 00000 & 00000 & 00000 \\ 00000 & 00000 & 10000 & 00000 & 00000 \\ 00000 & 00000 & 00000 & 00100 & 00000 \\ 00000 & 00000 & 00000 & 00001 & 00000 \\ 00000 & 00000 & 00000 & 00000 & 01000 \\ 01000 & 00000 & 00000 & 00000 & 00000 \\ 00000 & 00010 & 00000 & 00000 & 00000 \\ 00000 & 00000 & 10000 & 00000 & 00000 \\ 00000 & 00000 & 00000 & 00100 & 00000 \\ 00000 & 00000 & 00000 & 10000 & 00000 \\ 00000 & 00000 & 00000 & 00000 & 00100 \\ 00100 & 00000 & 00000 & 00000 & 00000 \\ 00000 & 00001 & 00000 & 00000 & 00000 \\ 00000 & 00000 & 01000 & 00000 & 00000 \\ 00000 & 00000 & 00000 & 00010 & 00000 \\ 00000 & 00000 & 00000 & 00000 & 10000 \end{bmatrix}$$

Number of rows and number of columns present in the above code set represent code size ($K=M^2$) and code length ($N=M*p$) of the code respectively. Therefore, from the prime number p we can generate $1, 2, \dots, p$ numbers of code families with size K and length N . However, only the p^{th} code family (shorter length code family) generates p^2 number of users with the length p^2 . The lower order code families have the code size which is slightly less than that of the code length. Thus, for analysis, we have considered only the odd numbers of code families.

The relationship between the cardinality of the code and code length of the code is given by

$$K = M^2 \tag{1}$$

$$N = M * p \tag{2}$$

where $M = 1, 2, 3, \dots, p$ number of code families and p is the prime number.

Lowered code weight of the proposed code reduces the power consumption and the complexity of the encoder / decoder structure. However, the drawback of the proposed code lies in that code selection depends only on the prime number p .

III. SYSTEM DESCRIPTION

The block diagram of the proposed SAC OCDMA system with fiber Bragg grating is shown in Fig.1, which consists of optical broad band source, spectral encoder, spectral decoder, star coupler and splitter circuits. The information being transmitted in the optical fiber communication system is in the form of digital. The optical broad band source emits an optical pulse if the information bit is '1' and it emits no pulse if the bit is '0'.

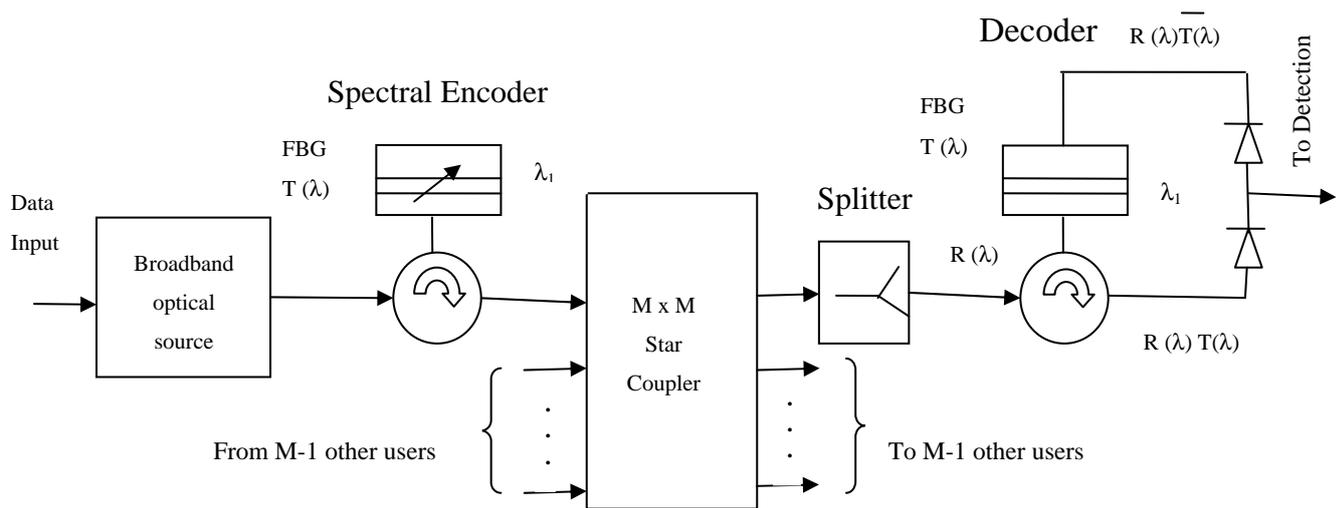


Fig.1. SAC optical CDMA system using CWVL prime code

The optical signal which is emitted by the source is then encoded into N spectral components according to the 'N' elements of the code used in the encoder [4] [7]. Since the weight of the proposed CWVL prime code is one, it requires only one fiber Bragg grating, thus, simplifying the encoder structure. The star coupler combines the encoded signal from all the 'M' number of users. At receiver, the received optical signal is divided into two parts by a splitter and then these signals are passed through a decoder. The decoder filters out the unwanted spectral components and passes the desired spectral components. The filtered signals are then passed through a detector followed by further electrical detection process.

IV. PERFORMANCE ANALYSIS

In general, the performance analysis of a SAC-OCDMA system is carried out by considering the phase induced intensity noise of source and the noises which occur at the receiver such as thermal noise and shot noise. In this work, we have proposed a CWVL prime code with zero cross correlation value. Since the cross correlation of the proposed code is always zero, it is assumed that the intensity noises are completely eliminated. Hence, the bit error rate performance of the system with the proposed code is analyzed by considering only the thermal and shot noise.

Therefore, an expression for the signal to noise ratio and the bit error rate performance of the system

can be written as [9]

$$SNR = \frac{\frac{R^2 P_{sr}^2 w^2}{N^2}}{2eBP_{sr}R\frac{wK}{N} + \frac{4K_B T_n B}{R_L}} \quad (3)$$

where P_{sr} is the effective source power at the receiver in watts. The bit error rate can be estimated using

$$BER = \frac{1}{2} \operatorname{erfc} \sqrt{\frac{SNR}{8}} \quad (4)$$

where erfc is the complementary error function. The numerator term in equation (3) represents the signal power and the denominator term denotes the effect of shot noise and thermal noise power respectively. The assumed system parameters are listed in Table II.

TABLE II
Parameters used for the Numerical Calculation

Parameters	Notations	Numerical Values
Responsivity of the photodiode	R	$(\eta e) / (hf_c) = 0.7495$
Plank's constant	H	6.626×10^{-34} Js
Quantum efficiency	H	0.6
Electronic charge	E	1.602×10^{-19} Column
Electrical bandwidth	B	80 MHz
Boltzmann's constant	K_B	1.379×10^{-23}
Load resistance	R_L	1030 Ω
Noise Temperature	T_n	300 K
Operating wavelength	λ	1550 nm

V. RESULTS AND DISCUSSION

Fig.2 shows the bit error rate performance of the proposed code and MQC for $p = 11$. For the given p , the proposed code supports p numbers of code families with code length Mp (M code family number) but MQC code supports only one code family with the code length p^2+p (for $p=11$; $N=132$). When $M=11$, the CWVL prime code supports 121 number of users with length 121 and MQC code supports the same 121 number of users with the length 132. Though the length of the proposed lower order code families (for $M=7$; $K=49$, $N=77$ and for $M=9$; $K=81$, $N=99$) is longer, the length of the higher order (for $M=11$; $K=121$, $N=121$) code family is slightly shorter than the MQC code. Further, the code weight of the proposed code is one independent of the code size. Therefore the bit error rate performance of the proposed code is much better than the MQC code. Further, for the given bit error rate of 10^{-9} the code supports 115 numbers of simultaneous users and this number is only 80 in MQC code.

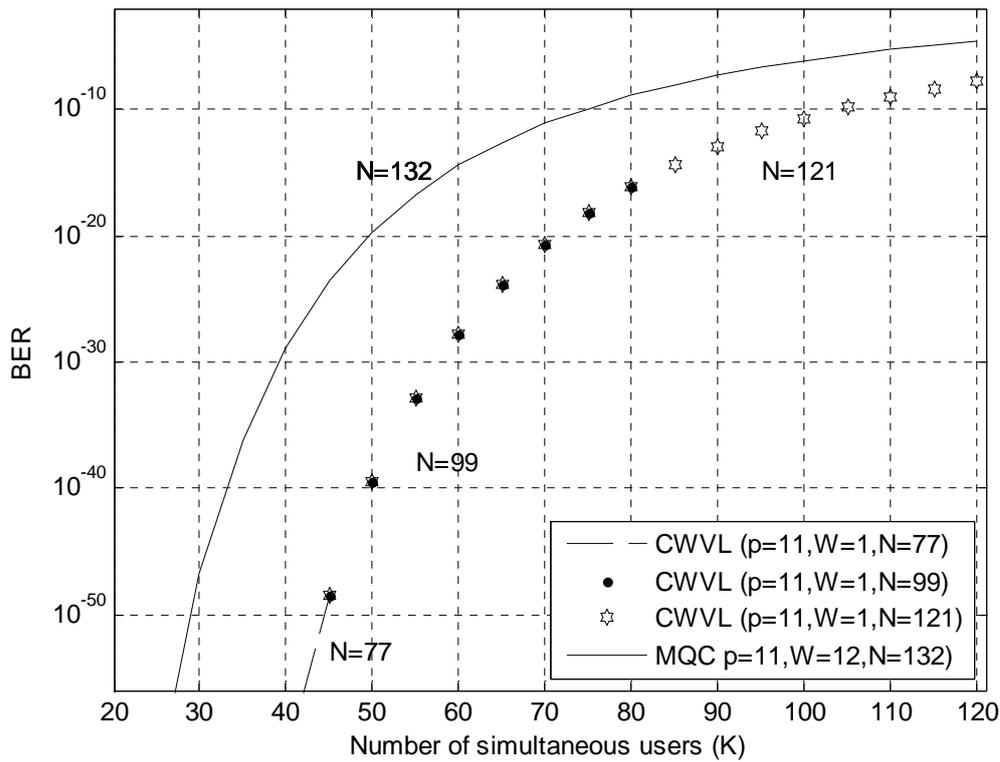


Fig.2.Number of simultaneous users versus BER for MQC and CWVLPC with the effective source power equal to -10dBm and for p=11.

Fig.3 and Fig. 4 shows that the bit error rate performance of the proposed code for $p = 17$ with effective source power equal to -10dBm and -5dBm respectively. From the figure 3 and 4 it is observed that for the sufficient amount of source power the system with the proposed CWVL prime code supports higher number of simultaneous users for the given bit error rate of 10^{-9} . When the effective source power is equal to -10dBm, the proposed code supports 110 numbers of simultaneous users and this number is slightly greater than the MQC code. If we use the sufficient amount of power, say $P_{sdBm} = -5dBm$, then the system with the proposed code supports 225 number of simultaneous users and it is only 100 in the MQC code. Since, the weight of the proposed code is always one independent of the order of the family, the proposed code supports higher number of users for the given sufficient amount of power.

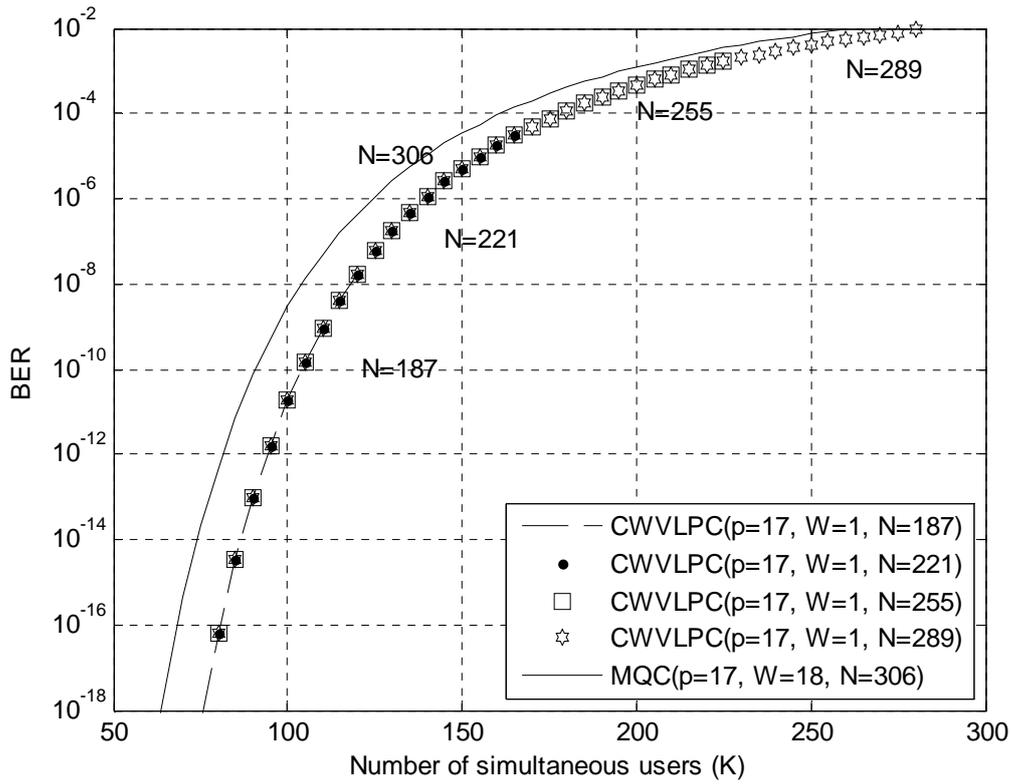


Fig.3. Number of simultaneous users versus BER for MQC and CWVLPC with the effective source power equal to -10dBm .

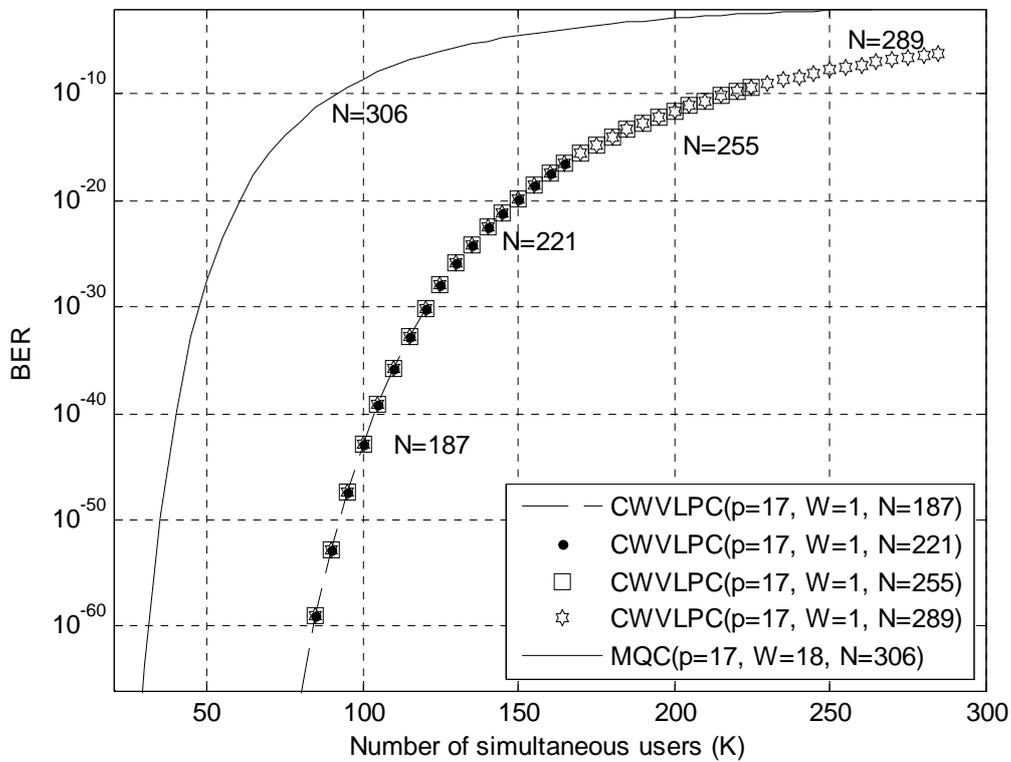


Fig.4. Number of simultaneous users versus BER for MQC and CWVLPC for the effective source power -5dBm.

Fig.5 shows the number of simultaneous users of the proposed system versus BER with the effective source power equal to -10dBm. The BER performance of the proposed code is compared with the performance of the system with MQC code for $p=7$. In Fig.5 the performance is analyzed for the electrical bandwidth of 311 MHz and the data transmission rate is 622 Mb/s. Lower the p value supports higher transmission rate and also it supports higher number of simultaneous users for the given bit error rate of 10^{-9} than the system with MQC code.

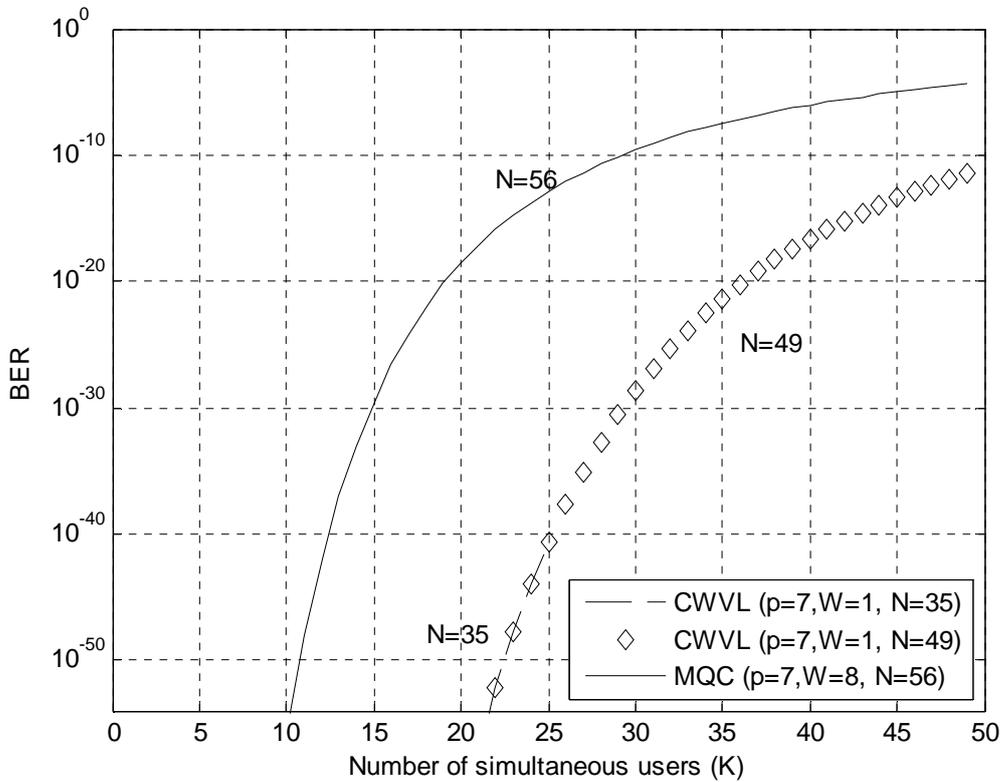


Fig.5. Number of simultaneous users versus BER for MQC and CWVLPC with the effective source power equal to -10dBm at a transmission rate of 622Mb/s.

Fig. 6 compares the effective source power and the BER performance of the system with CWVLPC and MQC code. The bit error rate performance of the system with the MQC code is better only for the lower source power and it is constant for the higher values. But, for the sufficient amount of power and for the given number of simultaneous users the system with the proposed CWVL prime code shows better performance than the MQC codes.

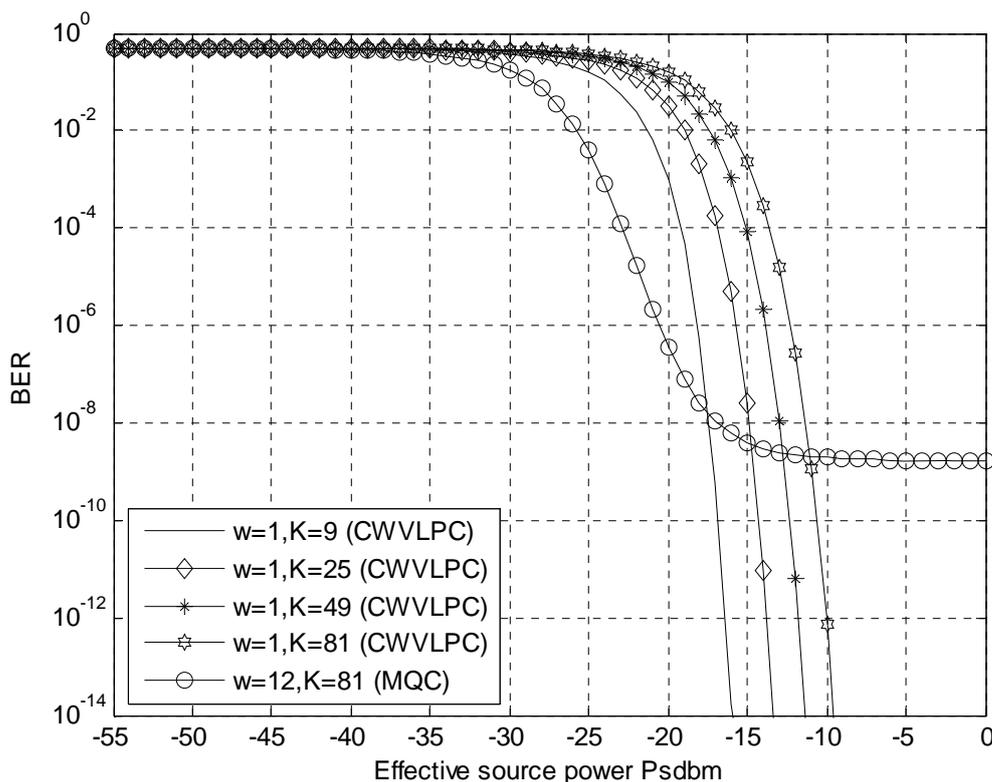


Fig.6. Effective source power versus BER for MQC code and CWVL prime code

VI. CONCLUSION

In this work, we have proposed a new simple modulo- p addition based constant weight variable length prime code family. Since the cross correlation value of the proposed CWVL prime code is zero always, the code completely eliminates the MUI and PIIN which are the two major degrading performance parameters in the SAC-OCDMA system. Another advantage is that the code weight of the code is always one independent of the prime number. Hence, it simplifies the encoder structure design of the system. In addition, for a given prime number, p we can generate p number of code families with different length but with code weight equal to one. Further, the bit error rate performance of the system is much better than the commonly used MQC code. Finally, the number of simultaneous users supported by the system with the proposed code for the given bit error rate of 10^{-9} is also much higher than MQC.

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