Experimental results for optimal placement of piezoelectric plates for active vibration control of a cantilever beam

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Abstract—**The fatigue phenomena correlated to the gas turbine blades vibrations can lead to catastrophic failure. To damp the vibrations amplitude typically damping passive systems are used. In the last years the interest in the piezoelectric materials, and their use as damping elements, has received considerable attention by many researchers. Recently different research groups have started to study their use in blades of turbomachinery. Because of their effectiveness strongly depends on their position, some of the authors have proposed ([15], [17]) a new model to find the optimal position to control the multimode vibrations. Such model has been corroborated by experimental results for different combinations of excited eigenmodes ([16], [18]]). In this paper the authors present new experimental results with the aim to increase the knowledge of the optimal position of the piezoelectric plates when different eigenmodes are involved.**

TABLE I
Nomenclature

a	axis position of the centre of the piezo plates	r	percentage coupling coefficient
$\mathbf{B}$	vector control	S	transversal area of the beam
c	beam width	T_a	piezoelectric thickness
d_{31}	piezoelectric coefficient	T_b	beam thickness
E_a	Young's modulus of the piezoelectric material	V	voltage applied to the piezoelectric plates
E_b	Young's modulus of the beam	w	vertical displacement
h	piezo plates length	$\tilde{w}$	virtual vertical displacement
a	axis position of the centre of the piezo plates	α	damping coefficient
I_b	inertia moment of the beam	ρ	density
L_b	beam length	ω_i	natural frequency
M_a	piezoelectric bending moment	S	transversal area of the beam

I. Introduction

The reduction of the blade's life, due to the fatigue phenomena, is a problem of great interest ([1]-[4]). Typically, passive damping systems are used but these, differently from the active damping, are not able to change their configuration depending on the applied loads. In the last decades the use of piezoelectric materials, as elements for active damping, has received increasing interest ([5]-[]6), and in the last years also for the blades of turbomachinery ([7]-[14]). Because of their effectiveness strongly depends on their configurations, some of the authors, have proposed a new model for the optimal placement of piezoelectric plates to control multimode vibrations ([15]). To test the proposed model an experimental apparatus, with a cantilever fixed beam, has been built and some results have been shown in a previous paper ([16]). The model has been also extended to a rotating beam ([17]) and a different experimental system has been built and tested ([18]). In this paper new results for a cantilever fixed beam will be shown.

II. Governing Equations for Piezoelectric Coupled Beam

In Fig. 1 an Euler-Bernouilli beam has been represented

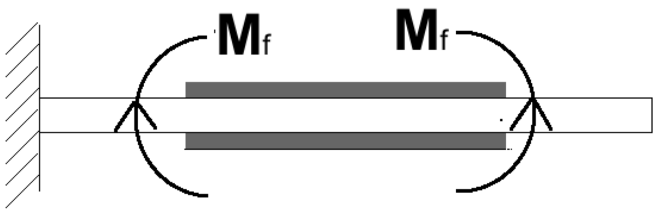

Fig. 1 Reference configurations and flexural moments applied by piezoelectric plates (Pin Force Model)

By the Pin Force Model ([5]) the action of the piezoelectric plates can be summarized by two flexural moments applied at the end of the plates (Fig. 1).

The equilibrium equations can be derived by the principle of the virtual work:

$$\delta L_e + \delta L_{in} + \delta L_a = 0 \tag{1}$$

where δL_e, δL_{in}, δL_a are, respectively, the virtual work of the elastic, inertial and piezoelectric forces.

Indicating with w the vertical displacement and with M_a the action of the piezoelectric elements, the different contributions can be written as:

$$\delta L_e = E_b I_b \int_0^{L_b} \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 \tilde{w}}{\partial x^2} dx \tag{2}$$

$$\delta L_{in} = -\rho S \int_0^{L_b} \frac{\partial^2 w}{\partial t^2} \tilde{w}\, dx \tag{3}$$

$$\delta L_a = M_a \left(\left. \frac{\partial \tilde{w}}{\partial x} \right|_{x=a+\frac{h}{2}} - \left. \frac{\partial \tilde{w}}{\partial x} \right|_{x=a-\frac{h}{2}} \right) \tag{4}$$

where:

$$M_a(t) = \frac{\psi}{6+\psi} E_a c T_a T_b \Lambda(t) \tag{5}$$

with:

$$\begin{cases} \Lambda(t) = \dfrac{d_{31}}{T_a} V(t) \\ \psi = \dfrac{E_b T_b}{E_a T_a} \end{cases} \tag{6}$$

Indicating with $\phi_i(x)$ is the *i*-th flexural modal displacement of the cantilever beam and $X_i(t)$ its amplitude, the vertical displacement can be approximated by:

$$w(x,t) = \sum_{i=1}^{N} X_i(t)\phi_i(x) \tag{7}$$

and the equilibrium equations (1) becomes:

$$\mathbf{M}\ddot{\mathbf{X}}(t) + \mathbf{K}\mathbf{X}(t) = \mathbf{B}V(t) \tag{8}$$

where **M, K** and **X** are, respectively, the mass matrix, the stiffness matrix and the amplitude mode vector. **B** is the vector control:

$$\begin{aligned} \mathbf{B} = \tilde{M}_a \Big[& \phi_1'\left(a+\frac{h}{2}\right) - \phi_1'\left(a-\frac{h}{2}\right), \phi_2'\left(a+\frac{h}{2}\right) + \\ & -\phi_2'\left(a-\frac{h}{2}\right), \ldots, \phi_N'\left(a+\frac{h}{2}\right) - \phi_N'\left(a-\frac{h}{2}\right) \Big] \end{aligned} \tag{9}$$

with $\tilde{M}_a = \dfrac{\psi}{6+\psi} E_a c T_a T_b \dfrac{d_{31}}{T_a}$.

If the Rayleigh damping model, with β=0: $\mathbf{C} = \alpha\mathbf{M}$, has applied the (8) becomes:

$$\mathbf{M}\,\mathbf{X}(t) + \mathbf{C}\,\mathbf{X}(t) + \mathbf{K}\mathbf{X}(t) = \mathbf{B}V(t) \tag{10}$$

In real applications, e.g. the gas turbine blades, the applied load, because of its spectrum, can excite different eigenmodes. An efficient damping can be obtained applying another load with the same spectrum but opposite in sign. Because of the flexural moments, due to the piezoelectric elements, depends on V(t) ((5), (6)) it is possible to obtain a damping effect choosing appropriately its spectrum. Obviously, for a chosen spectrum, more is the induced flexure more is the capability of the active elements to damp the vibrations and, as is evident from (4), it depends on their position. For a single mode the optimal placement is known ([19]) but for a multimode vibrations a new strategy needs to be developed. In ([15]) some of the authors have proposed a theoretical model so that, to corroborate its theoretical previsions, an experimental apparatus has been built. Three different configurations will be examined and different spectrums of V(t). The effctiveness of the chosen configuration will be measured by the amplitude of the vibrations of the free end, so that the most effective configuration (Fig.1), will be that which maximizes this amplitude. Therefore given a general spectrum to the to the V(t) with components $V_i(t)$ at the frequency ω_i:

$$V(t) = \sum_{i=1}^{Ns} V_i cos(\omega_i t) \tag{11}$$

where N_s is the number of the excited modes. It is possible to obtain an approximate expression for the amplitude of the vibrations of the free end ([15]) with:

$$| w(a,h,L_b) | = \sum_{i=1}^{N_s} \left| \frac{B_i(a,h) V_i \phi_i(L_b)}{\alpha\omega_i} \right| \tag{12}$$

III. EXPERIMENTAL RESULTS

Two coupled modes have been considered, and the following voltage *V(t)* has been used:

$$V(t) = (1-r)cos(\omega_i t) + rcos(\omega_j t) \tag{13}$$

here the parameter *r* ($0 \leq r \leq 1$) indicates the ratio of the *j*-th component.

The procedure is completely analogously to that described in ([16]), three different configurations have been considered:

i. all four piezoelectric plates active:1-4
ii. fixed end piezoelectric plates active: 2 and 3
iii. free end piezoelectric plates active: 1 and 4

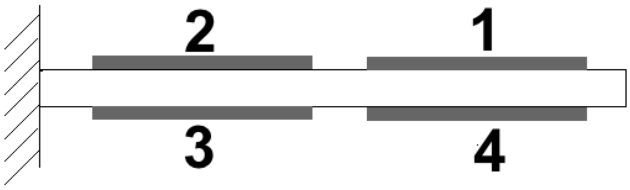

Fig. 2 PZT's plates configuration

The principal steps are: chosen two coupled modes, assigned a piezoelectric configuration and a value of *r*, the tip amplitude has been compared with that obtained with *r=0: w[0]/w[r]*. Analogously with the ([16]): w[0]/w[r] smaller than 1 implies that the chosen piezoelectric configuration is more efficient for the spectrum correspondent a $r \neq 0$. In [16] the results for the lower modes have been reported. In this paper we extend these results to the higher coupled modes and a completed spectrum range. In the Figs. 3-6 the results for various plates configuration and various coupled modes have been reported. As the tests conducted in [16] good results are maintained compared with the theoretical previsions.

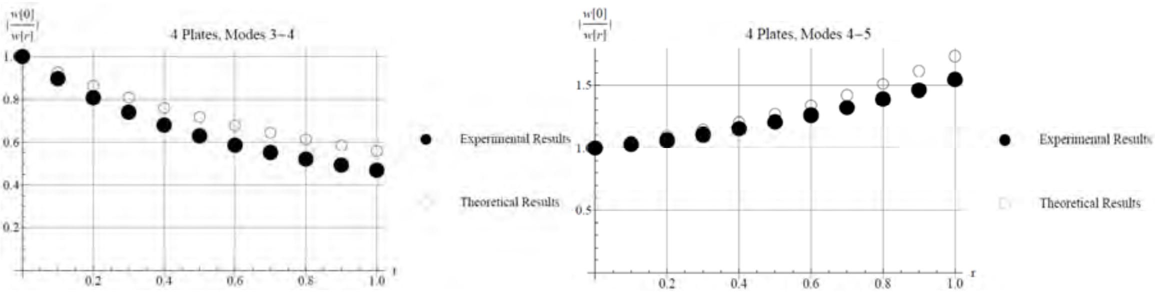

Fig. 3 Displacement ratios for four plates, coupled modes: 3-4 and 4-5

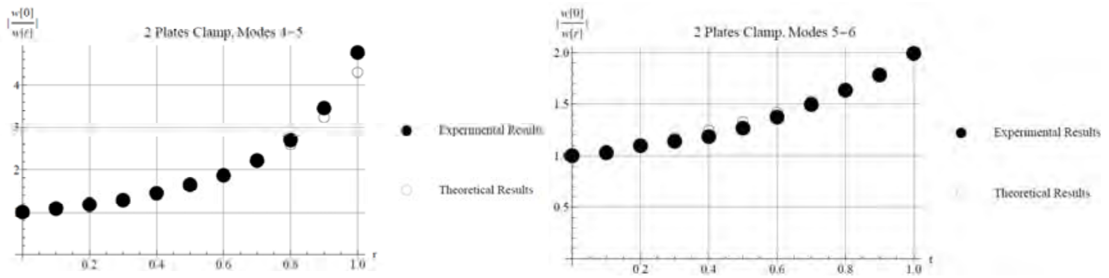

Fig. 4 Displacement ratios for two plates near the clamp: coupled modes: 4-5 and 5-6

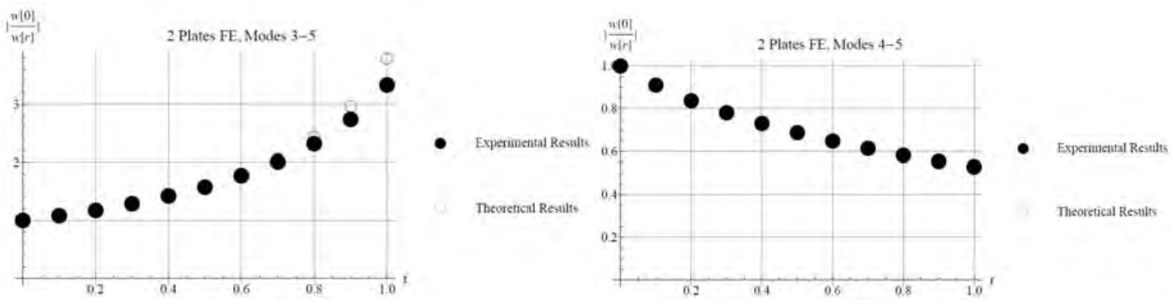

Fig. 5 Displacement ratios for two plates near the free end: coupled modes 3-5 and 4-5

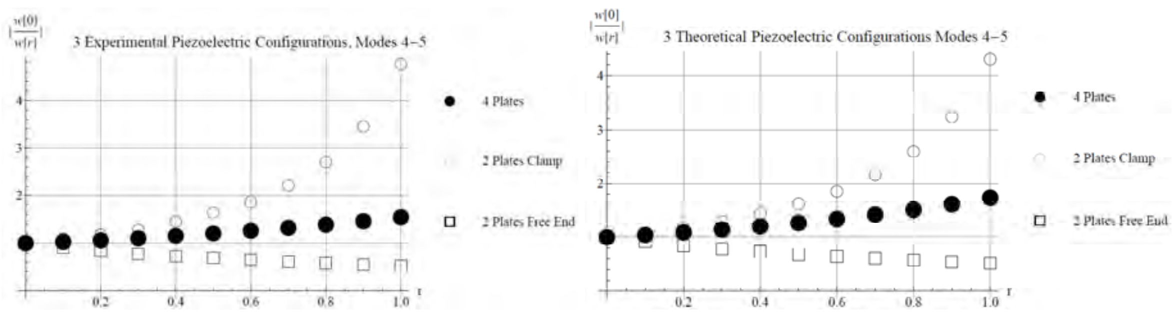

Fig. 5 Comparison between experimental and theoretical mode for: 4 plates, 2 plates near the clamp and 2 plates near the free end

IV. CONCLUSIONS AND FUTURE WORKS

In ([15], [17]) some of the authors have proposed a new theoretical model for the optimal placement of piezoelectric plates to control the multimodes vibrations. An experimental apparatus, to corroborate theoretical prevision, has been developed and the first tests have been performed for the lower coupled modes with a good agreement ([16]). In this paper the tests have been extended to higher modes and for a complete range spectrum. The new experimental analysis has strengthened the validity of the theoretical model proposed by the authors with a good correspondence between theoretical and experimental results. The research conducted has gathered a considerable amount of data and experimental knowledge which will be essential for future studies, laying out a foundation on which to build with the ultimate goal to reliably reduce the detrimental forces contributing to the material fatigue. Future works will focus in the studies about the realistic blades ([20]) and the response of the blades to the impulse load ([21]).

REFERENCES

[1] N.X. Hou, Z.X. Wen, Q.M. Yu, Z.F. Yue, “Application of a combined high and low cycle fatigue life model on life prediction of SC blade,” *International Journal of Fatigue* 31, pp. 616-619, 2009.

[2] L. Witek, “Experimental crack propagation and failure analysis of the first stage compressor blade subject to vibration,” *Engineering Failure Analysis* 16 (7) pp. 2163-2170, 2009.

[3] J. Kubiac Sz., B. Urquiza G., J. Garci C., F. Sierra E., “Failure analysis of steam turbine last stage blade tenon and shroud,” *Engineering Failure Analysis* vol. 14 (8) pp. 1476-1487, 2007.

[4] E. Poursaeidi, M. Salavatian, “Fatigue crack growth simulation in a generator fan blade,” *Engineering Failure Analysis* vol. 16, 888–898, 2009.

[5] E.F. Crawley, J. de Luis, J, “Use of piezoelectric actuators as elements of intelligent structures” *AIAA Journal* vol. 25, 1373-1385, 1987.

[6] F. Botta, G. Cerri, “Wave propagation in Reissner-Mindlin piezoelectric coupled cylinder with non-constant electric field through the thickness,” *International Journal of Solids and Structures*, vol. 44, Issues 18-19, pp. 6201-6219, September 2007.

[7] A. Hohl, M. Neubauer, S. M. Schwarzendahl , L. Panning, J. Wallaschek, “Active and semiactive Vibration Damping of Turbine Blades with Piezoceramics,” in *Proc. of SPIE*, 2009, Vol. 7288 72881H1-72881H10.

[8] I. Goltz, H. Bhmer, R. Nollau, J. Belz, B. Grueber, J. R. Seume, “Piezo-electric actuation of rotor blades in an axial compressor with Piezoceramics,” in *Proc. of 8th European Conference on Turbomachinery (ETC),* 23-27 March 2009, Graz, Austria.

[9] A. J. Provenza, C. R. Morrison, “Control of fan blade vibrations using piezoelectric and bi-directional telemetry,” in *Proc. of ASME Turbo Expo*, 2011, June 6-10, 2011, Vancouver, British Columbia, Canada

[10] S. M. Schwarzendahl, “On blade damping technology using passive piezoelectric dampers,” in *Proc. of ASME Turbo Expo*, 2012, June 11-15, 2012, Copenaghen, Denmark.

[11] B. Choi, J. Kauffman, K. Duffy, A. Provenza A., Morrison C., “Active Vibration Reduction of Titanium Alloy Fan Blades (FAN1) Using Piezoelectric Materials,” *Propulsion-Safety and Affordable Readiness (P SAR) Conference cosponsored by U.S. Army, Navy, and Air Force*, 2010, Jacksonville, Florida, March 16/18.

[12] B. Choi, C. Morrison, K. Duffy, “An Active Damping at Blade Resonances Using Piezoelectric Transducers,” *Propulsion-Safety and Affordable Readiness (PSAR) Conference cosponsored by the U.S. Army, Navy, and Air Force*, 2008, Myrtle Beach, South Carolina, March 18/ 20.

[13] C. P. Lawson, P. C. Ivey, “Turbomachinery blade vibration amplitude measurement through tip timing with capacitance tip clearance probes,” *Sensors and Actuators A* vol. 118, pp. 14–24, 2005.

[14] A. Hohl, M. Neubauer, S. M. Schwarzendahl, L. Panning, J. Wallaschek , “Active and semiactive Vibration Damping of Turbine Blades with Piezoceramics” in *Proc. SPIE*, 2009, 7288/ 72881H.

[15] F. Botta, Dini D., Schwingshackl C., di Mare L., Cerri G., “Optimal placement of piezoelectric plates to control multimode vibrations of a beam,” *Advances in Acoustics and Vibration*, vol. 2013 (in press).

[16] F. Botta, N. Marx, S. Gentili, C. W. Schwingshackl, L. Di Mare L., G. Cerri, D. Dini, “Optimal placement of piezoelectric plates for active vibration control of gas turbine blades: experimental results,” in Proc. of SPIE Vol. 8345, 2012, 83452H-183452H-11.

[17] F. Botta, D. Dini , de Lieto Vollaro R, “A new function for the optimal placement of piezoelectric plates to control multimode vibrations of a rotating beam,” *International Journal of Engineering and Technology*, vol. 5, issue 5, October 2013.

[18] F. Botta, N. Marx, C. W. Schwingshackl, G. Cerri, D. Dini, “A wireless vibration control technique for gas turbine blades using piezoelectric plates and contactless energy transfer,” in *Proc. of the ASME Turbo Expo*, 2013, June 3-7, San Antonio, Texas, USA.

[19] R. Barboni, A. Mannini, E. Fantini, P. Gaudenzi, “Optimal placement of PZT actuators for the control of beam dynamics” *Smart Mater. Struct.* Vol. 9, pp. 110–120, 2000.

[20] G. Cerri, M. Gazzino, F. Botta, C. Salvini, “Production Planning with Hot Section Life Prediction for Optimum Gas Turbine Management,” *International Journal of Gas Turbine, Propulsion and Power Systems*, vol. 2, no. 1, pp. 9-16, 2008.

[21] F. Botta, G. Cerri, “Shock response spectrum in plates under impulse loads,” *Journal of Sound and Vibration*, vol. 308, Issues 3-5, pp. 563-578, 4 December 2007.