A Review of Method in FDTD for the Analysis of Oblique Incident Plane Wave on Periodic Structures

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Abstract—**Thispaper introduce the comparison method in Finite Different Time-Domain (FDTD) for solving oblique incident plane wave at periodic structures. The method referred to as split-field transformation and constant transverse wavenumber (CTW) direct field transformation. In order to overcome the oblique incident where has the delay time in the transverse plane wave., the CTW tended to apply as the suitable method with the constant wavenumber in the transverse plane (x-direction) and by applying periodic boundary condition (PBC) can be directly implement in the time domain. The stability is same as conventional stability of FDTD where the time step not necessary to obtain stable result as the incident angle is increased. A set auxiliary variable (P and Q fields) is introduced for split-field to realize the discretization of transformed Maxwell's equation is difficult to derived equation for oblique incident because the formulation required extra memory during simulation. Split-field exhibit instability for the grazing incident affect from the stability criterion. The formulations for both methods are compared to authoritative solutions.**

FDTD, periodic structures, normal incident plane wave, oblique incident plane wave, PBC, split-field and CTW.

I. Introduction

Periodic structures presently consider discretizing the solution space within the unit cell using a traditional Yee lattice [1,2]. A unit cell of a plane wave incident can be enquiry by applying the periodic boundary condition (PBC). PBCs are formulated in the spectrum domain and that efficient executed by numerical technique frequency domain [3]. By applying the PBC in electromagnetic modeling, it can reduce the computational cost while the computational efficiency are improved and to prevent problems with boundary effects caused by finite size. The application of the PBC is straightforward for the case of normally incident plane waves since for the case of normally incident plane waves since the fields do not involvement any delay as they travel across the unit cell and it involve no phase shift between each periodic structure.

Typically, plane wave source propagating are not normally incident it also could be in oblique incident. Oblique incident will cause more difficult in implementation time domain because the variation phases of the cell to cell between corresponding points in different unit cells [4].

Thus, the phase difference leads in time advance or delay cannot be solved directly using time domain method. This delay requires knowledge of the future values of the fields at any step in the transverse plane. Based on problem, FDTD method difficult to simulate the oblique source because FDTD is time-domain method where simulation depends on the simultaneous time and space. The FDTD method belongs in the class of grid based differential time-domain Maxwell's equation for numerical modeling methods [5].

In order to solve this problem for modeling periodic structures, there are two type of method was introduced where it knows as field transformation method and direct field method. Field transformation required additional terms that required special handling in transformed field equation compared with the standard Maxwell's equation. The transformed equation used to remove the time gradient across the grid [6]. Split-field is the one of technique grouped in this method which split it to two components. Direct field method is quite contrast than the field transformation method. It directly works with Maxwell's equation and there is no need for any field transformation. Another technique are contains in direct field is Sin/Cos method implemented by excited two plane wave simultaneously which one with sine waveform and the other with cosine waveform [7]. Even though it is a single frequency technique, but this method stable for grazing incident (θ=90o) associated with sin and cosine excitation.

A CTW was representing as a direct field method to overcome the oblique incident source. This method was originated from an analysis of wave structure in excitation of plane wave scattering parameter. As approach to consider the excitation, it will kept constant in the direction of the transverse wavenumber. FDTD are used in determining the transverse wavenumber where it simulates in time domain techniques and uses the electric (E) and magnetic (H) fields directly since it is straightforward that no conversions must be made after the simulation has run to get these values [8].

II. FORMULATION

A. *Split-Field Transformation*

Split-field FDTD uses the field transformation techniques where it split to two components of direct electromagnetic fields E and H by introducing nonphysical P and Q fields. P and Q is representative as a phase shifted with applicable to actual fields. This also express that the transformed field of Maxwell equation contains additional terms and tended to some difficulties when modified equation are discretized [9]. Consider the periodic array along the x-direction and the electromagnetic field of this plane wave is expressed as follow:

$$E(x,y,t) = E_0(t)e^{(k_x x+k_y y)} \tag{1}$$

$$H(x,y,t) = H_0(t)e^{(k_x x+k_y y)} \tag{2}$$

Note that $k_x=\frac{\omega}{c}\sin(\theta)$, $k_y=\frac{\omega}{c}\cos(\theta)$, $k_0=\frac{\omega}{c}$ and c is the light of speedin the free space. A new set of additional terms by splitting the variables are defined as below:

$$P(x+x_p, y,t) = P(x,y,t) \tag{3}$$

$$Q(x+x_p, y,t) = Q(x,y,t) \tag{4}$$

In this case, 2D TM mode is considered in determining the Maxwell's equation in frequency domain that replaced with the fields equation follow:

$$Q_x=\eta_0 H_x \,.\, e^{jk_x X} \tag{5}$$

$$Q_y=\eta_0 H_y \,.\, e^{jk_x X} \tag{6}$$

$$P_z = E_z.\, e^{jk_x X} \tag{7}$$

Periodic boundary conditions are applied in this field transformation with their condition refer to periodic array in x-direction:

$$P_z(\mathrm{x}=0) = P_z(\mathrm{x}=\mathrm{x}_p) \tag{8}$$

$$Q_x(\mathrm{x}=0) = Q_x(\mathrm{x}=\mathrm{x}_p) \tag{9}$$

$$Q_y(\mathrm{x}=0) = Q_y(\mathrm{x}=\mathrm{x}_p) \tag{10}$$

The Maxwell's equation will transform into frequency domain as formulation follows:

$$j\omega\frac{\mu_r Q_x}{C} = \frac{\partial P_z}{\partial y} \tag{11}$$

$$j\omega\frac{\mu_r Q_y}{C} = -\frac{\partial P_z}{\partial x} + j\omega\frac{\sin\theta}{C}P_z \tag{12}$$

$$j\omega\frac{\epsilon_r P_z}{C} = \frac{\partial Q_y}{\partial x} - \frac{\partial Q_x}{\partial y} + j\omega\frac{\sin\theta}{C}Q_y \tag{13}$$

From the applicable equation, it stated that ($j\omega \leftrightarrow \partial/\partial t$) time derivative is arising on both sides of the equation. P and Q are calculated at each half time step to figure out the two grids technique that introduced in this component [10].

Based on the figure 1, it shows the sequence in updating the split-field transformed. This show how the iteration of discretization Maxwell's equation works as the transformation needed. Thus, the transformation introduces the additional update equation as follows:

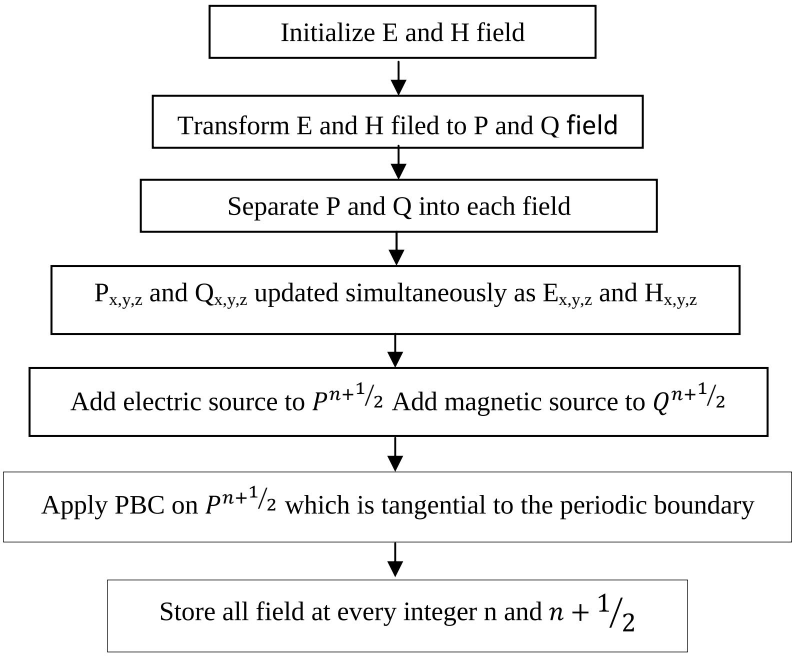

Figure 1: Updating procedure in split-field FDTD method

In split-field method used total field and scattered field (TF/SF) techniques to determining the scattering parameter. The formulation for split-field method is contains more extra formulation to transform the Maxwell's equation with the curl on the right involve the total field. By knowing the P and Q components, so the total field collected can be calculated in both values.

Applying the Maxwell's equation will obtain the following FDTD update equation where truncated the space in the x-direction [6]. These update equation develop only the total field values and are given as follow:

$$\frac{\varepsilon_x}{c}\frac{\partial P_x}{\partial t}+\sigma_x\eta_0 P_x=\frac{\partial Q_z}{\partial y}-\frac{\partial Q_y}{\partial z}+\frac{\sigma_x\eta_0 k_y}{c}Q_z-\frac{\sigma_x\eta_0 k_z}{c}Q_y$$
$$\frac{\varepsilon_y}{c}\frac{\partial P_y}{\partial t}+\sigma_y\eta_0 P_y=\frac{\partial Q_z}{\partial x}-\frac{\partial Q_x}{\partial z}+\frac{\sigma_y\eta_0 k_z}{c}Q_x$$
$$\frac{\varepsilon_z}{c}\frac{\partial P_z}{\partial t}+\sigma_z\eta_0 P_z=\frac{\partial Q_y}{\partial x}-\frac{\partial Q_x}{\partial y}+\frac{\sigma_z\eta_0 k_y}{c}Q_x \qquad (14)$$

$$\frac{\mu_x}{c}\frac{\partial Q_x}{\partial t}+\frac{\sigma_x}{\eta_0}Q_x=-\frac{\partial P_z}{\partial y}+\frac{\partial P_y}{\partial z}-\frac{\sigma_x k_y}{\eta_0 c}P_z-\frac{\sigma_x k_z}{\eta_0 c}P_y$$
$$\frac{\mu_y}{c}\frac{\partial Q_y}{\partial t}+\frac{\sigma_x}{\eta_0}Q_y=\frac{\partial P_z}{\partial x}+\frac{\partial P_x}{\partial z}-\frac{\sigma_x k_z}{\eta_0 c}P_x$$
$$\frac{\mu_z}{c}\frac{\partial P_z}{\partial t}+\frac{\sigma_x}{\eta_0}Q_z=-\frac{\partial P_y}{\partial x}+\frac{\partial P_x}{\partial y}-\frac{\sigma_x k_x}{\eta_0 c}P_x \qquad (15)$$

B. Constant Transverse Wavenumber (CTW)

CTW used as the incident wave in FDTD method that defining the constant value. In periodic structures for an oblique incident plane wave, CTW acts as a time domain wave with constant transverse wavenumber are expected to be constant number. Implementation of this method by considering the plane wave truncated in x direction over the wide range of frequencies. It is because the constant wavenumber in x direction has no delay in periodic plane and PBC can be used directly in the time method equations. The constant number is independent of frequency with a periodicity along the x direction so the PBC relating electric field and magnetic field at neighbouring cells in the frequency domain [11] can be expressed as below:

$$E(x=0,y,z)=E(x=0,y,z)\exp\,(jk_x) \qquad (16)$$
$$H(x=0,y,z)=H(x=0,y,z)\exp\,(jk_x) \qquad (17)$$

where$k_x = k_0 sin\,\theta = a\,sin\,\theta/C$. $k_x$is the horizontal wavenumber determined by both frequency incident angle that assuming the incident angle is θ that related each other, where the component transverse (horizontal) wavenumber kept constant in the direction of periodicity. Exp (jk_x) is also constant number in (16) and (17), therefore no time delay is required in this equation. This method is suitable for simulation of the incident angle radiance close to the grazing angles [12]. K_0 is determined from $k_0 = 2\pi f \sqrt{\epsilon_0\mu_0}$ where is the free space wave number and f is the operational frequency. Then it directly transformed into time domain using the Fourier transformation, future time domain $(t + a\, sin\, \theta/C)$data are needed in the updating equation as represented below:

$$E(x = 0, y, z, t) = E(x = 0, y, z, t + a\, sin\, \theta/C) \quad (18)$$

$$H(x = 0, y, z, t) = H(x = 0, y, z, t + a\, sin\, \theta/C) \quad (19)$$

Noted that C = $1/\sqrt{\varepsilon_0\mu_0}$ is the free space wave speed. The updating equations for both fields in current time (t) step are implemented to create the fundamentals of formulation PBC [13].

Figure 2 indicated the iteration from a single iteration of the updating procedure must be repeated to satisfy the updating equation of this method. Along the iteration, this equation used Fourier transform to allow implementation offrequency domain to time domain. Obviously, the final stage has stored the entire field to represent the value of both fields in every integer.

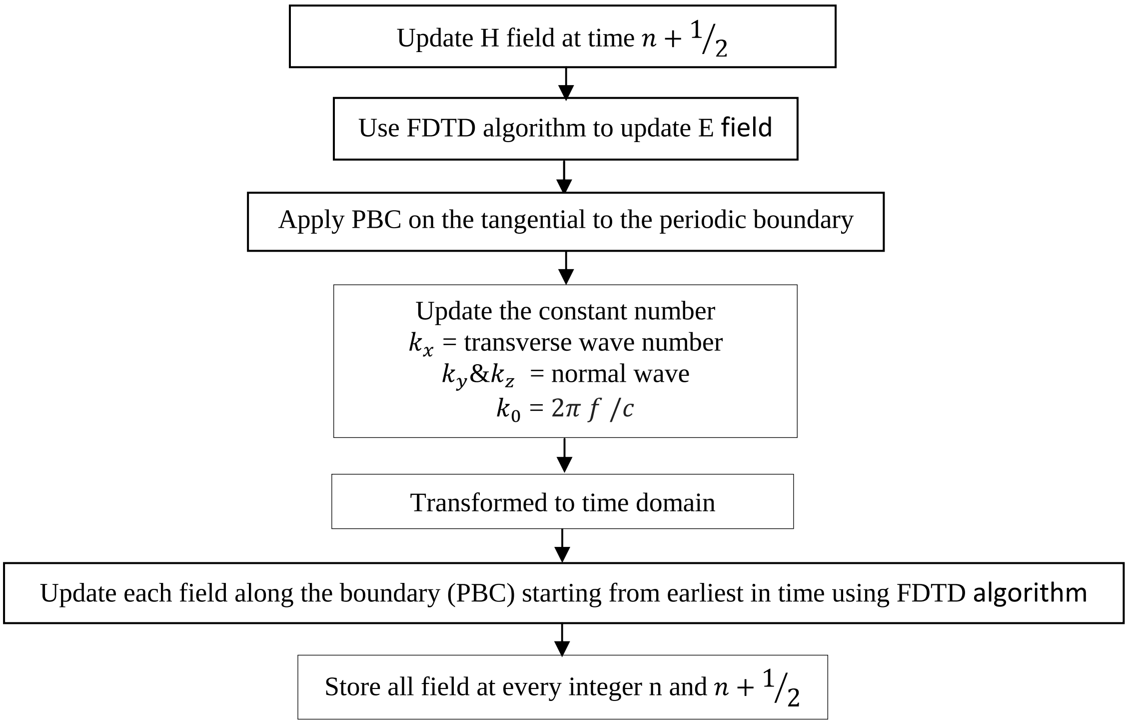

Figure 2: Updating for direct field transformation based on discretization Maxwell's equation

CTW Used TF/SF Technique

Excitations of plane waves in this method are proposed in two techniques composed of the total field/scattered field (TF/SF) technique and total field technique. Usually the TF/SF is considered when the constant k_x are considered as a fixed value and the incident angle θ varies with frequency. The $H_y$component is added on the excitation plane wave for the TM^x case as the split-field method. The plane wave is excited to propagate at x < x_0 and x >x_0 where the incident angle are denoted by θ (elevation) and   (azimuth) [14] to becomes the total field region and no scattered field region. Scattered field can be obtained by the difference of total field and incident field. In the time domain, the incident wave can be represented as follow:

$$E_t^{CTW}(x - x_0, y, z, t) \quad = \exp\,(-Jk_x\, x_0)\exp\,(Jk_y\, y)E_t\,(x, y, z, t) \quad (20)$$

$$H_t^{CTW}(x - x_0, y, z, t) = \exp\,(-Jk_x\, x_0)\exp\,(Jk_y\, y)H_t\,(x, y, z, t) \quad (21)$$

where equation (20) and (21) indicates that the fields of CTW wave at (x, y, z, t) are equal to its valueexp $(-Jk_x\, x_0)$. Therefore, it shows that the CTW has no delay as it travel in the x direction plane wave.

CTW Used Total Field Technique

Total field is transmitted signal that wavenumber instead of a specified incident angle in the FDTD based on one excitation plane wave. By choosing the constant wavenumber k_x , it recognized from the transformation of (14) and (15) will transform directly into time domain formulation as represented below in PBC:

$$E_t^{CTW}(x = 0, y, z, t) = E(x = 0, y, z, t)\exp\,(jk_x\, x) \quad (22)$$

$$H_t^{CTW}(x = 0, y, z, t) = H(x = 0, y, z, t)\exp\,(jk_x\, \mathrm{x}) \quad (23)$$

Note that $(jk_x\, x)$ is a constant number and it considered no time delay required. The formulation is Since the PBC is directly in time domain method, the over wide range of frequencies will be solved compare with the single frequency method which is Sin-Cosine method [15]. Incident angle (θ) is related to frequency (k_0) and transverse (horizontal) wavenumber as $k_x = k_0 sin\, \theta$. The incident wave in CTW is defined as a time domain wave with transverse wavenumber as shown in figure 3:

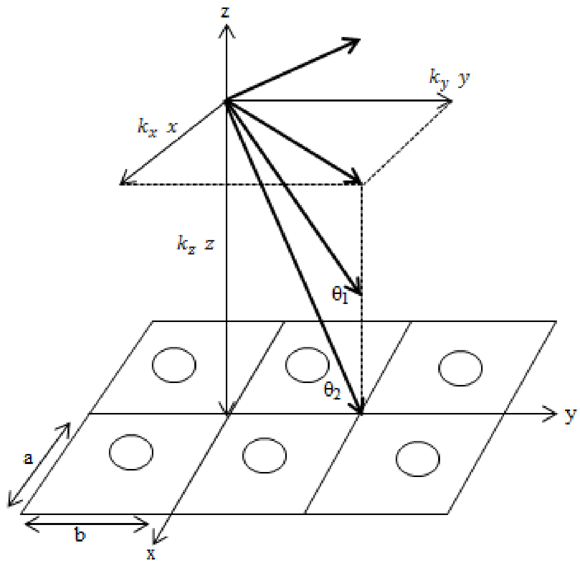

Figure 3: Multiple incident angles carried by CTW wave. The unit cell represented by dimension of a, b in the direction of x and y.

Updating equations for this method is same as the standard FDTD update equations. The update equations for the E and H fields are written as in (24) and (25) bottom. From the equation, the electric field components are calculated at in integer time steps and magnetic field component are calculated at half-integer time step [1].

$$E_{x_{i-1/2,j,k}}^{n+1} = E_{x_{i-1/2,j,k}}^{n} c\Delta t\left(\frac{H_{z_{i,j,k-1/2}}^{n+\frac{1}{2}} - E_{z_{i,j-1,k-1/2}}^{n+\frac{1}{2}}}{\Delta y} - \frac{E_{y_{i,j-1,k}}^{n+\frac{1}{2}} - E_{y_{i,j-1/2,k-1}}^{n+\frac{1}{2}}}{\Delta z}\right)$$

$$E_{y_{i,j-1/2,k}}^{n+1/2} = E_{y_{i,j-1/2,k}}^{n-1/2} c\Delta t\left(\frac{H_{x_{i,j-1/2,k+1/2}}^{n+\frac{1}{2}} - H_{x_{i,j+1/2,k-1/2}}^{n+\frac{1}{2}}}{\Delta z} - \frac{H_{z_{i-1/2,j-1/2,k}}^{n+\frac{1}{2}} - H_{z_{i-1/2,j-1/2,k}}^{n+\frac{1}{2}}}{\Delta x}\right)$$

$$E_{z_{i,j,k-1/2}}^{n+1/2} = E_{z_{i,j,k-1/2}}^{n-1/2} c\Delta t\left(\frac{H_{y_{i+1/2,j,k-1/2}}^{n+\frac{1}{2}} - H_{y_{i-1/2,j,k-1/2}}^{n+\frac{1}{2}}}{\Delta z} - \frac{H_{x_{i,j+1/2,k-1/2}}^{n+\frac{1}{2}} - H_{x_{i,j-1/2,k-1/2}}^{n+\frac{1}{2}}}{\Delta y}\right) \quad (24)$$

$$H_{x_{i,j-1/2,k-1/2}}^{n+1/2} = H_{x_{i,j-1/2,k-1/2}}^{n-1/2} c\Delta t\left(\frac{E_{z_{i,j,k-1/2}}^{n} - E_{z_{i,j-1,k-1/2}}^{n}}{\Delta y} - \frac{E_{y_{i,j-1,k}}^{n} - E_{y_{i,j-1/2,k-1}}^{n}}{\Delta z}\right)$$

$$H_{y_{i-1/2,j,k-1/2}}^{n+1/2} = H_{x_{i-1/2,j,k-1/2}}^{n-1/2} c\Delta t\left(\frac{E_{x_{i-1/2,j,k}}^{n} - E_{x_{i-1/2,j,k-1}}^{n}}{\Delta z} - \frac{E_{y_{i,j,k-1/2}}^{n} - E_{y_{i-1,j,k-1/2}}^{n}}{\Delta x}\right)$$

$$H_{z_{i-1/2,j-1/2,k}}^{n+1/2} = H_{x_{i-1/2,j-1/2,k}}^{n-1/2} c\Delta t\left(\frac{E_{y_{i,j-1/2,k}}^{n} - E_{z_{i-1,j-1/2,k}}^{n}}{\Delta y} - \frac{E_{x_{i-1/2,j,k}}^{n} - E_{x_{i-1/2,j-1,k}}^{n}}{\Delta z}\right) \quad (25)$$

III. NUMERICAL STABILITY

A. *Split Field Method*

The stability of this method tends to zero with increasing incidence angle, thus making simulation at large angles impossible. Courant stability criterion of this method is strict and angle-dependent. This equation expressed the numerical stability for this method for oblique incident plane wave.

$$\frac{C\Delta t}{\Delta x} \leq \frac{1 - sin\,\theta}{\sqrt{D}} \tag{22}$$

where Δt is the time step size that FDTD algorithm required. However, Δt must be decreased to obtain stale result as the incident angle is increased. This means that the time step is smaller than a specific bound determined by the space increments Δx, Δy, and Δz in an effort to avoid numerical instability.

A simple expression for the stability limit of the 2D split-filed update method in free space was derived as a function of the incident angle θ_{inc}[16].

$$S \leq \frac{\cos\theta_{inc}^2}{\sqrt{1 + \cos\theta_{inc}^2}} \tag{23}$$

B. CONSTANT TRANVERSE WAVENUMBER (CTW)

Numerical stability for this method is same as conventional FDTD stability can be applied without significant modification [6] and is angle independent. CTW is stable for grazing incident compare to split-field method because this method suitable for wideband frequency approach as compared for single frequency that using sin-cosine method.

$$c\Delta t < 1/\sqrt{1/\Delta x^2 + 1/\Delta y^2 + 1/\Delta z^2} \leq 1 \tag{24}$$

whereΔt is the time step and Δx, Δy, and Δz is the grid size of FDTD scheme.

This numerical stability of CTW method requires that the time increment Δt has a specific bound relative to the lattice space increments. Since the FDTD method based on sampling and updating the electric field and magnetic field in time and space, wave propagation in the discrete grid-space may differ from it in the continuous space. For numerically stable FDTD method, the time step Δt in ratio cΔt/Δu must be enclosed to a special limit, often called the Courant factor where the limit depends on the both dimension and the angle of incidence expressed as:

$$S = c\Delta t/\Delta u \tag{25}$$

IV. ADVANTAGES OF CONSTANT TRANSVERSE WAVENUMBER (CTW)

The difference between CTW and other method is a transformed method. In CTW, it directly transformed where the new algorithm in this method is very easy to implement and it can obtain the wide-band characteristic. Compare than the field transformed method that introduces the new additional field (auxiliary field). The additional field will compute and update the electric field and magnetic field transform to P field and Q field. Thus, complicated discretization in Maxwell's equation was derived.

CTW method is not depending on the incidence angle because the implementation of the total field technique which required only one field either E or H field, instead of the split - field method is depending on the angle incidence. Then the time step size (Δt) must be decreased to obtain stale results as the incident angle is increased. This causes more memory storage required because the incident angle close to grazing angle and the simulation time becomes grossly long.

The precise definition of stability depends on the context of CTW method and split-field method, where is derived from the accuracy of the both algorithms. Thus, the CTW is usually used the stability of conventional FDTD that no need to change the time step either in normal incident or oblique incident where it more related to the incident angle [17]. It is because incident angle that determined or introduced the source plane wave. But, in split-field method, it request difference stability for normal incident and oblique incident. When structures in oblique incident, the dimensionality of the problem have to know in order to calculate the stability of structures.

V. CONCLUSIONS

This paper presents a comparison between two methods to solve the periodic structures at the oblique incident plane wave. The methods refer to split field transformation and CTW transformation. CTW method is chosen based on direct transformation (no P and Q fields) from Maxwell's equation. This method is easy to implement and less memory consuming at simulation time. Constant value represent has no time delay in the transverse plane and no additional terms required as field transformation. Therefore, FDTD method it is simple to implement numerically which provides a flexible means for directly solving Maxwell's time-dependent based on the curl equation by using the finite differences to discretize them.

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