Fixed point theorem in Intuitionistic fuzzy-3 metric space using $weak^{**}$ compatibility

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ABSTRACT

The purpose of this paper is to prove a common fixed point theorem in intuitionistic fuzzy -3metric space by studying the relationship between reciprocal continuity for idempotent maps in intuitionistic fuzyy metric space. Our result generalize the Result of Jitendra et al [7] and many others

Key word: fixed point, IFM-space, $weak^{**}$ **compatibility**

1.INTRODUCTION

The concept of fuzzy sets was introduced by $zadeh$ [13] following the concept of fuzzy sets, fuzzy metric spaces have been introduced by kramosil and michlek [9] and George and veeramani [6] modified the notion of fuzzy metric space with the help of continuous t-norms

As a generalization of fuzzy sets , Atanassov [14] introduced and studied the concept of intuitionistic fuzzy sets park[11] using the idea of intuitionistic fuzzy sets defined the notion of intuionistic fuzzy metric spaces with the help of continuous t-norm and continuous t co-norm as a generalization of fuzzy metric space due to George & veeramani [6] had showed that every metric induces an intuitionistic fuzzy metric every fuzzy metric space is an intuitionistic fuzzy metric space and found a necessary and sufficient condition for an intuitionistic fuzzy metric space to be complete choudhary[15] introduced mutually contractive sequence of self maps and proved a fixed point theorem kramosil & michlek [9] introduced the notion of Cauchy sequences in an intuitionistic fuzzy metric space and proved the well known fixed point theorem of Banach [4], Turkoglu et al [12] gave the generalization of jungek's common fixed point theorem [19] to intuitionistic fuzzy metric spaces, they first formulate the definition of weakly commuting and R- weakly commuting mapping in intuitionistic fuzzy metric spaces and proved proved the intuitionistic fuzzy version of pant's theorem[20]

Here we will define $weak^{**}$ commuting in intuitionistic fuzzy metric space and reciprocal continuity for idempotent maps in intuitionistic fuzyy metric space. And prove a fixed point theorem in IF-3 metric space. Our result generalize the Result of Jitendra et al [7] and many others

2. Preliminaries

Definition 2.1 [7] A binary operation $* [0,1] \times [0,1] \rightarrow [0,1]$ is a continuous t norm if it satisfies the following condition

2.1.1 $*$ is commutative and associative

2.1.2 $*$ is continuous

2.1.3 $a * 1 = a$ for all $a \in [0,1]$

2.1.4 $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0,1]$

Definition 2.2 [7] A binary operation $\lozenge : [0,1] \times [0,1] \rightarrow [0,1]$ is continuous t-conorm if $\lozenge$ is satisfying the following condition

2.2.1 ◊ is commutative and associate

2.2.2 ◊ is continuous

2.2.3 $a \lozenge 0 = a$ for all $a \in [0,1]$

2.2.4 $a \lozenge b \le c \lozenge d$ Whenever $a \le c$ and $b \le d$ for all $a, b, c, d \in [0,1]$

Definition 2.3 [4] the 3-tuple (X, M, *) is called a fuzzy metric space (FM-space) if X is an arbitrary set * is a continuous t-norm and M is a fuzzy set in $X^2 \times [0,\infty]$ satisfying the following conditions for all $x, y, z \in X$ and $t, s > 0$.

2.3.1 $M(x,y,0) > 0$

2.3.2 $M(x,y,t) = 1, \forall\, t > 0 \; iff \; x = y$

2.3.3 $M(x,y,t) = M(y,x,t)$,

2.3.4 $M(x,y,t) * M(y,z,s) \le M(x,z,t+s)$

2.3.5 $M(x,y,\ .): [0,\infty] \to [0,1]$ is continuous.

Remark 2.4 since * is continuous, it follows from (2.3.4) that the limit of a sequence in FM-space is uniquely determined

Definition 2.5 [16] A five –tuple (X, M, N, ∗, ◊) is said to be an intuitionistic fuzzy metric space if X is an arbitrary set * is a continuous t – norm, ◊ is a continuous t-conorm and M, N are fuzzy sets on $X^2 \times (0,\infty)$ satisfying the following conditions for all $x, y, z \in X, s, t > 0$

2.4.1 $M(x,y,t) + N(x,y,t) \le 1$

2.4.2 $M(x,y,t) > 0$

2.4.3 $M(x,y,t) = M(y,x,t)$

2.4.4 $M(x,y,t) * M(y,z,s) \le M(x,z,t+s)$

2.4.5 $M(x,y,.): (0,\infty) \to (0,1)$ is continuous

2.4.6 $N(x,y,t) > 0$

2.4.7 $N(x,y,t) = N(y,x,t)$

2.4.8 $N(x,y,t) \lozenge N(y,z,s) \ge N(x,z,t+s)$

2.4.9 $N(x,y,.): (0,\infty) \to (0,1]$ is continuous

Then (M, N) is called an intuitionistic fuzzy metric On X , the function $M(x,y,t)$ and $N(x,y,t)$ denote the degree of nearness and the degree of non- nearness between x and y with respect to t respectively

Remark 2.6 Every fuzzy metric space (X, M, ∗) is an intuitionistic fuzzy metric space form$X, M, 1-M, *, \lozenge)$ such that t-norm * and t-conorm ◊ are associated ie $x \lozenge y = 1 - ((1-x) * (1-y))$ for any $x, y \in [0,1]$ but the converse is not true

Definition2.7 Two self mappings A and S of an IFM-space(X, M, N, ∗, ◊) is called $weak^{**}$ commuting if $A(X) \subset S(X)$ and for any x in X

$$M(A^2S^2x, S^2A^2x, t) \geq M(A^2x, S^2x, t)$$

And

$$N(A^2S^2x, S^2A^2x, t) \leq N(A^2x, S^2x, t)$$

Remark 2.8 If A and S are idempotent maps i.e. $A^2 = A$ and $S^2 = S$ then weak commutative reduced to weak commuting pair (A.S)

i.e $M(A^2S^2x, S^2A^2x, t) \geq M(A^2Sx, S^2Ax, t) \geq M(AS^2x, SA^2x, t) \geq M(ASx, SAx, t) \geq M(A^2x, S^2x, t)$

and

$N(A^2S^2x, S^2A^2x, t) \leq N(A^2Sx, S^2Ax, t) \leq N(AS^2x, SA^2x, t) \leq N(ASx, SAx, t) \leq N(A^2x, S^2x, t)$

However point wise R-weakly commuting mapping need not be compatible

Definition2.9 Two self mappings A and S which are idempotent maps i.e. $A^2 = A$ and $S^2 = S$ of IFM-space (X, M, N, $*$, $\Diamond$) are called reciprocally continuous on X if $lim_{n\to\infty} A^2S^2xn = A^2x$

And $lim_{n\to\infty} S^2A^2xn = S^2x$ whenever $\{x_n\}$ is a sequence in X such that $lim_{n\to\infty} A^2xn = lim_{n\to\infty} S^2xn = x$ for some x in X i.e

$$M(A^2S^2xn, S^2A^2xn, t) \geq M(A^2Sxn, S^2Axn, t) \geq$$
$$M(AS^2xn, SA^2xn, t) \geq M(ASxn, SAxn, t) \geq M(A^2xn, S^2xn, t) \geq M(A^2x, S^2x, t)$$

and

$$N(A^2S^2xn, S^2A^2xn, t) \leq N(A^2Sxn, S^2Axn, t) \leq$$
$$N(AS^2xn, SA^2xn, t) \leq N(ASxn, SAxn, t) \leq N(A^2xn, S^2xn, t) \leq N(A^2x, S^2x, t)$$

whenever $\{x_n\}$ is a sequence in X such that

$lim_{n\to\infty} M(A^2xn, S^2xn, t) = M(A^2x, S^2x, t)$ and $lim_{n\to\infty} N(A^2xn, S^2xn, t) = N(A^2x, S^2x, t)$ for all $t > 0$ thus if two self mappings are $weak^{**}$ commuting then they are reciprocall continuous as well

Lemma 2.13[16]. Let $\{y_n\}$ be a sequence in IFM space $(X, M, N, *, \Diamond)$ with the condition $lim_{t\to\infty} M(x, y, t) = 1$ and $lim_{t\to\infty} N(x, y, t) = 0$. If there exist a number $k \in (0,1)$ such that $M(y_{2n+2,} y_{n+1}, kt) \geq M(y_{2n+1,} y_n, t)$ and $(y_{2n+2,} y_{n+1}, kt) \leq N(y_{2n+1,} y_n, t)$, for all $t > 0$, then $\{y_n\}$ is a Cauchy sequence in X

Lemma 2.14. Let A and B be two self-maps on a complete IFM-space $(X, M, N, *, \Diamond)$ such that for somek $\in (0,1)$, for all $x, y \in X$, for all $t > 0$

$$M(Ax, Bx, t) \geq \min\{M(x, y, t), M(Ax, x, t)\}$$

and

$$N(Ax, Bx, t) \leq \max\{N(x, y, t), N(Ax, x, t)\}$$

Then A and B have a unique common fixed point in X

Proof Let $p \in X$. taking $x_0 = p$, define sequence $\{x_n\}$ in X by $Ax_{2n} = x_{2n+1}$ and $x_{2n+1} = x_{2n+2}$. By taking $x = x_{2n}, y = x_{2n+1}$ *and* $x = x_{2n}, y = x_{2n-1}$, respectively, in the contractive condition , we obtain that

$$M(x_{n+1}, x_n, kt) \geq M(x_n, x_{n-1}, t),$$

And (2.3)

$$N(x_{n+1}, x_n, kt) \leq N(x_n, x_{n-1}, t) \text{ for all } t > 0 \text{, for all n}$$

Therefore by lemma 2.7, $\{x_n\}$ is a Cauchy sequence in X, which is complete. Hence $\{x_n\}$ converges to some u in X. Taking $x = x_{2n}$ and $y = u$ and letting $n \to \infty$ in the contractive condition, we get $Bu = u$. Similarly, by putting $x = u$ and $y = x_{2n+1,}$ we get $Au = u$. Therefore, u is the common fixed point of the maps A and B. The uniqueness of the common fixed point follows from the contractive condition

3Main Result

Let $(X, M, N, *, \Diamond)$ be a complete IF-3 metric space and let F and T be continuous mappings of X in X. Let A be a self mappings of X satisfying $[A, F]$ and $[A, T]$ are $weak^{**}$ commuting and

(1) $A(X) \subseteq F(X) \cap T(X)$

(2) $M(A^2x, A^2y, a, b, t) \geq$

$$\varphi[min\begin{Bmatrix} M(F^2x, T^2y, a, b, t), M(F^2x, A^2x, a, b, t), M(F^2x, A^2y, a, b, t), \\ M(T^2y, A^2y, a, b, t), M(A^2x, T^2y, a, b, t), \\ , M(F^2y, A^2y, a, b, t) \end{Bmatrix}$$

and

$N(A^2x, A^2y, a, b, t) \leq$

$$\Psi[max\begin{Bmatrix} N(F^2x, T^2y, a, b, t), N(F^2x, A^2x, a, b, t), N(F^2x, A^2y, a, b, t), \\ N(T^2y, A^2y, a, b, t), N(A^2x, T^2y, a, b, t), \\ , N(F^2y, A^2y, a, b, t) \end{Bmatrix}]$$

For all $x, y \in X$, where φ and Ψ: $[0.1] \to [0,1]$ are continuous functions such that $\varphi(t) > t$ and $\Psi(t) < t$ for each $0 \leq t \leq 1$ and $\varphi(1) = 1$ and $\Psi(0) = 0$ and $a, b \in X$. The sequence $\{x_n\}$ and $\{y_n\}$ in X are such that $x_n \to x$, $y_n \to y$
$\Rightarrow M(x_{n,} \, y_n$,a, b, t)$\to$ $M(x, y, a, b, t)$ and $N(x_{n,} \, y_n$,a, b, t)$\to$ $N(x, y, a, b, t)$ where $t > 0$ then F, T and A have a unique common fixed point in X

Proof we define sequence $\{x_n\}$ and $\{y_n\}$ such that $y_{2n} = A^2x_{2n} = F^2x_{2n+1}$ and $y_{2n+1=}A^2x_{2n} = F^2x_{2n+1}$ and $y_{2n+1} = A^2x_{2n+1} = T^2x_{2n+2}$ for $n = 1,2 \dots \dots$ now we shall prove that $\{y_n\}$ is a Cauchy sequence

Let $G_n = M(y_n, y_{n+1}, a, b, t) < 1$ and $H_n = N(y_n, y_{n+1}, a, b, t) > 0$ for $n = 1,2 \dots \dots$ then by (2) we have

$$G_{2n} = M(y_{2n}, y_{2n+1}, a, b, t) =$$
$$M(A^2x_{2n}, A^2x_{2n+1}, a, b, t) \geq$$
$$\varphi[min\begin{Bmatrix} M(F^2x_{2n+1}, T^2x_{2n}, a, b, t), M(F^2x_{2n+1}, A^2x_{2n+1}, a, b, t), M(F^2x_{2n+1}, A^2x_{2n}, a, b, t), \\ M(T^2x_{2n}, A^2x_{2n}, a, b, t), M(A^2x_{2n+1}, T^2x_{2n}, a, b, t), \\ , M(F^2x_{2n}, A^2x_{2n}, a, b, t) \end{Bmatrix}$$
$$= \varphi[min\begin{Bmatrix} M(y_{2n}, y_{2n-1}, a, b, t), M(y_{2n}, y_{2n+1}, a, b, t), M(y_{2n}, y_{2n}, a, b, t), \\ M(y_{2n-1}, y_{2n}, a, b, t), M(y_{2n+1}, y_{2n-1}, a, b, t), \\ , M(y_{2n-1}, y_{2n}, a, b, t) \end{Bmatrix}$$
$$= \varphi[min\begin{Bmatrix} M(y_{2n}, y_{2n-1}, a, b, t), M(y_{2n}, y_{2n+1}, a, b, t), M(y_{2n}, y_{2n}, a, b, t), \\ M(y_{2n-1}, y_{2n}, a, b, t), M(y_{2n+1}, y_{2n}, a, b, t), \\ , M(y_{2n}, y_{2n-1}, a, b, t), M(y_{2n-1}, y_{2n}, a, b, t) \end{Bmatrix}$$
$$= \varphi[min\{G_{2n-1,} G_{2n}, 1, G_{2n-1,} G_{2n}, G_{2n-1,} G_{2n-1,}\} \qquad \text{(i)}$$

and

$$H_{2n} = N(y_{2n}, y_{2n+1}, a, b, t) =$$
$$N(A^2x_{2n}, A^2x_{2n+1}, a, b, t) \leq$$
$$\Psi[max\begin{Bmatrix} N(F^2x_{2n+1}, T^2x_{2n}, a, b, t), N(F^2x_{2n+1}, A^2x_{2n+1}, a, b, t), N(F^2x_{2n+1}, A^2x_{2n}, a, b, t), \\ N(T^2x_{2n}, A^2x_{2n}, a, b, t), N(A^2x_{2n+1}, T^2x_{2n}, a, b, t), \\ , N(F^2x_{2n}, A^2x_{2n}, a, b, t) \end{Bmatrix}$$
$$= \Psi[max\begin{Bmatrix} N(y_{2n}, y_{2n-1}, a, b, t), N(y_{2n}, y_{2n+1}, a, b, t), N(y_{2n}, y_{2n}, a, b, t), \\ N(y_{2n-1}, y_{2n}, a, b, t), N(y_{2n+1}, y_{2n-1}, a, b, t), \\ , N(y_{2n-1}, y_{2n}, a, b, t) \end{Bmatrix}$$
$$\leq \Psi[max\begin{Bmatrix} N(y_{2n}, y_{2n-1}, a, b, t), N(y_{2n}, y_{2n+1}, a, b, t), N(y_{2n}, y_{2n}, a, b, t), \\ N(y_{2n-1}, y_{2n}, a, b, t), N(y_{2n+1}, y_{2n}, a, b, t), \\ , N(y_{2n}, y_{2n-1}, a, b, t), N(y_{2n-1}, y_{2n}, a, b, t) \end{Bmatrix}$$
$$= \Psi[max\{H_{2n-1,} H_{2n}, 0, H_{2n-1,} H_{2n}, H_{2n-1,} H_{2n-1,}\} \qquad \text{(ii)}$$

If $G_{2n-1,} \geq G_{2n}$ then $G_{2n} \geq \varphi[G_{2n-1,}] > G_{2n-1,}$

And if $H_{2n-1,} \leq H_{2n}$ then $H_{2n} \leq \Psi\,[H_{2n-1,}] < H_{2n-1,}$ a contradiction therefore $G_{2n-1,} \leq G_{2n}$ and $H_{2n-1,} \geq H_{2n}$ therefore from (1) and (2) we have

$G_{2n} \geq \varphi[G_{2n-1,}] > G_{2n-1,}$ and $H_{2n} \leq \Psi\,[H_{2n-1,}] < H_{2n-1,}$ (iii)

Thus $\{G_{2n}: n \geq 0\}$ is increasing and $\{H_{2n}: n \geq 0\}$ is decreasing sequence of positive real numbers in

[0, 1] and therefore tend to limits $l_1 \leq 1$ and $l_2 \geq 0$, it is clear that $l_1 = 1$ and $l_2 = 0$ because if $l_1 < 1$ and $l_2 > 0$ then on taking limit as n→ ∞ in (iii) we get $l_1 \geq \varphi[l_1] > l_1$ and $l_2 \leq \Psi\, [l_2] < l_2$,a contradiction hence $l_1 = 1$ and $l_2 = 0$

Now for any integer m

$$M(y_n, y_{n+m}, a, b, t) \geq M\left(y_n, y_{n+1}, a, b, \frac{t}{m}\right) * \dots\dots\dots\dots\dots * M\left(y_{n+m-1}, y_{n+m}, a, b, \frac{t}{m}\right)$$

$$\geq M\left(y_n, y_{n+1}, a, b, \frac{t}{m}\right) * \dots\dots . . * M\left(y_n, y_{n+1}, a, b, \frac{t}{m}\right)$$

and

$$N(y_n, y_{n+m}, a, b, t) \leq N\left(y_n, y_{n+1}, a, b, \frac{t}{m}\right) \lozenge \dots\dots\dots\dots\dots \lozenge N\left(y_{n+m-1}, y_{n+m}, a, b, \frac{t}{m}\right)$$

$$\leq N\left(y_n, y_{n+1}, a, b, \frac{t}{m}\right) \lozenge \dots\dots . . \lozenge N\left(y_n, y_{n+1}, a, b, \frac{t}{m}\right)$$

Therefore $lim_{n\to\infty} M(y_n, y_{n+m}, a, b, t) \geq 1 * 1 * \dots\dots . . * 1$

Because $lim_{n\to\infty} M(y_n, y_{n+1}, a, b, t) = 1, for\ t > 0$

and

$lim_{n\to\infty} N(y_n, y_{n+m}, a, b, t) \leq 0 \lozenge 0 \lozenge \dots\dots . . \lozenge 0$, since

$lim_{n\to\infty} N(y_n, y_{n+1}, a, b, t) = 0 for\ t > 0$

Thus $\{y_n\}$ is a Cauchy sequence and by the completeness of X. $\{y_n\}$ converges to $u \in X$. So its subsequences $\{A^2 x_{2n+1}\}, \{T^2 x_{2n}\}$ and $\{F^2 x_{2n+1}\}$ also converges to same point u.

Since $[A, F]$ is $weak^{**}$ commuting so

$$M(A^2F^2x_{2n+1}, F^2A^2x_{2n+1}, a, b, t) \geq M(A^2x_{2n+1}, F^2x_{2n+1}, a, b, t)$$

and

$$N(A^2F^2x_{2n+1}, F^2A^2x_{2n+1}, a, b, t) \leq N(A^2x_{2n+1}, F^2x_{2n+1}, a, b, t)$$

On taking limit as → ∞ , $A^2F^2x_{2n+1} = F^2A^2x_{2n+1} = F^2u$, now we will prove that $F^2u = u$

First suppose that $F^2u \neq u$ then there exist $t > 0$ such that $M(F^2u, u, a, b, t) < 1\ and\ N(F^2u, u, a, b, t) > 0$

Now

$$M(A^2F^2x_{2n+1}, A^2x_{2n}, a, b, t) \geq \varphi[min \left\{ \begin{array}{c} M(F^3x_{2n+1}, T^2x_{2n}, a, b, t), M(F^3x_{2n+1}, A^2F^2x_{2n+1}, a, b, t), M(F^3x_{2n+1}, A^2x_{2n}, a, b, t), \\ M(T^2x_{2n}, A^2x_{2n}, a, b, t), M(A^2F^2x_{2n+1}, T^2x_{2n}, a, b, t), \\ , M(F^2x_{2n}, A^2x_{2n}, a, b, t) \end{array} \right\}$$

This implies

$$M(F^2u, u, a, b, t) \geq \varphi[min \left\{ \begin{array}{c} M(F^2u, u, a, b, t), M(F^2u, F^2u, a, b, t), M(F^2u, u, a, b, t), \\ M(u, u, a, b, t), M(F^2u, u, a, b, t), \\ , M(F^2u, u, a, b, t) \end{array} \right\}$$

and

$$N(A^2F^2x_{2n+1}, A^2x_{2n}, a, b, t) \leq$$
$$\Psi[max\begin{Bmatrix} N(F^3x_{2n+1}, T^2x_{2n}, a, b, t), N(F^3x_{2n+1}, A^2F^2x_{2n+1}, a, b, t), N(F^3x_{2n+1}, A^2x_{2n}, a, b, t), \\ N(T^2x_{2n}, A^2x_{2n}, a, b, t), N(A^2F^2x_{2n+1}, T^2x_{2n}, a, b, t), \\ , N(F^2x_{2n}, A^2x_{2n}, a, b, t) \end{Bmatrix}$$

This implies

$$N(F^2u, u, a, b, t) \leq \Psi[max\begin{Bmatrix} N(F^2u, u, a, b, t), N(F^2u, F^2u, a, b, t), N(F^2u, u, a, b, t), \\ N(u, u, a, b, t), N(F^2u, u, a, b, t), \\ , N(F^2u, u, a, b, t) \end{Bmatrix}$$

This implies that $M(F^2u, u, a, b, t) \geq \varphi[M(F^2u, u, a, b, t)] > M(F^2u, u, a, b, t)$

and $N(F^2u, u, a, b, t) \leq \Psi[N(F^2u, u, a, b, t)] < N(F^2u, u, a, b, t)$ which is a contradiction therefore $F^2u = u$. Thus u is a fixed point of F. Similarly we can show that u is also a fixed point of A. Now we claim that u is a fixed point of T. suppose it is not so then for any $t > 0$ $M(u, T^2u, a, b, t) < 1$ *and* $N(u, T^2u, a, b, t) > 0$ now

$$M(A^2u, A^2T^2x_{2n}, a, b, t) \geq \varphi[min\begin{Bmatrix} M(F^2u, T^3x_{2n}, a, b, t), M(F^2u, A^2u, a, b, t), M(F^2u, A^2T^2x_{2n}, a, b, t), \\ M(T^3x_{2n}, A^2T^2x_{2n}, a, b, t), M(A^2u, T^2x_{2n}, a, b, t), \\ , M(F^2T^2x_{2n}, A^2T^2x_{2n}, a, b, t) \end{Bmatrix}$$

and

$$N(A^2u, A^2T^2x_{2n}, a, b, t) \leq \Psi[max\begin{Bmatrix} N(F^2u, T^3x_{2n}, a, b, t), N(F^2u, A^2u, a, b, t), N(F^2u, A^2T^2x_{2n}, a, b, t), \\ N(T^3x_{2n}, A^2T^2x_{2n}, a, b, t), N(A^2u, T^2x_{2n}, a, b, t), \\ , N(F^2T^2x_{2n}, A^2T^2x_{2n}, a, b, t) \end{Bmatrix}$$

This implies that

$$M(u, T^2u, a, b, t) \geq \varphi[min\begin{Bmatrix} M(u, T^2u, a, b, t), M(u, u, a, b, t), M(u, T^2u, a, b, t), \\ M(T^2u, T^2u, a, b, t), M(u, T^2u, a, b, t), \\ , M(T^2u, T^2u, a, b, t) \end{Bmatrix}$$

and

$$N(u, T^2u, a, b, t) \leq \Psi[max\begin{Bmatrix} N(u, T^2u, a, b, t), N(u, u, a, b, t), N(u, T^2u, a, b, t), \\ N(T^2u, T^2u, a, b, t), N(u, T^2u, a, b, t), \\ , N(T^2u, T^2u, a, b, t) \end{Bmatrix}$$

This implies that $M(u, T^2u, a, b, t) \geq \varphi[M(u, T^2u, a, b, t)] > M(u, T^2u, a, b, t)$

and $N(u, T^2u, a, b, t) \leq \Psi[N(u, T^2u, a, b, t)] < N(u, T^2u, a, b, t)$

Which is a contradiction therefore $T^2u = u$ hence u is a fixed point of T.

I.E. u is a common fixed point of T, F and A

Uniqueness suppose there is another fixed point $v \neq u$ then

$$M(A^2u, A^2v, a, b, t) \geq$$

$$\varphi[min\begin{Bmatrix} M(F^2u, T^2v, a, b, t), M(F^2u, A^2u, a, b, t), M(F^2u, A^2v, a, b, t), \\ M(T^2v, A^2v, a, b, t), M(A^2u, T^2v, a, b, t), \\ , M(F^2v, A^2v, a, b, t) \end{Bmatrix}$$

and

$$N(A^2u, A^2v, a, b, t) \le \Psi[max \begin{Bmatrix} N(F^2u, T^2v, a, b, t), N(F^2u, A^2u, a, b, t), N(F^2u, A^2v, a, b, t), \\ N(T^2v, A^2v, a, b, t), N(A^2u, T^2v, a, b, t), \\ , N(F^2v, A^2v, a, b, t) \end{Bmatrix}$$

This implies that

$$M(u, v, a, b, t) \ \ge \varphi[min \begin{Bmatrix} M(u, v, a, b, t), M(u, u, a, b, t), M(u, v, a, b, t), \\ M(v, v, a, b, t), M(u, v, a, b, t), \\ , M(v, v, a, b, t) \end{Bmatrix}$$

and

$$N(u, v, a, b, t) \ \le \Psi[max \begin{Bmatrix} N(u, v, a, b, t), N(u, u, a, b, t), N(u, v, a, b, t), \\ N(v, v, a, b, t), N(u, v, a, b, t), \\ , N(v, v, a, b, t) \end{Bmatrix}$$

This implies that $M(u, v, a, b, t) \ge \varphi(M(u, v, a, b, t))$ and $N(u, v, a, b, t) \ \le \Psi(N(u, v, a, b, t))$

A contradiction so $u = v$ hence A, F and T have unique common fixed point

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