

A New Technique in Ranking of Alternatives; Keen Analysis Surge of Advantage States

A. M. El-Kassas

Department of Production Engineering and Mechanical Design,
Faculty of Engineering, Tanta University, Tanta, Egypt
ahmed.elkassas@f-eng.tanta.edu.eg

Abstract—This paper presents a new dimensionless technique for decision-making. The new technique is the Keen Analysis Surge of Advantage States of Alternatives with respect to its criterion for appreciates the incontestable value. This technique depends on the real advantage of all alternatives with the new definitions of concordance and discordance indices especially that have not the same unit. The advantages can be summarized as follows; don't cause a redundant evaluation and also provide a clear ranking of alternatives with priority vector, capability in introducing the actual comparisons between alternative, clarify the effective relations between alternatives, gives a priority vector with actual outranking relations and robust ranking of alternatives. Numerical example introduced with actual estimation.

Keyword - Concordance, Discordance, Ranking of alternatives, Outranking

I. INTRODUCTION

MCDM techniques which based on pairwise comparisons and physical or monetary values of alternatives are non-compensatory techniques. It starts by comparing each action to each of the others, in relation to each criterion [1]-[3], [5], [6]. The comparisons is based on calculating the concordance measure which represents the degree of dominance of alternative A_k over alternative A_l for all criterion and discordance measure is carried out in concordance analysis for each pair of alternatives. It aggregates the results of all comparisons and builds the model for outranking relations according to the notion of concordance and discordance. These methods may be uses this results in the second phase of fuzzy relation exploitation, to construct two complete preorders through a descending and an a sending distillation procedure [7]. A partial preorder is elaborated as the intersection of the two complete preorders. The comparative analysis of preorders leads to a final robust result or a model re-analysis. Most of researchers have directed the intention to choosing a realistic values of indifference, preference and veto thresholds, Rogers and Bruen [8], weighting the criteria [9].

II. CRITICISM OF MCDM

MCDM techniques which depends on physical or monetary values of alternatives have some contradictions in most steps of analysis. *The first contradiction* was appeared on the condition of overlapping of any alternative to other with respect to each criterion in concordance index. If $g_j(A_k)$ is more than or equal $g_j(A_l)$, this means that A_k is at least as good as A_l with respect to j^{th} criterion and also, the concordance value becomes one. This sequence of estimations causes of a weakness and redundancy in concordance index, where activate the equality of alternatives in the direction of concordance, and also don't differentiate between the degree of advantages between alternatives as long as it validates the condition of concordance. The concordance values are interpreted simple as the percentage of criteria where one alternative is at least as good as the other without clear evidence to neglect the contribution of each criteria with respect to others. *The second contradiction* was observed in discordance index which expresses the degree that an alternative A_k is worse than a competing alternative A_l . The concordance index is the maximum element of the set that contains the ratios of the amount of discordance between A_k and A_l for each criterion indicating discordance, to the maximum discordance between any pair of actions for the corresponding criterion. In ELECTRE III, calculation of discordance depends on a further threshold called a veto threshold, where concordance depends on another threshold and unlike concordance, no aggregation over criteria takes places; one discordant criterion is sufficient to discard outranking. The units of discordance do not compatible with the units of concordance, furthermore, unfairness in the choice of max discordance in the set of discordance. Therefore, this paper presents a **new technique** able to correct the weakness in concept of concordance and discordance estimations and its units, moreover the improvements of flaw in ELECTRE methods. **The new technique is the Keen Analysis Surge of Advantage States (KASAS)** of alternatives with respect to its criterion. This technique depends on the real advantage of all alternatives with the new constraints of concordance and discordance indices. Keen Analysis Surge of Advantage States able to introduce the actual comparisons between alternative, clarify the effective relations between alternatives, gives a priority vector with actual outranking relations and robust ranking of alternatives. Numerical example is introduced with actual estimations.

III. MATERIALS AND METHODS

A. Performance Matrix

Most of the MCDM methods require that assign the weights of importance for each attribute. Usually, these weights normalized to add up to one. Therefore, MCDM problems has expressed in matrix format. a decision matrix (performance matrix) A is an $(M * N)$ matrix in which elements a_{ij} indicate the performance of alternative A_i with respect to j^{th} criteria [10].

The performance of alternatives is evaluated in terms of decision criterion C_j (for $i = 1, 2, 3, \dots, M$ and $j = 1, 2, 3, \dots, N$). It is also assumed that the decision maker has determined the weights of relative performance of the decision criteria (denoted as W_j , for $j = 1, 2, 3, \dots, N$). This information is best summarized in the performance matrix as indicated in Figure 1.

	<i>Criteria</i>				
	C_1	C_2	C_3	...	C_N
<i>Alt</i>	W_1	W_2	W_3	...	W_N
A_1	a_{11}	a_{12}	a_{13}	...	a_{1N}
A_2	a_{21}	a_{22}	a_{23}	...	a_{2N}
A_3	a_{31}	a_{32}	a_{33}	...	a_{3N}
\vdots	\vdots	\vdots	\vdots	\ddots	\vdots
A_M	a_{M1}	a_{M2}	a_{M3}	...	a_{MN}

Figure 1: A Typical performance matrix.

After construction of performance matrix of alternatives with respect to its criterion the used technique has been applied.

B. Keen Analysis Surge of Advantage States (KASAS)

The major difference of any MCDM technique is the pholosogy in performance of alternatives with respect to its criterion. Therefore, this technique depends on the outcoming steps.

1) Detrmination of concordance set

The concordance set C_{kl} of two alternatives A_k and A_l , where $k \in M$ and $l \geq 1$, is defined as the set of all criteria for which A_k is preferred to A_l as indicated in equation 1:

$$C_{kl} = \{j\} \text{ such that } a_{kj} > a_{lj} \quad \begin{matrix} \text{for } j = 1, 2, 3, \dots, N \\ \text{and } k \neq l \end{matrix} \quad (1)$$

From equation 1 it is shown that when the comparison occurs between A_k and A_l , then the alternative A_k is preferred than A_l at criterion j only if $a_{kj} > a_{lj}$. Therefore, if comparison occurs between A_l and A_k , then the alternative A_l is not preferred than A_k at criterion j .

Equation 1 neglects the equality between any two alternatives with respect to each criterion to delete aredundnt of weights on each comparison and permits the correct preference relations between any two alternatives due to controversial without advantages.

2) Discordance Sets

The complementary subset of concordance is called the discordance set and it is described as in equation 2:

$$D_{kl} = \{j\} \text{ such that } a_{kj} < a_{lj} \quad \begin{matrix} \text{for } j = 1, 2, 3, \dots, N \\ \text{and } k \neq l \end{matrix} \quad (2)$$

From equations 1 and 2, it is shown that two complementary sets must be estimated without any intersections that resulted from the equality and the decision will be depends on the computation between two sets.

3) Construction of concordancematrix

The new technique suggested that the degree of difference between alternatives under each criterion must be known. This degree is the contribution in concordance between any two alternatives. So, the degree of difference between any two alternatives A_k and A_l under criterion j is denoted by $D_j.A_k/A_l$ and defined by equation 3 as follows:

$$D_j.A_k/A_l = a_{kj} - a_{lj} \quad \begin{matrix} \text{for } j = 1, 2, 3, \dots, N \\ \text{and } k \neq l \end{matrix} \quad (3)$$

The positive difference between any two alternatives belongs to concordance set and this degree of difference is the contribution in concordance of these alternatives under its criterion. Therefore, the max difference under each criterion ($|M.D_j|$) between any two alternatives should be estimated. The contribution of relation between any two alternatives A_k and A_l under any j criterion in concordance index is defined in equation 4.

The outranking relation of concordance of A_k/A_l is $(C_j A_k/A_l) = \frac{D \cdot D_j \cdot A_k/A_l}{|M.D_j|}$ and so,

$$C_j A_k/A_l = \frac{a_{kj} - a_{lj}}{|M.D_j|} \quad \text{for } k = 1, 2, 3, \dots, M \quad (4)$$

$k \neq l \text{ and } C_j A_k/A_l > 0$

This contribution is the partial outranking between A_k and A_l in concordance index. Thus, the concordance value C_{kl} that indicates the relative importance of alternative A_k over A_l with respect to all criterion is the sum of multiplication of contributions with its criterion weights as defined in equation 5 as follows:

$$C_{kl} = \sum_{j=1}^{j=N} W_j (C_j A_k/A_l) \quad \text{for } k; l = 1, 2, 3, \dots, M \quad (5)$$

$k \neq l \text{ and } C_j A_k/A_l > 0$

Therefore, the concordance matrix C is constructed from concordance values $C_{kl} (0 < C_{kl} < 1)$ and is defined as follows:

$$C = \begin{bmatrix} 0 & C_{12} & C_{13} & \dots & C_{1M} \\ C_{21} & 0 & C_{23} & \dots & C_{2M} \\ C_{31} & C_{32} & 0 & \dots & C_{3M} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ C_{M1} & C_{M2} & C_{M3} & \dots & 0 \end{bmatrix}$$

It should be noted that here the entries of matrix are equal zero due to the equality.

4) Construction of discordance matrix

The discordance matrix D expresses the degree that a certain alternative A_k is worse than a competing alternative A_l relative to each criterion. From the discordance set D_{kl} in equation 2, it is shown that the complementary subset of concordance is the set that has the negative difference between alternatives as indicated in equation 3. The same philosophy of the degree of difference in concordance is also applied in discordance with the negative difference. Thus, the contribution of relation between the alternatives A_k and A_l in discordance index under each criterion is defined in equation 6 as follows:

$$d_j A_k/A_l = \frac{a_{kj} - a_{lj}}{|M.D_j|} \quad \text{for } k; l = 1, 2, 3, \dots, M \quad (6)$$

$k \neq l \text{ and } d_j A_k/A_l > 0$

The discordance value d_{kl} that indicates the poorness of alternative A_k with respect to A_l as shown in equation 7 is the sum of multiplication of contribution under criterion j with criterion weight for all criterion and is indicated as follows:

$$d_{kl} = \sum_{j=1}^N W_j (d_j A_k/A_l) \quad \text{for } k; l = 1, 2, 3, \dots, M \quad (7)$$

$k \neq l \text{ and } d_j A_k/A_l > 0$

So, discordance matrix D is constructed from discordance values $d_{kl} (-1 < d_{kl} < 0)$ and is constructed as follows:

$$D = \begin{bmatrix} 0 & d_{12} & d_{13} & \dots & d_{1M} \\ d_{21} & 0 & d_{23} & \dots & d_{2M} \\ d_{31} & d_{32} & 0 & \dots & d_{3M} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ d_{M1} & d_{M2} & d_{M3} & \dots & 0 \end{bmatrix}$$

It should be noted that here the entries of matrix are equal zero due to the equality.

5) Preference of alternatives

From the concordance matrix, the outranking of any alternative to others was identified. Where, row one indicates the outranking of alternative number one to all other alternatives with the effects of criterion weights. Thus, sum of rows is the score of dominance of row alternative to all others. So, the Preference of dominance of any alternative A_k is constructed as follows:

$$P.A_k = \sum_{j=1}^M C_{kl} \quad \text{for } k; l = 1, 2, 3, \dots, M \quad (8)$$

and $k \neq l$

The priority values of dominance are the priority vector of dominance which has one column and M rows as indicated as in equation (9). The maximum value in priority vector of dominance candidates the alternative to be an optimal alternative depends on the required order.

$$P = [P.A_1, P.A_2, P.A_3, \dots, P.A_{M-1}, P.A_M]^T \quad (9)$$

In the discordance matrix, each row indicates the degree of outranked of A_k by other alternatives. Therefore, the sum of each row is the score of outranked of each alternative by others. Thus, the discordance of any alternative A_k is constructed as follows:

$$D.A_k = \sum_{j=1}^M d_{kl} \quad \text{for } k; l = 1, 2, 3, \dots, M \quad (10)$$

and $k \neq l$

The Total discordance ($T.D$) is the priority vector of discordance which has one column and M rows. The minimum value of discordance candidates the alternative to be optimal where all members are negative.

$$T.D = [D.A_1, D.A_2, D.A_3, \dots, D.A_{M-1}, D.A_M]^T \quad (11)$$

6) Ranking of alternatives

The Ranking of alternatives generally depends on the required decision. If the required decision is the optimal alternative that outrank all other alternatives, the alternative that has high score in priority vector of dominance is the optimal alternative (profit). If the the required decision is the optimal alternative that has less outranked from other alternatives (cost), the alternative that has the minimum value (negative value) in the priority vector of discordance is the optimal alternative.

Globally, the optimal ranking is estimated by summation of the corresponding values in the priority vector of dominance and in the priority vector of discordance and is represented in ranking vector ($R.V$) as follows:

$$R.V = [P.A_1, P.A_2, P.A_3, \dots, P.A_{M-1}, P.A_M]^T + [D.A_1, D.A_2, D.A_3, \dots, D.A_{M-1}, D.A_M]^T \quad (12)$$

Therefore, the alternative that has high score is the optimal alternative than others and vice versa. Ranking of alternative is constructed depends on the priority vector values. Ranking of alternatives indicates the outranking of any alternative to others, so, the outranking relation will be discussed in the next division.

7) Outranking relations

The outranking relations is constructed by means of a threshold value in the concordance index. For example, A_k will only have a chance to dominate A_l if its corresponding concordance index C_{kl} exceeds at least a certain threshold value \underline{C} . That is, the following is true: $C_{kl} > \underline{C}$. The threshold value \underline{C} can be determined as the average concordance index. That is, the following relation is true:

$$\underline{C} = \frac{1}{M(M+1)} \sum_{k=1}^M \sum_{l=1}^M C_{kl} ; \text{ where } l \neq k \quad (13)$$

The threshold value \underline{d} can be determined as the average discordance index, where \underline{d} is defined as follows:

$$\underline{d} = \frac{1}{M(M+1)} \sum_{k=1}^M \sum_{l=1}^M d_{kl} \quad \text{where } l \neq k \quad (14)$$

Once the two indices are defined, an outranking relation S was defined by:

$$S = A_k S A_l \quad \text{if and only if } \begin{cases} C_{kl} \geq C_{lk} \\ d_{kl} < d_{lk} \end{cases} \text{ and also } C_{kl} > -d_{lk} \quad (15)$$

Equation 15 assures that A_k outranks A_l if the concordance of A_k exceeds concordance of A_l and the discordance of A_k is lower than discordance of A_l .

IV. NUMERICAL EXAMPLE

It is assumed that there are ten options (A, B, C, \dots, J) to be compared using six criteria [11]. These criteria are assumed on numerical scales, and much that high values are deemed preferable to low ones. Details of performance matrix are contained in Table I with its criterion weights.

To easy understanding the estimations, we will begin with the degree of deference of alternative A with respect to all other alternatives as indicated in Table II. Max degrees of deference with respect to six criterions are indicated in Table III. The concordance contribution of alternative A with respect to all other alternatives is illustrated in Table IV with the application of equations 4 and 6.

TABLE I. Performance matrix of alternatives with respect to criterion

Criterion		C_1	C_2	C_3	C_4	C_5	C_6
CR.WE		0.25	0.1	0.15	0.25	0.05	0.2
Alternative	A	6	300	27	18	570	12
	B	2	450	21	19	400	23
	C	16	350	27	12	420	18
	D	10	500	20	12	450	20
	E	11	380	23	20	400	16
	F	5	250	31	10	430	18
	G	16	390	24	18	510	21
	H	17	400	22	26	380	23
	I	10	410	16	23	410	20
	J	5	250	18	21	400	22

TABLE II. DEGREE OF DIFFERENCE ($D.D. A_k/A_l$) OF ALTERNATIVES

	C_1	C_2	C_3	C_4	C_5	C_6
$D.D. A/B$	4	-150	6	-1	170	-11
$D.D. A/C$	-10	-50	0	6	150	-6
$D.D. A/D$	-4	-200	7	6	120	-8
$D.D. A/E$	-5	-80	4	-2	170	-4
$D.D. A/F$	1	50	-4	8	140	-6
$D.D. A/G$	-10	-90	3	0	60	-9
$D.D. A/H$	-11	-100	5	-8	190	-11
$D.D. A/I$	-4	-110	11	-5	160	-8
$D.D. A/J$	1	50	9	-3	170	-10

TABLE III. MAX DEGREE OF DIFFERENCE ($M.D_j. A_k, A_l$) UNDER EACH CRITERION

	C_1	C_2	C_3	C_4	C_5	C_6
$M.D_j$	15	250	15	14	190	11

Application of equations 8 and 10 are constructed in Table IV to extract the weighted concordance and discordance of alternative A with respect to others. The weighted concordance and discordance of alternatives from B to j with respect to other alternatives are defined From Table VI up to Table XIV respectively.

TABLE IV
 The contribution of concordance and discordance of A_k and A_l under each criterion

	C_1	C_2	C_3	C_4	C_5	C_6
$C_j \cdot A / B$	0.27	-0.6	0.4	-0.07	0.89	-1
$C_j \cdot A / C$	-0.67	-0.2	0	0.43	0.79	-0.55
$C_j \cdot A / D$	-0.27	-0.8	0.47	0.43	0.63	-0.73
$C_j \cdot A / E$	-0.33	-0.32	0.27	-0.14	0.89	-0.36
$C_j \cdot A / F$	0.07	0.2	-0.27	0.57	0.74	-0.55
$C_j \cdot A / G$	-0.67	-0.36	0.2	0	0.32	-0.82
$C_j \cdot A / H$	-0.73	-0.4	0.33	-0.57	1	-1
$C_j \cdot A / I$	-0.27	-0.44	0.73	-0.36	0.84	-0.73
$C_j \cdot A / J$	0.07	0.2	0.6	-0.21	0.89	-0.91

TABLE V
 The weighted concordance and discordance values with respect to alternative A

	C_1	C_2	C_3	C_4	C_5	C_6
$W_j(C_j A / B)$	0.0 7	-0.0 6	0.0 6	-0.0 2	0.0 5	-0.2
$W_j(C_j A / C)$	-0.1 7	-0.0 2	0	0.1 1	0.0 4	-0.1 1
$W_j(C_j A / D)$	-0.0 7	-0.0 8	0.0 7	0.1 1	0.0 3	-0.1 5
$W_j(C_j A / E)$	-0.0 8	-0.0 3	0.0 4	-0.0 4	0.0 5	-0.0 7
$W_j(C_j A / F)$	0.0 2	0.0 2	-0.0 4	0.1 4	0.0 4	-0.1 1
$W_j(C_j A / G)$	-0.1 7	-0.0 4	0.0 3	0	0.0 2	-0.1 6
$W_j(C_j A / H)$	0.1 8	-0.0 4	0.0 5	-0.1 4	0.0 5	-0.2
$W_j(C_j A / I)$	-0.0 7	-0.0 4	0.1 1	-0.0 9	0.0 4	-0.1 5
$W_j(C_j A / J)$	0.0 2	0.0 2	0.0 9	-0.0 5	0.0 5	-0.1 8

TABLE VI
 Degree of difference and concordance values of alternative B with respect to other alternatives under each criterion

	C_1	C_2	C_3	C_4	C_5	C_6
$D \cdot D \cdot B / A$	-4	150	-6	1	-170	11
$D \cdot D \cdot B / C$	-14	100	-6	7	-20	5
$D \cdot D \cdot B / D$	-8	-50	1	7	-50	3
$D \cdot D \cdot B / E$	-9	70	-2	-1	0	7
$D \cdot D \cdot B / F$	-3	200	-10	9	-30	5
$D \cdot D \cdot B / G$	-14	60	-3	1	-110	2
$D \cdot D \cdot B / H$	-15	50	-1	-7	20	0
$D \cdot D \cdot B / I$	-8	40	5	-4	-10	3
$D \cdot D \cdot B / J$	-3	200	3	-2	0	1

TABLE VII
 Degree of difference and concordance values of alternative C with respect other alternatives under each criterion

	C_1	C_2	C_3	C_4	C_5	C_6
$D \cdot D \cdot C / A$	10	50	0	-6	-150	6
$D \cdot D \cdot C / B$	14	-100	6	-7	20	-5
$D \cdot D \cdot C / D$	6	-150	7	0	-30	-2
$D \cdot D \cdot C / E$	5	-30	4	-8	20	2
$D \cdot D \cdot C / F$	11	100	-4	2	-10	0
$D \cdot D \cdot C / G$	0	-40	3	-6	-90	-3
$D \cdot D \cdot C / H$	-1	-50	5	-14	40	-5
$D \cdot D \cdot C / I$	6	-60	11	-11	10	-2
$D \cdot D \cdot C / J$	11	100	9	-9	20	-4

TABLE VIII

Degree of difference and concordance values of alternative *D* with respect to other alternatives under each criterion

	<i>C</i> ₁	<i>C</i> ₂	<i>C</i> ₃	<i>C</i> ₄	<i>C</i> ₅	<i>C</i> ₆
<i>D.D.D</i> / <i>A</i>	4	200	-7	-6	-120	8
<i>D.D.D</i> / <i>B</i>	8	50	-1	-7	50	-3
<i>D.D.D</i> / <i>C</i>	-6	150	-7	0	30	2
<i>D.D.D</i> / <i>E</i>	-1	120	-3	-8	50	4
<i>D.D.D</i> / <i>F</i>	5	250	-11	2	20	2
<i>D.D.D</i> / <i>G</i>	-6	110	-4	-6	-60	-1
<i>D.D.D</i> / <i>H</i>	-7	100	-2	-14	70	-3
<i>D.D.D</i> / <i>I</i>	0	90	4	-11	40	0
<i>D.D.D</i> / <i>J</i>	5	250	2	-9	50	-2

TABLE IX

Degree of difference and concordance values of alternative *E* with respect to other alternatives under each criterion

	<i>C</i> ₁	<i>C</i> ₂	<i>C</i> ₃	<i>C</i> ₄	<i>C</i> ₅	<i>C</i> ₆
<i>D.D.E</i> / <i>A</i>	5	80	-4	2	-170	4
<i>D.D.E</i> / <i>B</i>	9	-70	2	1	0	-7
<i>D.D.E</i> / <i>C</i>	-5	30	-4	8	-20	-2
<i>D.D.E</i> / <i>D</i>	1	-120	3	8	-50	-4
<i>D.D.E</i> / <i>F</i>	6	130	-8	10	-30	-2
<i>D.D.E</i> / <i>G</i>	-5	-10	-1	2	-110	-5
<i>D.D.E</i> / <i>H</i>	-6	-20	1	-6	20	-7
<i>D.D.E</i> / <i>I</i>	1	-30	7	-3	-10	-4
<i>D.D.E</i> / <i>J</i>	6	130	5	-1	0	-6

TABLE X

Degree of difference and concordance values of alternative *F* with respect to other alternatives under each criterion

	<i>C</i> ₁	<i>C</i> ₂	<i>C</i> ₃	<i>C</i> ₄	<i>C</i> ₅	<i>C</i> ₆
<i>D.D.F</i> / <i>A</i>	-1	-50	4	-8	-140	6
<i>D.D.F</i> / <i>B</i>	3	-200	10	-9	30	-5
<i>D.D.F</i> / <i>C</i>	-11	-100	4	-2	10	0
<i>D.D.F</i> / <i>D</i>	-5	-250	11	-2	-20	-2
<i>D.D.F</i> / <i>E</i>	-6	-130	8	-10	30	2
<i>D.D.F</i> / <i>G</i>	-11	-140	7	-8	-80	-3
<i>D.D.F</i> / <i>H</i>	-12	-150	9	-16	50	-5
<i>D.D.F</i> / <i>I</i>	-5	-160	15	-13	20	-2
<i>D.D.F</i> / <i>J</i>	0	0	13	-11	30	-4

TABLE XI

Degree of difference and concordance values of alternative *G* with respect to other alternatives under each criterion

	<i>C</i> ₁	<i>C</i> ₂	<i>C</i> ₃	<i>C</i> ₄	<i>C</i> ₅	<i>C</i> ₆
<i>D.D.G</i> / <i>A</i>	10	90	-3	0	-60	9
<i>D.D.G</i> / <i>B</i>	14	-60	3	-1	110	-2
<i>D.D.G</i> / <i>C</i>	0	40	-3	6	90	3
<i>D.D.G</i> / <i>D</i>	6	-110	4	6	60	1
<i>D.D.G</i> / <i>E</i>	5	10	1	-2	110	5
<i>D.D.G</i> / <i>F</i>	11	140	-7	8	80	3
<i>D.D.G</i> / <i>H</i>	-1	-10	2	-8	130	-2
<i>D.D.G</i> / <i>I</i>	6	-20	8	-5	100	1
<i>D.D.G</i> / <i>J</i>	11	140	6	-3	110	-1

TABLE XII

Degree of difference and concordance values of alternative *H* with respect to other alternatives under each criterion

	<i>C</i> ₁	<i>C</i> ₂	<i>C</i> ₃	<i>C</i> ₄	<i>C</i> ₅	<i>C</i> ₆
<i>D.D.H/A</i>	11	100	-5	8	-190	11
<i>D.D.H/B</i>	15	-50	1	7	-20	0
<i>D.D.H/C</i>	1	50	-5	14	-40	5
<i>D.D.H/D</i>	7	-100	2	14	-70	3
<i>D.D.H/E</i>	6	20	-1	6	-20	7
<i>D.D.H/F</i>	12	150	-9	16	-50	5
<i>D.D.H/G</i>	1	10	-2	8	-130	2
<i>D.D.H/I</i>	7	-10	6	3	-30	3
<i>D.D.H/J</i>	12	150	4	5	-20	1

TABLE XIII

Degree of difference and concordance values of alternative *I* with respect to other alternatives under each criterion

	<i>C</i> ₁	<i>C</i> ₂	<i>C</i> ₃	<i>C</i> ₄	<i>C</i> ₅	<i>C</i> ₆
<i>D.D.I/B</i>	8	-40	-5	4	10	-3
<i>D.D.I/C</i>	-6	60	-11	11	-10	2
<i>D.D.I/D</i>	0	-90	-4	11	-40	0
<i>D.D.I/E</i>	-1	30	-7	3	10	4
<i>D.D.I/F</i>	5	160	-15	13	-20	2
<i>D.D.I/G</i>	-6	20	-8	5	-100	-1
<i>D.D.I/H</i>	-7	10	-6	-3	30	-3
<i>D.D.I/J</i>	5	160	-2	2	10	-2

TABLE XIV

Degree of difference and concordance values of alternative *J* with respect to other alternatives under each criterion

	<i>C</i> ₁	<i>C</i> ₂	<i>C</i> ₃	<i>C</i> ₄	<i>C</i> ₅	<i>C</i> ₆
<i>D.D.J/A</i>	-1	-50	-9	3	-170	10
<i>D.D.J/B</i>	3	-200	-3	2	0	-1
<i>D.D.J/C</i>	-11	-100	-9	9	-20	4
<i>D.D.J/D</i>	-5	-250	-2	9	-50	2
<i>D.D.J/E</i>	-6	-130	-5	1	0	6
<i>D.D.J/F</i>	0	0	-13	11	-30	4
<i>D.D.J/G</i>	-11	-140	-6	3	-110	1
<i>D.D.J/H</i>	-12	-150	-4	-5	20	-1
<i>D.D.J/I</i>	-5	-160	2	-2	-10	2

TABLE XV. CONCORDANCE MATRIX, *C* = 0.19624

	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>	<i>G</i>	<i>H</i>	<i>I</i>	<i>J</i>	<i>P.A_k</i>
<i>A</i>	0	0.17	0.15	0.21	0.08	0.22	0.05	0.10	0.15	0.17	1.30
<i>B</i>	0.28	0	0.26	0.19	0.16	0.33	0.08	0.03	0.07	0.13	1.51
<i>C</i>	0.30	0.30	0	0.17	0.16	0.26	0.03	0.06	0.21	0.32	1.81
<i>D</i>	0.29	0.17	0.10	0	0.13	0.26	0.04	0.06	0.09	0.22	1.36
<i>E</i>	0.22	0.19	0.15	0.19	0	0.33	0.04	0.02	0.09	0.20	1.43
<i>F</i>	0.15	0.16	0.04	0.11	0.12	0	0.07	0.10	0.16	0.14	1.05
<i>G</i>	0.37	0.29	0.20	0.28	0.22	0.46	0	0.05	0.22	0.33	2.42
<i>H</i>	0.57	0.39	0.38	0.44	0.34	0.64	0.20	0	0.28	0.41	3.64
<i>I</i>	0.35	0.21	0.26	0.20	0.14	0.42	0.10	0.01	0	0.19	1.86
<i>J</i>	0.24	0.09	0.23	0.20	0.13	0.27	0.07	0.01	0.06	0	1.28

TABLE XVI. DISCORDANCE MATRIX, $d = -0.1968$

	A	B	C	D	E	F	G	H	I	J	D. A_k
A	0	-0.28	-0.30	-0.29	-0.22	-0.15	-0.37	-0.57	-0.35	-0.24	-2.75
B	-0.17	0	-0.30	-0.17	-0.19	-0.16	-0.29	-0.39	-0.21	-0.09	-1.95
C	-0.15	-0.26	0	-0.10	-0.15	-0.04	-0.20	-0.38	-0.26	-0.23	-1.77
D	-0.21	-0.19	-0.17	0	-0.19	-0.11	-0.28	-0.44	-0.20	-0.20	-1.98
E	-0.08	-0.16	-0.16	-0.13	0	-0.12	-0.22	-0.34	-0.14	-0.13	-1.49
F	-0.22	-0.33	-0.26	-0.26	-0.33	0	-0.46	-0.64	-0.42	-0.27	-3.18
G	-0.05	-0.08	-0.03	-0.04	-0.04	-0.07	0	-0.20	-0.10	-0.07	-0.67
H	-0.10	-0.03	-0.06	-0.06	-0.02	-0.10	-0.05	0	-0.01	-0.01	-0.43
I	-0.15	-0.12	-0.21	-0.09	-0.09	-0.16	-0.22	-0.28	0	-0.06	-1.38
J	-0.17	-0.13	-0.32	-0.22	-0.20	-0.14	-0.33	-0.41	-0.19	0	-2.10

After construction of the concordance and discordance matrices from the previous tables, the requirements are the priority of alternatives, outranking relations between alternatives. From Tables XV and XVI, the priority vectors of concordance and discordance are estimated ($P.A_k$ and $D.A_k$) and then the overall score of alternatives can be constructed from $R.V$ (equation 15) as indicated in Table XVII. In Table XVII, the priority of concordance and discordance was summed to be the $R.V$ vector that is the overall score of priority of all alternatives.

Thus, it is now necessary to start to identify patterns of dominance among the options, using the conditions in equation 19. Here, this yields the following initial dominance pattern for each alternative and the number of dominated alternatives with it and identification as in Table XVIII.

The new technique has one advantage than other methods where it constructs a complete relation between all alternatives and there is not a relation that contradict with them. Each relation has number of dominations varying than others, which is the road of ranking and outranking. In begins with the lowest domination, then F is the lowest alternative, the next is the relation that has one dominated. This is found in relation one, which dominates F , so A is the previous of F . thus alternative F becomes number ten and A has number nine in ranking. The next relation is the relation that has two dominated which is relation number nine which J dominates A and F . So, alternative J becomes number eight in ranking. This procedure gives that same ranking from $R.V$ vector and extract that the alternative, which has a high score, outranks all alternatives lower it.

When we subtract the number of alternatives from the dominated alternatives from each alternative, this extracts the ranking of alternative. As example in relation 10, A and F are dominated from J so, alternative J has ranking equal $10 - 2 = 8$, where 10 is the number of alternatives. Also, in relation 8 the alternative H has a ranking number one ($10 - 9 = 1$). Finally, ranking of alternatives as indicated in Table XVII arranged as follows HGICEBDJAF. This analysis is true for alternatives more than three.

TABLE XVII. RANKING OF ALTERNATIVES HGICEBDJAF

	P	ID/P	T.D	ID/T.D	R.V	ID/R.V
A	1.30	9.00	-2.75	9.00	-1.45	9.00
B	1.51	5.00	-1.95	6.00	-0.44	6.00
C	1.81	4.00	-1.77	5.00	0.04	4.00
D	1.36	7.00	-1.98	7.00	-0.62	7.00
E	1.43	6.00	-1.49	4.00	-0.06	5.00
F	1.05	10.00	-3.18	10.00	-2.13	10.00
G	2.42	2.00	-0.67	2.00	1.75	2.00
H	3.64	1.00	-0.43	1.00	3.21	1.00
I	1.86	3.00	-1.38	3.00	0.48	3.00
J	1.28	8.00	-2.10	8.00	-0.81	8.00
SUM	17.66	SUM	-17.71			

TABLE XVIII. INITIAL DOMINANCE PATTERN FOR EACH ALTERNATIVE

No.	Alternative	Dominated alternatives	Number of dominated alternatives
1	A	F	(1)
2	B	A, D, F, J	(4)
3	C	A, B, D, E, F, J	(6)
4	D	A, F, J	(3)
5	E	A, B, D, F, J	(5)
6	F	Non	(0)
7	G	A, B, C, D, E, F, I, J	(8)
8	H	A, B, C, D, E, F, G, I, J	(9)
9	I	A, B, C, D, E, F, J	(7)
10	J	A, F	(2)

V. DISCUSSION

ELECTRE methods yield a whole system of binary outranking relations between the alternatives. Because the system is not necessarily complete, the ELECTRE method is sometimes unable to identify the preferred alternatives. It only produces a core of leading alternatives [16]. ELECTRE method has a clear view of alternatives by eliminating less favourable ones, especially convenient while encountering few criteria with large number of alternatives in a decision-making problem [18]. Through a series of consecutive assessments of the outranking relations of alternatives, ELECTRE elicits the so-called concordance index, defined as the amount of evidence to support the conclusion that A_k outranks or dominates A_l , as well as, the discordance, the counter-part of concordance index.

The basic concept of the ELECTRE method deal with outranking relations by using pairwise comparisons among alternatives under each one of the criteria separately to extract the concordance and discordance sets. The suggested new technique, Keen Analysis Surge of Advantage States [KASAS] shows that the concordance sets are based on unfairness condition that make redundant scores for alternative which also gives an imprecise advantage for one or more alternative than others. These advantages are indicated in the equality of the classification of concordance and discordance sets, where $a_{kj} \geq a_{lj}$, then A_k primarily is preferred than A_l and the criterion j becomes in the concordance set. The condition makes redundant scores for alternatives A_k and A_l during equality of performance and score of concordance is added in two evaluations A_k/A_l and A_l/A_k without distinguishes between two alternatives. Therefore, the new technique suggests that deleting the equality from concordance set condition. As a result, from this suggestion is indicated in Table II when comparing A/C with respect to C_3 and A/G with respect to C_4 . From this analysis, the concordance set is restricted with the elements that have advantages than others.

The second step in ELECTRE technique is the construction of concordance and discordance indices. The concordance index equals the sum of weights with the members of the concordance set anyhow the type of outranking relation that decided to the nature of concordance relation (power and strongest). The concordance or discordance assures with the validation of relation of outranking and ranging with respect to outranking relation regardless recognizing the radical degree of performance of each alternative with respect to others concerning each criterion. Therefore, the new technique suggests that known the degree of difference between alternatives under each criterion. If these degrees are known for each alternative with respect to others, so, it is normalized with respect to maximum difference about its criteria. Normalization operation gives the contribution of relations between one alternative to others for all concordance and discordance indices as indicated in each column in Table IV. the concordance and discordance degrees not only depend on the normalized degree of difference but also depends on the criterion weights that enhance and constitute the effect of difference between alternatives. The weighted concordance C_{kl} and discordance d_{lk} members give a monitoring of the outranking relations between one alternative to others. From the nature of estimations inside the new technique it is shown that, measuring units of concordance and discordance are the same units and there is not a contradiction of units which also candidates this technique to be a dimensionless analysis with conflicting criteria. From concordance and discordance matrices, it is able to validate the preference of alternatives and get the actual outranking and ranking relations.

VI. CONCLUSION

From the weakness in the ELECTRE techniques and its shortages that have been indicated in most researches, this paper introduced a new dimensionless technique with conflicting criteria. This technique is a Keen Analysis Surge of Advantage States [KASAS] which able to deals with the decision problems under the presence of number of decision criteria. This technique has a power to deal with the conflicting criteria and incommensurable units. The new technique used a new adjustment of nature of concordance and discordance indices. It is indicated the contribution of advantage of one alternative in related to other alternatives contributions in the direction of dominance by the degree of difference. This technique is called dimensionless analysis because its structure eliminates any units of measure, which uses relative ratios instead of actual ones. Thus, it can be used in single or multi-dimensional decision making problems. This technique has a capability to identify the outranking relations between any two alternatives without shortage, delete the old understanding of impossibility of getting an outranking relation. Priority score is the advantage of this method offered through computation of composite priorities of the alternatives by linearly adding the weighted concordance and discordance indices values. The priority vector of alternatives in this technique is the effective tool to make a correct and complete ranking of alternatives.

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AUTHOR PROFILE

Ahmed M. El-kassas is an associate professor at Tanta University in Egypt. He teaches the operations research and industrial engineering courses in the Department of Production Engineering and Mechanical Design at Faculty of Engineering – Tanta University. He is author and co-author of various international publications.