Comparative Study for DC Motor Speed Control Using PID Controller

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Abstract—this paper presents a comparative study between Proportional-Integral-Differential (PID) tuning methods which they are commonly used in industrial application. Modified Ziegler-Nichols, Chien-Hrones-Reswick (CHR), Tyreus–Luyben, Damped Oscillation tuning methods have been chosen for this purpose. The parameters of these methods have been demonstrated. A permanent magnet direct current (PMDC) motor has been introduced as speed control target. The parameters of the PMDC have been calculated practically. Atmega328 microcontroller and IBT-2 driver that control the speed of the selected PMDC using pulse width modulation (PWM) have been adopted as a practical control circuit. For simulation the mathematical models of all PID tuning methods by using Matlab Simulink. All the results show that the Modified Ziegler-Nichols is the most suitable tuning method for targeted system, since this tuning method provides more dynamic response to load disturbance with acceptable overshoot.

Keyword: - Modified Ziegler-Nichols, Chien-Hrones-Reswick (CHR), Tyreus–Luyben, Damped Oscillation, PID Controller

I. INTRODUCTION

Process control industry has seen many advances in the past two decades in terms of the controller design and its implementation methods [1]. Proportional-Integral-Derivative (PID) controller has been used for several decades since 1940 in industries for process control applications. PID controllers tend to bring down the difference between the process variable and set point by comparing the response with the desired value [2]. PID controller is the most common control algorithm used in process control applications [3]. The performance of PID controller mostly depends on the precision of system models and their parameters. A setting of the proportional, integral and derivative values of a controller to get the best response for a process using a tuning algorithm is called tuning of a PID controller [4],[5]. In spite of these advances classical PID controller is undoubtedly the most popular controller in the industry because of its simple structure and robust performance in different operating conditions. It has been reported that 97% controllers in refining paper and pulp industries have PID structure [6].

To implement a PID controller effectively, tuning process of its parameters plays a vital role. J. G. Zeigler and N. B. Nichols were the first to present the simple tuning rules for PID controller [7], [8]. Most of controlled systems became poor in characteristics and even it becomes unstable, if improper values of the controller tuning constants are used [9].

DC motors are the oldest types of electro-mechanical machines. They were invented after the creation of the first sources of DC current. They are more advantageous over other AC machines regarding controlling the speed regulation, they could be found in many applications which require high-speed control accuracy and reliable effective dynamic responses such as industrial, medical and military purposes where the speed must be variable in wide range [10]. DC motor speed is directly proportional to armature voltage, by adjusting the armature voltage. So, it is important to design a proper controller to control the speed of a DC motor. The parameters of the DC motor should be known since they are essential to build the mathematical model [11], and then get the PID controller tuning gains [12]. The organization of this paper is presented mathematical modelling structure in section II, tuning methods of PID controller parameters are presented in section III, simulink model of a dc motor introduced in section IV, comparison of simulation and experimental results are given in section V and conclusions are explained in section VI.

II. MODELLING CONSTRUCTION OF PMDC MOTOR

To represent the DC motor mathematical model, the construction and working should be presented in similar and simpler than the DC motor system [13]. Generally, the PMDC models are composed of two parts: electrical
and mechanical equations. The electrical equation shows the main parameters which they are armature resistance \(R_a\) and inductance \(L_a\). On the other hand, the main parameters of mechanical equation are moment of inertia \(J_m\) and the friction coefficient \(B_m\). The targeted plant is PMDC motor (model: YA-070), which has the parameters shown Table I

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Armature resistance (R_a)</td>
<td>7 Ω</td>
</tr>
<tr>
<td>Armature inductance (L_a)</td>
<td>0.008436 H</td>
</tr>
<tr>
<td>Torque constant (K_t)</td>
<td>0.094 Nm/A</td>
</tr>
<tr>
<td>Back EMF constant (K_b)</td>
<td>0.094 v/rad/sec</td>
</tr>
<tr>
<td>Rotor inertia (J_m)</td>
<td>2.2097e-04 Nm/rad/ sec^2</td>
</tr>
<tr>
<td>Viscous friction constant (B_m)</td>
<td>1.65e-04 Nm/ rad/ sec</td>
</tr>
</tbody>
</table>

The equations below represent the transfer function of PMDC:

\[
V_a = R_a I_a(t) + L_a \frac{dI_a}{dt} + E_b(t) 
\]

(1)

\[
E_b = K_b \omega(t) 
\]

(2)

\[
T_m = K_t I_a(t) 
\]

(3)

\[
T_m(t) - T_l(t) = J_m \frac{d\omega}{dt} + B_m \omega(t) 
\]

(4)

where, \(E_b\) represents back electromotive force, \(\omega\) is the angular velocity, \(T_m\) is the motor torque, \(T_l\) is the load torque, \(J_m\) is the rotor inertia, \(B_m\) is the viscous friction coefficient, \(K_t\) is the torque constant, \(K_b\) is the back electromotive force constant and \(V_a\), \(R_a\), \(L_a\), \(I_a\) represent the voltage, resistance, inductance and current of armature respectively. By Substituting (2) in (1), (3) in (4) and \(T_l = 0\) result the following equations:

\[
V_a = R_a I_a(t) + L_a \frac{dI_a}{dt} + K_b \omega(t) 
\]

(5)

\[
K_t I_a(t) = J_m \frac{d\omega}{dt} + B_m \omega(t) 
\]

(6)

The angular velocity transfer function of the PMDC is written in Eq. (7).

\[
\frac{\omega(S)}{V_a(S)} = \frac{K_t}{L_a I_m S^2 + (R_a I_m + L_a B_m) S + (R_a B_m + K_t K_b)} 
\]

(7)

By integrating Eq. (7), the angular position is written in Eq. (8).

\[
\theta(S) = \frac{K_t}{L_a I_m S^2 + (R_a I_m + L_a B_m) S + (R_a B_m + K_t K_b)} 
\]

(8)

where \(\theta\) is the angular position.

### III. Tuning Methods and Optimization

Through the elapsed decades, the techniques of the industrial controlling process have made remarkable advances. Adaptive control, neural control, and fuzzy control have been studied extensively. The proportional-integral-derivative (PID) controller proved worthiness among of these controlling processes, which has been widely implemented in an industrial process due to its simple structure and robust performance for a wide operation range. The Ziegler-Nichols tuning formula is perhaps the mostly known tuning method [14]. Regrettably, it has been difficult to tune the PID gains in a proper way since many industrial plants are often encumbrances with problems such as high order, time delays, and nonlinearities [15]. Moreover fixed gain feedback controllers fail to provide the desired control performance over a wide range of operating conditions.

PID controller has three principle effects, the proportional (P) action which gives a change in the input (manipulated variable) directly proportional to the control error, the integral (I) action which gives change in the input that proportional to the integrated error and its main purpose to eliminate offset point and the less commonly used Derivative (D) action which controls the speed up response or to stabilize the system and gives a change in the input that proportional to the derivative of the controlled error. The overall controller output is the sum of the contributions from these three terms. The corresponding three adjustable PID parameters are shown in [16] and the references cited therein:

1) **Controller gain \(K_p\):** increasing \(K_p\) value gives more proportional action and faster control.
2) **Integral time \(T_i\):** decreasing \(T_i\) value gives more integral action and faster control.
3) **Derivative time \(T_d\):** increasing \(T_d\) value gives more derivative action and faster control.

The following controlling processes which they are discussed and compared in this paper:

**A. Modified Ziegler-Nichols Tuning method**
A conservative tuning method which is preferred for control loops at which the measure of oscillation provide \( \frac{1}{4} \) decay ratio and the corresponding large and undesirable overshoots for set point changes. The modified settings are shown in Table II which shows the adjustable PID parameters with/without overshooting of the controller [17].

**TABLE II. Modified Ziegler-Nichols Tuning Method**

<table>
<thead>
<tr>
<th>Controller type</th>
<th>Kp</th>
<th>Ti</th>
<th>Td</th>
</tr>
</thead>
<tbody>
<tr>
<td>With less overshooting</td>
<td>0.33Ku</td>
<td>Pu/2</td>
<td>Pu/3</td>
</tr>
<tr>
<td>Without overshooting</td>
<td>0.2Ku</td>
<td>Pu/2</td>
<td>Pu/3</td>
</tr>
</tbody>
</table>

where \( K_t = \frac{K_p}{T_i} \), \( K_d = K_p \times T_d \), \( K_u \) is the ultimate proportional gain, \( P_u \) is the corresponding period [18].

**B. Chien-Hrones-Reswick (CHR) Tuning Method**

CHR method has been introduced to enhance set-point regulation and the disturbance rejection. It presents one qualitative specification for the overshoot accommodation. The CHR tuning formula uses the time constant (T), delay time (L) and gain (K) which presented in Fig.1, [19].

![Fig. 1 Step response curve](image)

The CHR P, PI and PID controllers tuning formulas for set-point regulation have been presented in Table III. The quickest response without overshoot is titled “without overshoot,” and the quickest response with 20% overshoot is titled “with 20% overshoot”.

**TABLE III. CHR tuning formula for set point regulation**

<table>
<thead>
<tr>
<th>Controller type</th>
<th>Without overshoot</th>
<th>With 20% overshoot</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Kp</td>
<td>Ti</td>
</tr>
<tr>
<td>P</td>
<td>0.3/(kL/T)</td>
<td>-</td>
</tr>
<tr>
<td>PI</td>
<td>0.35/(kL/T)</td>
<td>1.2T</td>
</tr>
<tr>
<td>PID</td>
<td>0.6/(kL/T)</td>
<td>T</td>
</tr>
</tbody>
</table>

Similarly, Table IV could use to design P, PI and PID controllers for disturbance rejection purposes.

**Table IV. CHR tuning formula for disturbance rejection**

<table>
<thead>
<tr>
<th>Controller type</th>
<th>Without overshoot</th>
<th>With 20% overshoot</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Kp</td>
<td>Ti</td>
</tr>
<tr>
<td>P</td>
<td>0.3/(kL/T)</td>
<td>-</td>
</tr>
<tr>
<td>PI</td>
<td>0.6/(kL/T)</td>
<td>4L</td>
</tr>
<tr>
<td>PID</td>
<td>0.95/(kL/T)</td>
<td>2.4L</td>
</tr>
</tbody>
</table>

**C. Tyreus–Luyben Tuning Method**

The Tyreus-Luyben has been improved procedure for PID controller tuning. It’s quite similar to the Ziegler–Nichols method but the final controller settings are not similar [20]. Also this method only proposes settings for PI and PID controllers. This tuning method tends to reduce oscillatory effects and improve robustness. These
settings are depending on ultimate gain and period which they are given in Table V. This method is time consuming and forces the system to margin if instability similarly to ZN method [17], [21].

<table>
<thead>
<tr>
<th>Controller type</th>
<th>Kp</th>
<th>Ti</th>
<th>Td</th>
</tr>
</thead>
<tbody>
<tr>
<td>PI</td>
<td>Ku/3.2</td>
<td>2.2Pu</td>
<td>-</td>
</tr>
<tr>
<td>PID</td>
<td>Ku/3.2</td>
<td>2.2Pu</td>
<td>Pu/6.3</td>
</tr>
</tbody>
</table>

D. Damped Oscillation Tuning Method

In some processes which are not allowable to sustained oscillations and solving stability problem, Harriott has proposed a slight modification of the standard tuning procedure. In this modification of the ultimate method, the gain (proportional control only) is adjusted, using step input analogous to those used in the ultimate method, until a response curve with 1/4 of the decay ratio is obtained, [17],[21]. The controller parameters are calculated as in Table VI.

<table>
<thead>
<tr>
<th>Controller type</th>
<th>Kp</th>
<th>Ti</th>
<th>Td</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>1.1 Ku</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>PI</td>
<td>1.1 Ku</td>
<td>Pu/2.6</td>
<td>-</td>
</tr>
<tr>
<td>PID</td>
<td>1.1 Ku</td>
<td>Pu/3.6</td>
<td>Pu/9</td>
</tr>
</tbody>
</table>

IV. Time Domain Performance Indices

The output of PID controller given by Equation 9 contains an error e(t), which is the only variable that affects the control system for a constant Kp, Ki and Kd.

\[ u(t) = k_pe(t) + k_i \int e(t) dt + k_d \frac{de(t)}{dt} \]  

The various time domain performance indices (J) are therefore considered error e(t) as the main variable in their cost function. The cost functions for the integral of absolute error (IAE) as written in Equation 10 and for the integral of squared error ISE Equation 11 criteria contain absolute error and squared error in while the integral of time multiplied squared error (ITSE) Equation 12 and for integral of time multiplied absolute error (ITAE) Equation 13 contain time multiplied by the squared error and absolute error, respectively. The mathematical description of the cost function J of the above time domain performance indices are as follows:

\[ J(IAE) = \int_0^\infty |e(t)| dt \]  
\[ J(ISE) = \int_0^\infty e^2(t) dt \]  
\[ J(ITSE) = \int_0^\infty te^2(t) dt \]  
\[ J(ITAE) = \int_0^\infty t|e(t)| dt \]

It should be noted that the presence of time and its higher power in the cost function generally enhances the overshoot but causes fast rise time and reduces settling time, whereas only error term in the cost function results in low rise time and hence gives very less overshoot [22].

V. Simulation Results

The overall mathematical model has been built using Matlab Simulink is shown in Fig.2
Fig. 2: DC motor mathematical model with PID controller structure

PID controller using different tuning approaches will be analysed and compared to find the optimal parameters of DC motor speed control system. Table VII has been implicated to find the simulation results, for evaluating the performance of the proposed controller and its robust performance based on different tuning approaches.

<table>
<thead>
<tr>
<th>Tuning method</th>
<th>Kp</th>
<th>Ki</th>
<th>Kd</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modified ZN(CL)</td>
<td>1.6356</td>
<td>35.82913472070099</td>
<td>0.04977676</td>
</tr>
<tr>
<td>CHR Set point track 20% Mp</td>
<td>8.874607417590573</td>
<td>5.758340046345543</td>
<td>0.0048426070295566</td>
</tr>
<tr>
<td>Tyreus-Luyben</td>
<td>2.5597</td>
<td>12.7987</td>
<td>0.0373</td>
</tr>
<tr>
<td>Damped Oscillation</td>
<td>3.85</td>
<td>0.4720708446866485</td>
<td>0.0188393333</td>
</tr>
</tbody>
</table>

Modified-ZN tuning method required to set the PID parameters K_i and K_d to zero and set the K_p to an ultimate value K_u to sustain oscillation according to Eq. (8). Suitable K_p which will sustain oscillation at the output will represent K_u. P_u is the corresponding period for any symmetrically consequent points of the oscillation output as shown in Fig. 3.
The ultimate gain $K_u = 8.178$, by using the measurement tool in Matlab scope window the corresponding ultimate period has been found $P_u = 0.0913$, then applying these values to Table II to calculate the required PID controller gains. The same procedure could be applied to Tyreus–Luyben tuning method in Table V and damped oscillation tuning method in Table VI. Unlike CHR tuning method, it’s required to find $K$, $L$ and $T$, and then to find these required values step input will be applied to the system. Then inspect the output to measure the required parameters from the curve Fig. 4.

Using the measurement tool in the Matlab scope, the required parameters could be found: $K=101.5$, $L=0.001161$ and $T=1.100839$, hence $KL/T=0.107$. Then apply these values to Table III or Table IV to find the required PID controller. A comparison has been made for the proposed PID controller tuning methods by using the given model Fig.2. The simulation results of Modified Ziegler-Nichols Tuning method is shown in Fig. 5. Chien-Hrones-Reswick PID Tuning with Set point track 20% $M_p$ is shown in Fig. 6. Tyreus – Luyben Method is shown in Fig. 7 and Damped Oscillation Tuning Method is shown in Fig. 8.
Fig. 5 Modified Ziegler-Nichols Tuning method

Fig. 6 Chien-Hrones-Reswick PID Tuning Set point track 20% Mp
The comparison of the overall response results are presented in Table VIII.
TABLE VIII. The comparison of the response results

<table>
<thead>
<tr>
<th>Tuning Method</th>
<th>Tr (s)</th>
<th>Ts (s)</th>
<th>Mp(%)</th>
<th>Ess (%)</th>
<th>On-load speed(RPM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modified- ZN</td>
<td>0.0571</td>
<td>0.874</td>
<td>74.56</td>
<td>0.009</td>
<td>999.991</td>
</tr>
<tr>
<td>CHR set point 20% overshoot</td>
<td>0.0583</td>
<td>1.66</td>
<td>1.531</td>
<td>0.002</td>
<td>1002</td>
</tr>
<tr>
<td>Tyreus-Luyben</td>
<td>0.0573</td>
<td>0.714</td>
<td>15.69</td>
<td>0.0012</td>
<td>998.8</td>
</tr>
<tr>
<td>Damped Oscillation</td>
<td>0.0578</td>
<td>0.0578</td>
<td>0.02</td>
<td>0.002</td>
<td>1002</td>
</tr>
</tbody>
</table>

Simulation responses represent transient (settling time), steady state, and disturbance of the speed and input current period which shows the response ability of the controller.

In order to analyse, study and comparison the behaviour of the system under different tuning methods is presented in Table IX.

TABLE IX. Performance indices

<table>
<thead>
<tr>
<th>Tuning method</th>
<th>ISE</th>
<th>IAE</th>
<th>ITAE</th>
<th>ITSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modified -ZN</td>
<td>871.2</td>
<td>20.54</td>
<td>5.98</td>
<td>173.4</td>
</tr>
<tr>
<td>CHR, set point 20%Mp</td>
<td>256.5</td>
<td>7.121</td>
<td>4.978</td>
<td>7.853</td>
</tr>
<tr>
<td>Tyreus-Luyben</td>
<td>285.4</td>
<td>7.772</td>
<td>2.327</td>
<td>11.2</td>
</tr>
<tr>
<td>Damped oscillation</td>
<td>252.4</td>
<td>4.812</td>
<td>2.721</td>
<td>5.002</td>
</tr>
</tbody>
</table>

VI. PRACTICAL IMPLEMENTATION

Matlab simulink model has been used for tuning of the PID controller which has the parameters as in Table VII. The gains from Table VII have been applied to the practical circuit which consists of microcontroller Atmega328 which has the specifications in [23], PMDC, IBT-2 motor driver that controls the speed by using pulse width modulation (PWM) and tacho-generator which is works as a feedback device that coupled to the motor armature for measuring the actual speed of the motor shaft as shown in Fig. 9. The PMDC motor has the parameters in Table I. Power supply unit has been used to supply the motor with the suitable voltage with the sufficient current. The practical performance can be achieved by made disturbance on the motor shaft under stable condition. Plotting corresponding system responses for each proposed tuning method are presented in Fig. 10 to Fig. 13.

![Practical implementation system](image-url)
Fig. 10 Modified Ziegler-Nichols tuning method

Fig. 11 Chien-Hrones-Reswick PID Tuning Set point track 20% Mp

Fig. 12 Tyreus-Luyben tuning method
The response curves have been studied, it has been noticed that the simulation results in Table 8 of damped oscillation tuning method for the targeted plant (PMDC) because it has low rise time (Tr), low settling time (Ts), low overshoot (Mp), low steady state error (Ess), low performance index. Unlike the Modified Ziegler-Nichols Tuning method which shows high (Tr), high (Ts), high (Mp), high (Ess), high performance index, however CHR tuning method which shows high (Tr), high (Ts), low (Mp), low (Ess), low performance index. Tyreus Luyben tuning method shows low (Tr), less (Ts), low (Mp), low (Ess), low performance index.

In practical implementation of the studied tuning methods Modified Ziegler-Nichols Tuning method which shows high dynamic response to load disturbance but high overshoot, however CHR tuning method which shows less dynamic with higher settling time, Tyreus Luyben tuning method shows less overshoot and less dynamic to load disturbance, finally damped oscillation tuning method shows higher settling time with no overshoot.

**REFERENCES**


