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SUPER (a, d)-EDGE-ANTIMAGIC GRAPHS

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ABSTRACT- A graph G of order p and size q is called (a,d)-edge-antimagic total if there exists a one-to-one and onto mapping f from $V(G) \cup E(G) to \{1, 2, \ldots, p + q\}$ such that the edge weights $w(xy) = f(x) + f(y) + f(xy, xy \in E(G)$ form an AP progression with first term 'a' and common difference 'd'. The graph G is said to be Super (a,d)-edge-antimagic total labeling if the $f(V(G)) = \{1, 2, \ldots, p\}$. In this paper we obtain Super (a,d)-edge-antimagic properties of certain classes of graphs, including Fans graph, Single fan graph, Half Kite graph and Ambrela graph.

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I. INTRODUCTION

All graphs in this paper are finite, undirected and without loops or multiple edges. For a graph G,V(G) and E(G) denote the vertex set and the edge-set respectively. A (p,q) graph G is a graph such that $|V(G)| = p$ and $|E(G)| = q$. We refer the readers to [16] or [17] for all other terms and notation not provided in this paper.

A labeling of a graph G is any mapping that sends some set of graph elements to a set of non-negative integers. If the domain is the vertex-set or edge-set, the labeling are called vertex labelings or edge labelings respectively. Moreover if the domain is $V(G) \cup E(G)$then the labeling is called total labeling.

Let f be a vertex labeling of a graph G, we define the edge-weight of $uv \in E(G)$ to be $w(uv) = f(u) + f(v)$. If f is a total labeling then the edge-weight of uv is $w(uv) = f(u) + f(uv) + f(v)$.

Let G be a (p,q) graph, a bijective function $f: V(G) \cup E(G) \rightarrow \{1,2, \ldots, p + q\}$ is called an (a,d)- edge antimagic total labeling of G, if the set of all edge-weights : $\{w(uv): E(G)\} = \{a, a + d, \ldots, a + (n - 1)d\}$ where $a > 0$ and $d \geq 0$ are two fixed integers called 1st term and common difference respectively of an Arithmetic Progression (AP). In his Ph.D. thesis, Hegde called this labeling a strongly (a,d)-indexable labeling [1]. If such a labeling exists, then G is said to be an (a,d)-edge antimagic total labeling. Moreover, f is a super (a,d)-edge-antimagic total graph is a graph that admits a super (a,d)-edge antimagic total labeling. The (a,0)-edge-antimagic total labelings are usually called edge-magic in the literature [2,3,6,7]. Definition of (a,d)-edge-antimagic total labeling and super (a,d)-edge-antimagic total labeling where introduced by Simanjuntak et.al [14]. These labelings are natural extensions of the notions of edge magic labeling studied by Kotzig and Rosa [10]. Also see [3,9,13,15] and the concept of super edge-magic labeling defined by Enomoto et.al [11]. Mac Dougall and Wallis [12].

Many other researchers obtained different forms of antimagic graphs. For example see Bodendiek et.al [2], Harts field et.al [8], Baca et.al [3] established some relationships between (a,d)-edge-antimagic vertex labeling and (a,d)-edge-antimagic total labeling. Also Bac^{v} a et.al studied super (a,d) edge-antimagic total labelings of mK_n [4] and super (a,d)-edge-antimagic properties of certain classes of graphs, including friendship graphs, Wheels, fan, complete graphs and complete bipartite graphs [5].

In this paper we establish super (a,d)-edge-antimagic properties of certain classes of graphs, including Fans graph, Single fan graph, Half Kite graph and Ambrela graph.

II. FAN GRAPHS

The Fans graph F_n is a set of n triangles having a common vertex as a centre c and joined by a pendent edge cx where x is pendent vertex. For the $\boldsymbol{i}^{th}$ triangle denote the other two vertices x_i and y_i

Theorem2.1. Suppose that the Fans graph F_n $(n \geq 1)$ is super (a,d)-edge-antimagic total then $d < 3$.

Proof . Suppose that F_n, $n \geq 1$ has a super (a,d) edge-antimagic total labeling

$f: V(F_n) \rightarrow \{1,2, \dots, 5n+3\}$.

Thus $W = \{w(uv): w(uv) = f(u) + f(v) + f(uv), uv \in E(F_n) = \{a, a+d, \dots, a+3nd\}$ is set of edge-weights. One can easily see that the minimum possible edge-weight in super (a,d)-edge-antimagic total labeling is at least 2n+6. On the other hand, the maximum edge weight is no more than $9n+5$

Thus, $a + 3nd \leq 9n+5$

and $d \leq \frac{9n+5-a}{3n} < \frac{7n-1}{3n} < 3$

The following result is interesting because it Characterizes (a,1)-edge-antimagicness of Fans graphs.

Lemma2.1. The Fans graph F_n has (a,1)-edge-antimagic vertex labeling for n=1,2,...,6,7.

Proof . First we verify that F_n has (a,1)-edge-antimagic vertex labeling for n=1,2,...,6,7.

Trivially F_1 has (a,1) edge antimagic vertex labeling f_1 with $f_1(c) = 1, f_1(x_1) = 2$, $f_1(y_1) = 4$, $f(x) = 3$ or $f_1(c) = 3, f(x_1) = 2$, $f(y_1) = 4$, $f(x) = 1$

In the case n=2, label $f_2(c) = 3$, $f_2(x_1) = 1$, $f_2(y_1) = 5$, $f_2(x_2) = 4$, $f_2(y_2) = 6, f(x) = 2$.

If n=3, then label $f_3(c) = 5$, $f_3(x_1) = 1$, $f_3(y_1) = 4$, $f_3(x_2) = 3$, $f_3(y_2) = 7$, $f_3(x_3) = 6$, $f_3(y_3) = 8$, $f_3(x) = 2$

If n=4, then label $f_4(c) = 7$, $f_4(x_1) = 1$, $f_4(y_1) = 6$, $f_4(x_2) = 2$, $f_4(y_2) = 4$, $f_4(x_3) = 5$, $f_4(y_3) = 9$, $f_4(x_4) = 8$, $f_4(y_4) = 10$, $f_4(x) = 3$

If n=5, then the construct the vertex labeling f_5 in the following way:

$f_5(c) = 9$, $f_5(x_1) = 1$, $f_5(y_1) = 8$, $f_5(x_2) = 2$, $f_5(y_2) = 6$, $f_5(x_3) = 3$, $f_5(y_3) = 4$, $f_5(x_4) = 7$, $f_5(y_4) = 11$, $f_5(x_5) = 10$, $f_5(y_5) = 12$, $f_5(x) = 5$,

For n=6, put $f_6(c) = 11$, $f_6(x_1) = 1$, $f_6(y_1) = 10$, $f_6(x_2) = 2$, $f_6(y_2) = 7$, $f_6(x_3) = 3$, $f_6(y_3) = 5$, $f_6(x_4) = 4$, $f_6(y_4) = 6$, $f_6(x_5) = 9$, $f_6(y_5) = 13$, $f_6(x_6) = 12$, $f_6(y_6) = 14$.

Lastly for n=7, put $f_7(c) = 13$, $f_7(x_1) = 1$, $f_7(y_1) = 10$, $f_7(x_2) = 2$, $f_7(y_2) = 7$, $f_7(x_3) = 3$, $f_7(y_3) = 9$, $f_7(x_4) = 4$, $f_7(y_4) = 6$, $f_7(x_5) = 5$, $f_7(y_5) = 8$, $f_7(x_6) = 11$, $f_7(y_6) = 15$, $f_7(x_7) = 14$, $f_7(y_7) = 16, f(x) = 12$

It is a matter of routine checking to see that the vertex labeling f_i, $1 \leq i \leq 7$ are (a,1)-edge antimagic.

Conversely suppose that there exists a one to one function $f: V(F_n) = \{1, 2, \dots, 2n+2\}$ with the set of edge-weights of all edges in F_n is $W(F_n) = \{a, a+1, \dots, a+3n\}$. Let $f(c) = k, f(x) = l$, $l \leq k \leq 2n+2$ and $f(V(F_n)) = S_1 \cup S_2 \cup \{k\} \cup \{l\}$ where $S_1 = \{1,2, \dots, k-2, k-1\}$ *and* $S_2 = \{k+1, k+2, \dots, 2n, 2n+1\}$ is a set consecutive integers.

Let $W_1 = \{w(cx_i): 1 \leq i \leq n\} \cup \{w(cy_i): 1 \leq i \leq n\} \cup \{x\}$

$W_1 = \{k+1, k+2, \dots, 2k-2, 2k-1, 2k+1, 2k+2, \dots, k+2n+2x\}$

where as $W_2 = \{a, a+1, \dots, k-1, k\}$ and $W_3 = \{2k, 2k+4, \dots, a+3n-2\}$ as the sets of edge weights where W_2 and W_3 are obtained as sum of two distinct elements in $S_1 - S_1$ and $S_2 - S_1$ respectively. There exist an pendent edge cx such that $W(cx) = S_1 + S_2$ ie c+x where $S_1 \in S_1$, $S_2 \in S_2$. Set $S - \{S_1\}$ contains $> k - 3$ distinct elements and $\frac{k-3}{2}$ pairs of edge weight which implies k must be odd and $|W_2| = \frac{k-3}{2}$

The sum of the values in the set $S - S_1$ is equal to the sum of the edge weight in W_2

Thus

$\text{k(k-1)/2} - S_1 = \frac{(k-2)a}{2} + \left(\frac{k-2}{4}\right)\left(\frac{k-2}{2} - 1\right)$ where $1 \leq S_1 \leq k-1$

or $\frac{3k}{4} \leq a \leq \frac{3k+8}{4}$

The value of the c is used (2n+1) times and the value of other vertices are used twice in the computation of the edge-weights. The sum of all the vertex labels used to calculate the edge-weight of F_n is equal to $2\sum_{i=1}^{n} f(x_i) + 2\sum_{i=1}^{n} f(y_i) + (2n+1)f(x) + f(x) = 4n^2 + 10n + 6 + 2nk - 2k - f(x)$

The sum of the edge-weights in the set W is

$\sum_{n=1}^{n} w(cx_i) + \sum_{i=1}^{n} w(cy_i) + \sum_{i=1}^{n} w(x_i y_i) + w(cx) = 3na + \frac{\boldsymbol{9n^2+5n+4}}{\boldsymbol{2}}$

Thus the following equation holds

$2\sum_{i=1}^{n} f\ (x_i)$ +2 $\sum_{i=1}^{n} f\ \ (y_i) + (2n+1)k - 2k - f(x) = 3na + \frac{\mathbf{9n^2+5n+4}}{\mathbf{2}}$

or 4nk-4-2x-n^2 +15n+8=6na

or 4nk-2k-2x- n^2 +15n+8=6na.

Since k is odd from $3 \le k \le\ 2n+1$, $\frac{3k}{4} \le a \le\ \frac{3k+8}{4}$

and from last equation we get all possible integers of parameters n,k,a,x which are

(n, k, a, x) = (1, 1, 3, 3), (2, 3, 4, 2), (3, 5, 5, 2), (4, 7, 6, 3), (5, 9, 7, 5), (6, 11, 8, 8), (7, 13, 19, 12).

Theorem2.2. The fan graph $F_n$has super (a,d) antimagic total labeling where d = 0, 2 and n = 1,2,...,7.

Proof . Label the vertices of F_n, n = (1,2,...,7) by the vertex labeling $f_i,\ 1 \le i \le\ 7$. From the previous lemma it follows that each labeling $f_i,\ 1 \le i \le\ 7$ . . successively suppose the value 1,2,...,2n+2 and the edge-weight of all the edges of F_n constitute an AP of common difference 1. If for each F_n, n = {1, 2, ..., 7}, we make the edge labeling from the set {$2n$ + 3, $2n$ + 4, ...$5n$ + 3} then resulting total labeling can be

(1) Super (a,0)-edge-antimagic with the common edge-weight a or

(2) Super (a,2)-edge-antimagic where edge-weights constitute an AP of common difference 2.

III. SINGLE FAN

A Single fan $F_n, n \ge 2$ is a graph obtained by joining a vertex c to all the vertices of a path P_n and the vertex $x \notin P_n$. Thus F_n, = ($P_n \cup \{x\}$) + {c} where cx_{n+1} is stand. F_n has n+2 vertices say $c, x, x_1, x_2, \dots, x_n, \boldsymbol{x_{n+1}}$ and 2n edges say $cx_i, 1 \le i \le\ n$, and stand $\boldsymbol{cx_{n+1}}$ and $x_i x_{i+1}$, where $1 \le i \le\ \ n-1$

We obtain a least upper bound for super (a,d) edge antimagic total labeling of Single Fan.

Theorem3.1. If F_n = **(**P_n $\cup \{x_{n+1}\}$**)** + **{c}** is super (a,d)-edge-antimagic total labeling then d < 3.

Proof . Let : f : V (F_n) ∪ E (F_n) → {1, 2, ..., 3n + 2}, f is super (a,d)-edge-antimagic total Labeling.

The set of edge weights W(F_n) = {$w(uv) : uv \in$ E(F_n) } = {$a, a+d, \dots, a+(2n-1)d$}

The total edge weight of set is $\sum_{uv\in E(F_n)} w(uv) = 2na + n(2n-1)d$ ……………..(1)

The sum of all vertex labels and edge labels used to calculate the edge-weights is thus equal to $3\sum_{i=2}^{n-1} f\ (x_i) + (n+1)f(c) + f(x_{n+1})$+2{$f(x_1)$+$f(x_n)$} + $\sum_{uv\in E(F_n)} f(uv)$

=3$\sum_{i=1}^{n+2} f\ (x_i) + (n-2)f(c) - 2f(x_{n+1}) -\ \{f(x_1) +\ f(x_n\ )\} + \sum_{uv\in E(E)} f(uv)$

= 3{$1 + 2 + \dots + (n+2)$} + $(n-2)f\ (c) - 2f(x_{n+1}) -\ f(x_1)$ - $f(x_n)$ + { n+3,…,$3n+2$}

= $\frac{\{11n^2+25n+18\}+2(n-2)f(c)-4f(x_{n+1})-2f(x_1)-2f(x_n)}{2}$ ……………………………..…(2)

From (1) and (2) we have the following equation

$\frac{\{11n^2+25n+18)\}+2(n-2)f(c)-4f(x_{n+1})-2f(x_1)-2f(x_n)}{2}$ = $2na + n(2n-1)d$

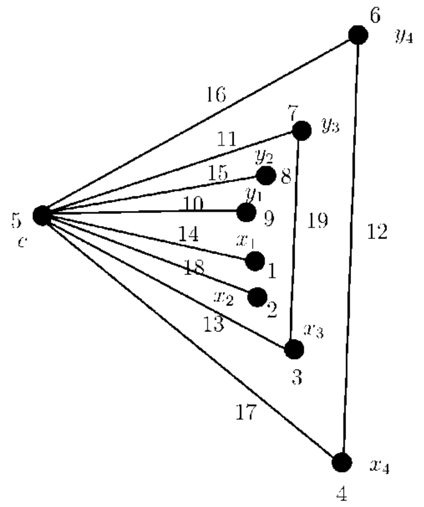
$d = \frac{11n^2+25n+18+2(n-2)f(c)-4f(x_{n+1})-2f(x_1)-2f(x_n)-4na}{2n(2n-1)}$

The minimum possible edge-weight is a = 1 + 2 +n + 3. The label of centre is $f(c) \le n+2, f(x_{n+1}) \ge 1$ and $f(x_1) + f(x_n) \ge\ 3$

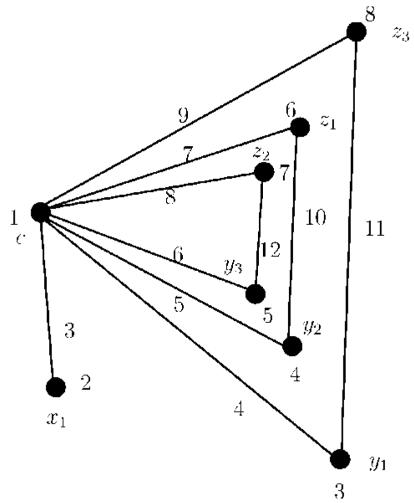
$d \le \frac{9n^2+n}{2n(2n-1)} < 3$

IV. HALF KITE

The half kite graph is a set of n triangles and 2n tails (pendent edges) having a common centre vertex 'c'. For i^{th} triangle, pendent edges are denoted by x_i and y_i denote the other two vertices see fig 1.



$n = 2$, *Fig*. 1



$n = 3$, *Fig*. 2

Theorem 4.1. Every half kite graph K_n, $n \geq 1$ has super (a,1)-edge-antimagic total labeling.

Proof . Now define the vertex labeling: V (K_n) = {1, 2, ...,, $4n + 1$} and the edge labeling E(K_n) = {$4n + 2$, ..., $5n$} in the following way

f (c) $= 2n + 1$

$f\ (x_i) = i,\ 1 \leq i \leq 2n$

$f\ (y_i) = 4n + 2 - i,\quad 1 \leq i \leq 2n$

$$f\ (cx_i) = \begin{cases} 6n + 3 - \dfrac{i+1}{2}, & if\ i\ is\ odd; \\ 8n + 3 - \dfrac{i}{2}, & if\ i\ is\ even \end{cases}$$

$$f\ (cy_i) = \begin{cases} 4n + 1 + \dfrac{i+1}{2}, & if\ i\ is\ odd; \\ 6n + 2 + \dfrac{i}{2}, & if\ i\ is\ even \end{cases}$$

$$f\ (x_i y_i) = \begin{cases} 8n + 2 + i, & if\ 1 \leq i \leq n-1 \\ 5n + 2, & if\ \mathrm{i} = \mathrm{n} \end{cases}$$

Now we study the super (a,1)-edge-antimagic total labeling of half kite K_n set of n triangles having common centre vertex with $\frac{n-1}{2}$ tails (pendent edges) at centre vertex c, let x_i denote the pendent vertices and y_i and z_i denote the other two vertices (see fig 2.)

Theorem 4.2. Every half kite K_n, $n \geq 1$ with $\frac{n-1}{2}$ pendent edges when n is odd has super (a,1)-edge-antimagic total labeling.

Proof . Let $f : V(K_n) \rightarrow \{1,2, \ldots, \frac{5n+1}{2}\}$ be the vertex labeling and $f: E(K_n) \rightarrow \{\frac{5n+1}{2} + 1 \ldots + 6n\}$ We define f as follows:

$f(c) = 1$

$fx_i = i + 1, \qquad 1 \leq i \leq \dfrac{n-1}{2}$

$f(y_i) = \dfrac{n+1}{2} + i, \qquad 1 \leq i \leq n$

$f(z_i) = \dfrac{(3n+1)}{2} + i, \qquad 1 \leq i \leq n$

$f(cx_i) = 2 + i, \quad 1 \le i \le \frac{n-1}{2}$

$f(cy_i) = \frac{n+1}{2} + 1 + i, \quad 1 \le i \le n$

$f(cz_i) = \frac{n+1}{2} + i, \quad n+1 \le i \le 2n$

$f(y_i z_{n-2(i-1)}) = 3n + 3 - i, \quad 1 \le i \le \frac{n+1}{2}$

$f(y_i z_{2n-2(1-i)}) = 4n + 3 - i, \quad \frac{n+3}{2} \le i \le n$

V. AMBRELA

A wheel $W_n, n \ge 3$ is a graph obtained by joining all vertices of a cycle C_n to a another vertex c called the centre. An Ambrela $A_n, n \ge 3$ is a graph obtained by joining c to y_1 of a path P_n. A_n contains 2n+1 vertices say $x_1, x_2 \dots \dots x_n, c, y_1, y_2, \dots, y_n$ and 3n edges say $cx_i,\ 1 \le i \le n, x_i x_{i+1},\ 1 \le i \le n-1,\ x_n x_1,\ y_i y_{i+1},\ 1 \le i \le n-1$ and cy_1.

Theorem 5.1. If Ambrela $A_n, n \ge 3$ is super (a,d)-edge-antimagic total labeling then $d < 3$.

Proof . Suppose that there exist a bijection $f: V(A_n) \cup E(A_n) \to \{1,2,\dots,n+1,n+2,\dots 5n+1\}$

Which is a super (a,d)-edge-antimagic total

And W = $\{ w(uv): w(uv) = f(u) + g(v) + g(uv), uv \in E(A_n)\} = \{a, a+d, \dots, a+(3n-1)d\}$

is the set of all edge weights. The maximum edge weight is no more than $2n + (2n+1) + (5n+1)$

Thus

$a + (e-1)d = a + (3n-1)\,d \le 9n + 2$ ………………………………….. … (3)

on the other hand, the maximum possible edge weight is at least

1 + 2 + $(2n + 2)$ ie $a \ge 2n + 5$ ……………………………………….. … ....(4)

From (3) and (4) for ambrela A_n we have

$a + (3n-1)d \le 9n + 2$

$d \le \frac{9n+2-a}{3n-1} \le \frac{7n+3}{3n-1} < 3$

Theorem 5.2. The Ambrela A_n An has super (a,d)-edge-antimagic total labeling with $f(x_i) = i$, $1 \le i \le n,\ f(c) = n+1,\ f(y_i) = n+1+i,\ 1 \le i \le n,$ if and only if $d = \frac{10n+9}{3}$.

Proof . Assume that a one-one and onto function f: V $(A_n) \cup$ E $(A_n) \to \{1,2,\dots,5n+1\}$ is a super (a,1)-edge-antimagic total labeling. In the computation of the edge weight of A_n under the one-one and onto function f the label of the centre is used n+1 times, the label of each vertex $x_i,\ 1 \le i \le n$ is used 3 times, label of each vertex y_i, $1 \le i \le n$ - 1 is used 2 times and y_n once.

Thus

$3\sum_{i=1}^{n} f(x_i) + (n+1)f(e) + 2\sum_{i=1}^{n-1} f(y_i) + f(y_n) + \sum_{e \in E(A_n)} f(e)$

$= 3(1 + 2 + \cdots 2n + 1) + (n-2)f(e) - \sum_{i=1}^{n-1} f(y_i) - 2f(y_n) + [2n + 2 \dots 5n + 1]$

$= \frac{33n^2+27n+6}{2} + (n-2)f(e) - \sum_{i=1}^{n} f(y_i) - f(y_n)$ ……………………………..……(5)

The sum of the edge weights under the one-one and onto mapping f is

$\sum_{e \in E(A_n)} w(e) = \frac{3n}{2}\{2a + (3n-1)\}$ ………………………………………………... (6)

From (5) and (6)

$\frac{3n}{2}\{2a + (3n-1)\} = \frac{33n^2+27n+6}{2} + (n-2)f(c) - \sum_{i=1}^{n} f(y_i) - f(y_n)$

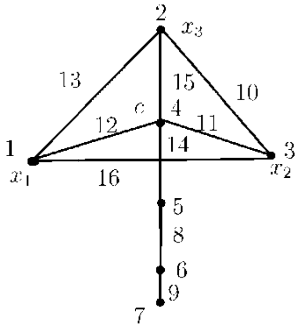
By putting $f(c) = n + 1$

$3na + \frac{3n}{2}(3n-1) = \frac{33^2 + 27n+6}{2} + (n-2)(n+1) - [(n+2) + \cdots + (2n+1)] - (2n+1)$

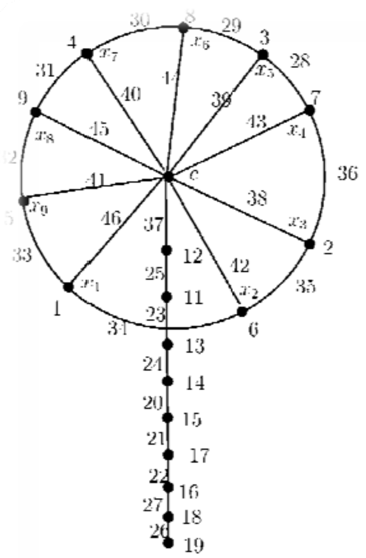
$3na = \frac{33n^2+18n}{2} - \frac{3n}{2}(3n-1) + (n-2)(n+1) - \frac{(2n+1)(2n+2)}{n} - \frac{(n+1)(n+2)}{2} - (2n+1)$

$a = \frac{10n+9}{3}$ where n = 3,9,15,21.....

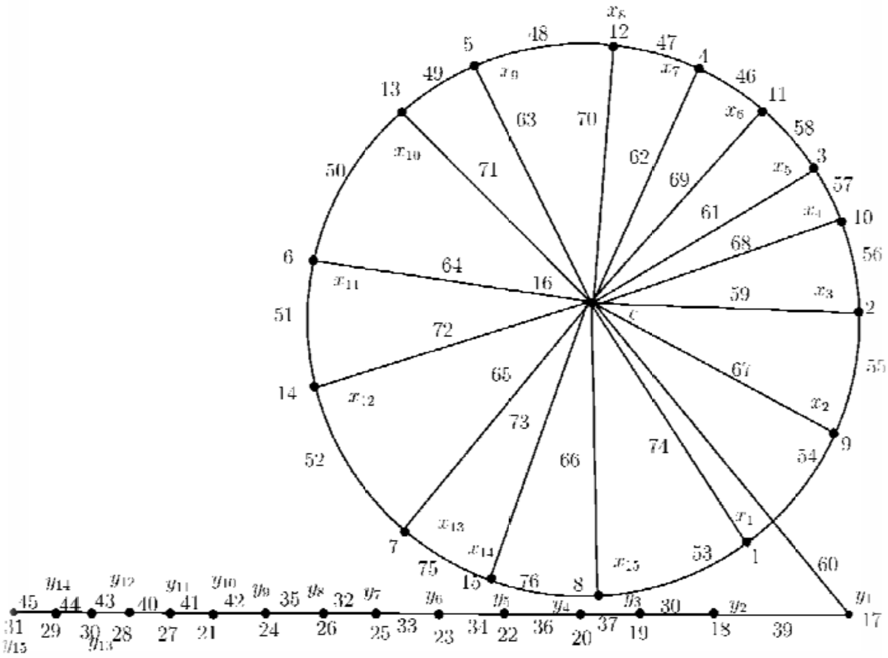
The following figures (3),(4),(5) and (6) are drawn for n=3,9,15,21. The general representation is left to the reader.



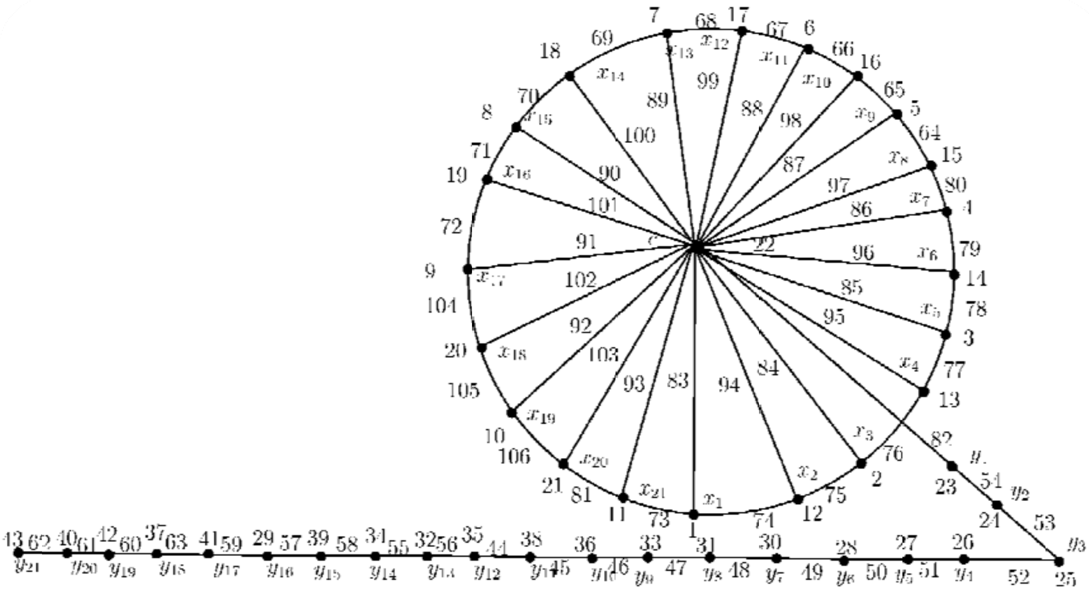
$Fig.\ 3, n = 3, a = 15$



$Fig.\ 4.\ n\ \ 9, a\ \ 38$



$Fig.\ 5.\ n\ \ 15, a\ \ 61$



$Fig. 6. n = 21, a = 84$

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