NUMERICAL ANALYSIS OF SINGLE STAGE PULSE TUBE REFRIGERATOR

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***Abstract*—A numerical model based on adiabatic flow behaviour has been developed for predicting performance of a single stage Pulse Tube Refrigerator (PTR). The pressure oscillations of compressor, PTand reservoir have been derived with assumption that compressor volume varies sinusoidally. The relationships of mass rates (cold and hot ends, orifice and Double Inlet (DI) valves) are studied for the PTR system. Refrigeration powers are arrived at by considering the effect of void volumes and phase changes between PT pressure and flow of mass at cold zone. Cooling powers predicted by the model have been compared with experimental data of a single stagePTR developed in our laboratory. Analysis shows that theoretical predictions are in reasonable agreement with experimental data.**

Keywords - Cryocooler, Pulse tube, Numerical modelling, Helium Refrigerator, Refrigeration power.

I. INTRODUCTION

The cryocoolers produce a known cooling power at a specific operating temperature in the cryogenic range. Different types of refrigerators are available namely GM, Stirling, Pulse tube (PT) and JT whose performances have been significantly improved over the years. Cryocoolers are used for a variety of applications such as magnet cooling, radiation shield cooling, cryosorption etc. As on date, commercial cryocoolers are available from various suppliersbut they are quite expensive. The cryocooler used for an application should have high efficiency, long term reliability, low cost, low maintenance, low noise level and low electromagnetic interference with the applications. The absence of moving components at cryogenic temperatures makes PTRquite reliable for long term use in applications.

In spite of the several of advancements in the area of PT coolers, the working mechanism in a PT cooler is not fully understood and hence different theoretical models are being used to understand PTR. The theoretical analysis has an important role to play towards understanding dynamic characteristics and internal processes of fluid flows in a PTR.

In this work, single stage PTR systems have been analysed assuming adiabatic fluid flow through PTR. Working fluid gas is treated as anideal gas and respective equations for pressure and volume variations with time have been used. The above analysis has led to optimization of different PT configurations as well as prediction of mass-flow rates at different locations in aPTR system. The present analysis also predicts cooling powers with respect to cold zone temperature. Theoretically predicted cooling powers are compared with experimental data of a single stagePTR developed in our laboratory. These are presented in this work.

II. MODELS FOR PTR

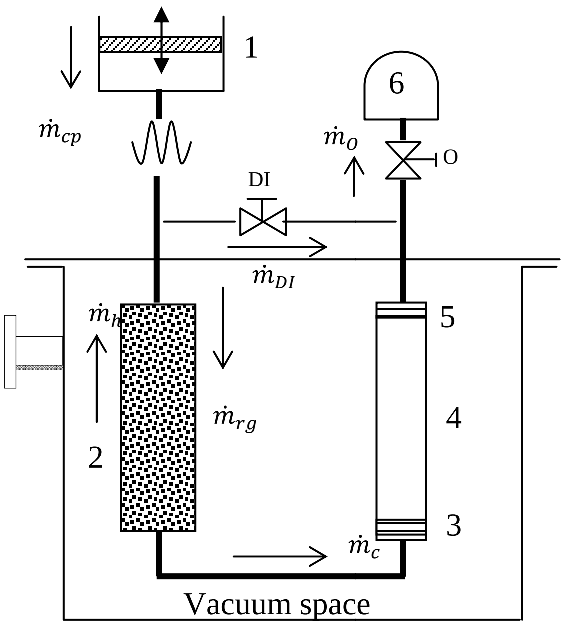
Since the discovery of the Basic Pulse Tube Refrigerator (BPTR) in 1963 by Gifford andLongsworth considerable developments have been made on PTRs and their performances have been improved over the last three decades. The cooling mechanism of a BPTR is surface heat pumping i.e. exchange of heat between working fluid and PT wall. Orifice Pulse Tube Refrigerator (OPTR) developed in 1984 by Mikulin led to improved performance over BPTR by introduction of an orifice valve and a reservoir. Subsequently Double inlet Pulse Tube Refrigeration(DPTR) developed by Zhu & Chen in 1990 led to further improvements of PTR by addition of Double Inlet (DI) valve by connecting compressor directly toPThot zone. Different models have been used for prediction of performances of PTRs and some of them are a) isothermal model, b) enthalpy flow theory, c) model based on solutions of energy, momentum and continuity equations, d) thermo-acoustic theory etc. In this work, we have attempted analysis of Pulse Tube performances using a model based on adiabatic flow of gas through PT, the details of which are discussed in this work.

A. *Adiabatic model for pulse tube refrigerator*

The adiabatic model of PTR assumes;

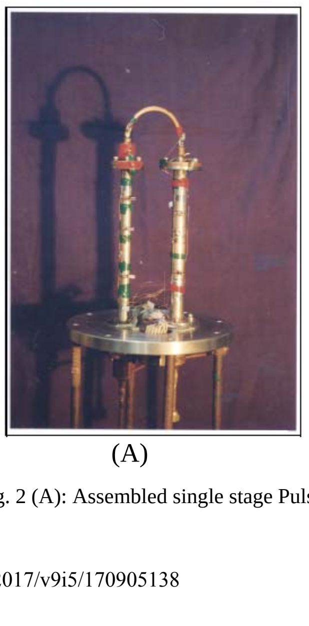
- Flow in the system is one-dimensional.
- Regenerator is 100 % effective.
- There are no pressure drops in regenerator, PT and heat-exchangers.
- Gas temperatures are same as that of wall temperature at cold & hot zones heat exchangers.
- Heat exchanger effectiveness is 100 %.
- Gas flow through valves and PT are adiabatic.
- Temperature of the reservoir is constant and is at ambient temperature.
- Working fluid (helium)istreated as ideal.

Fig. 1 shows diagram of DPTR. When DI valve is closed system becomes an OPTR when both orifice and DI valves are closed system becomes a BPTR. The mass flows through compressor, regenerator, cold end, hot end, orifice and DI valves are also shown in figure. Fig. 2(A) shows assembled single stage PT cryocooler and Fig. 2(B) shows photo of complete experimental set up ofsingle stage PTR.Refrigeration powers have been measured on this set up of single stage PTR.

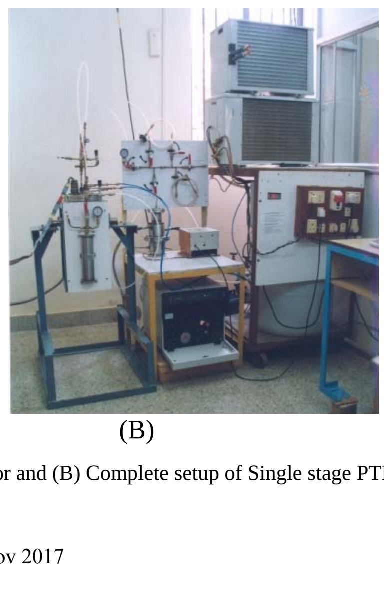


1. Compressor, 2. Regenerator, 3. Cold end heat exchanger, 4. Pulse tube, 5. Hot end heat exchanger, 6. Reservoir, O. Orifice Valve, DI. Double Inlet valve.

Fig. 1: Pulse Tube Refrigerator diagram.



(A)



(B)

Fig. 2 (A): Assembled single stage Pulse Tube Refrigerator and (B) Complete setup of Single stage PTR.

In our numerical model, the compressor volume variation is assumed to be sinusoidal and is given by,

$$V_{cp} = Vo + \frac{Vs}{2}(1 + sin\, 2\pi f t) \quad (1)$$

Here, Vo is dead volume of compressor and Vs is swept volume at oscillating frequency ƒ.

From first law thermodynamics it can be shown,

$$\mathrm{dQ = dW + du + hdm} \quad (2)$$

For an adiabatic process, $dQ = 0$.

Using the perfect gas equation, $PV = mRT$it is given by,

$$\frac{dm}{m} = \frac{dP}{P} + \frac{dV}{V} + \frac{dT}{T} \quad (3)$$

Using,$\frac{dT}{T} = \frac{R}{Cp}\frac{dP}{P}$the rate of change of mass flow and pressure are derived as:

$$\frac{dm}{dt} = \frac{1}{RT}\left[P\frac{dV}{dt} + \frac{1}{k}V\frac{dp}{dt}\right] \quad (4)$$

and$\frac{dP}{dt} = \frac{k}{V}\left[\dot{m}RT - P\frac{dV}{dt}\right]$ (5)

Mass flow through compressor can be written as

$$\dot{M}_{cp} = \dot{m}_{rg} + \dot{M}_{di} \quad (6)$$

The regenerator mass flow is proportional to pressure difference between compressor and PT,

$$\dot{m}_{rg} = C_{rg}(P_{cp} - P_t) \quad (7)$$

Where, $C_{rg} = \frac{\rho\pi {d_{rg}}^2 {d_h}^2}{600\, L_{rg}\mu}\frac{\emptyset^3}{(1-\emptyset)^2}$ (8)

Using the following equations the pressure variation of the compressor and reservoir can be calculated:

$$\frac{dP_{cp}}{dt} = \frac{k}{Vcp}\left[\dot{M}_{cp}RT - P_{cp}\frac{dVcp}{dt}\right] \quad (9)$$

$$\frac{dP_r}{dt} = -\frac{k}{V_r}[\dot{m}RT] \quad (10)$$

Nozzle flow equation has been assumed for flow rates through valves. Mass flows through orifice and DI valves can be obtained by equations,

$$\dot{m} = -\mu A_o\sqrt{\left(\frac{2k}{(k-1)}\frac{{P_{PT}}^2}{RT_H}\left[\frac{P}{P_{PT}}\right]^{\frac{2}{k}} - \left[\frac{P}{P_{PT}}\right]^{\frac{k+1}{k}}\right)} for\ P_{PT} > P \quad (11)$$

$$\dot{m} = -\mu A_o\sqrt{\left(\frac{2k}{(k-1)}\frac{P^2}{RT_H}\left[\frac{P_{PT}}{P}\right]^{\frac{2}{k}} - \left[\frac{P_{PT}}{P}\right]^{\frac{k+1}{k}}\right)} for\ P > P_{PT} \quad (12)$$

In equations (11) and (12), P is equal to P_{Cp} for flow through the DI valve and P is equal to P_r for flow through the orifice valve. For the case of DPTR, the mass flow rate through the DI valve is bypressure difference between the compressorand Pulse Tube, whereas in the case OPTR, the mass flow rate through orifice is by the pressuredifference between the Pulse Tube and the reservoir.

Using these equations, flow rate ofmass through hot end PTcan be calculated as,

$$\dot{M}_o = \dot{M}_h + \dot{M}_{dι} \quad (13)$$

$$\dot{M}_h = \dot{M}_o - \dot{M}_{dι} \quad (14)$$

The flow rateof mass through cold end of the Pulse Tube is given by:

$$\dot{M}_c = \frac{1}{RTc}\left[\frac{Vpt}{k}\frac{dPpt}{dt} + \dot{M}_h * T_h\right] \quad (15)$$

Using equations (4) to (14), the pressure variation in PT can be obtained as,

$$\frac{dP_{pt}}{dt} = \frac{[\dot{M}cp - \dot{M}o]*R*tc - [\dot{M}o - \dot{M}dι]*R*th - Vac*\frac{dPcp}{dt}*\frac{tc}{th}}{[\frac{Vpt}{k} + Vc + Vh + \frac{Vrg}{Trg}]} \quad (16)$$

Where $T_{rg} = \frac{Th - Tc}{ln\frac{Th}{Tc}}$ (17)

The above time varying differential equations are solved using Fourth order Runge-Kutta method. The Refrigeration effect produced by oscillating gas in PT is equal to enthalpy flow rate at cold end of PT averaged over a complete cycle and is given by,

$$Q = \frac{1}{\tau}\int_0^t\left[T_c * C_p * \dot{M}_c\right]dt \quad (18)$$

III. Solving Methodology

The equations relating to mass flow through regenerator, orifice and DI valves etc. are algebraic equations. Pressure variations of compressor, PT and reservoir are time varying ordinary differential equations and need initial values for solving them. From equation (1), the initial condition for the swept volume of compressor can be chosen at t=0, which indicates that compressor piston is in middle position with the pressure being the average value. Starting from this initial value of pressure at a given time, 4^{th} order Runge-Kutta method has been used to obtain the pressures at next time for compressor, PT and reservoir. This procedure is continued overa complete cycle and also till consistent results of pressure, mass flow rates are obtained at every instant of time. Usingmass flow rates at cold end over a complete cycle, refrigeration power produced by the PT cooler is obtained using equation (18). MATLAB has been used as the language for developing the program for solving the above equationsby applying Runge-Kutta procedure and obtaining solutions in the numerical analysis. The performances of PTR under the influence of different parameters are examined in the following section.

IV. Results and discussion

In this section we discuss results of numerical analysis using adiabatic model for single stagePTR. In the initial part, typical behaviour of pressures at different locations, PV diagram and mass flow rates are presented. In the later part, refrigeration powers predicted by model arecompared with experimental data of single stage PTR developed in our laboratory (shown in Fig. 2).

A. Pressure Variation with respect to Time.

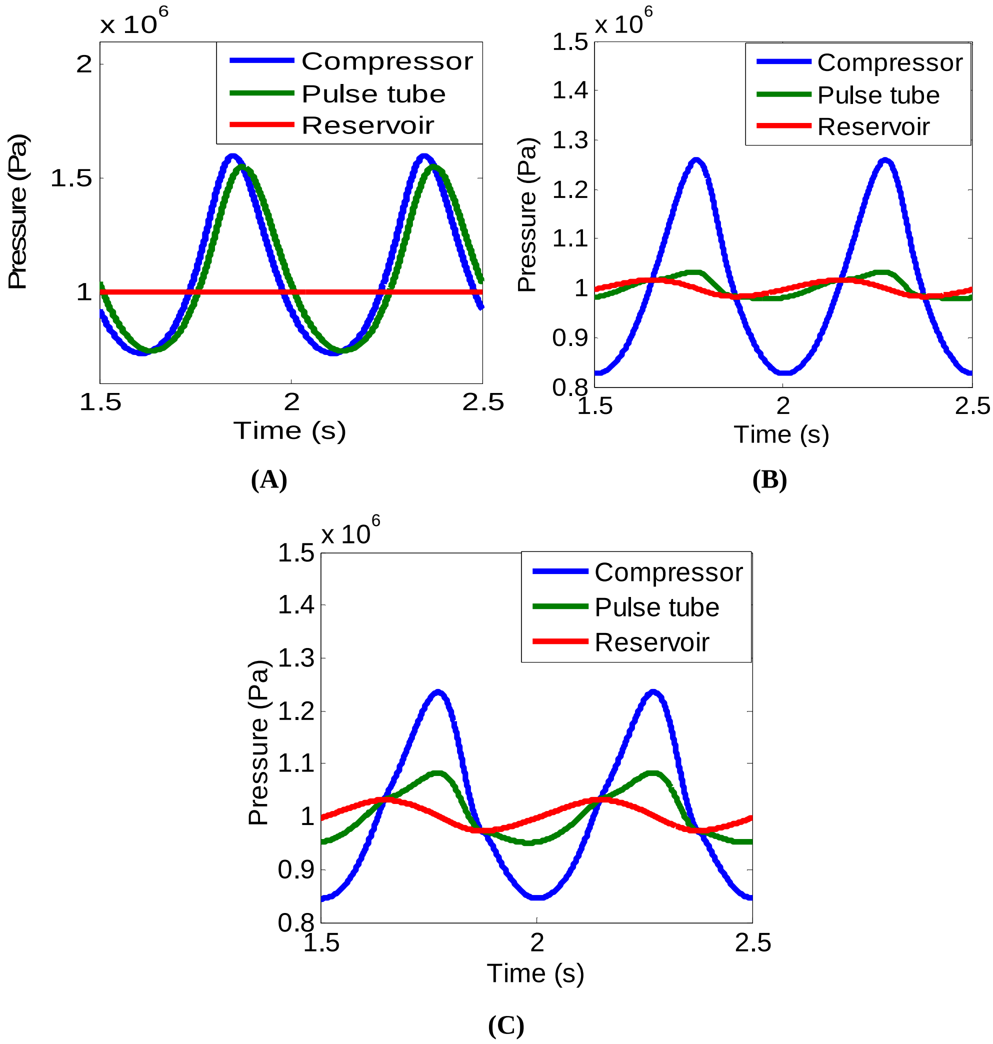
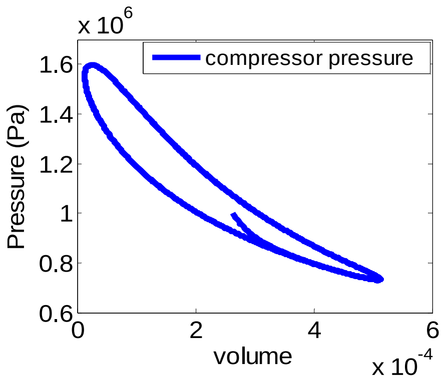


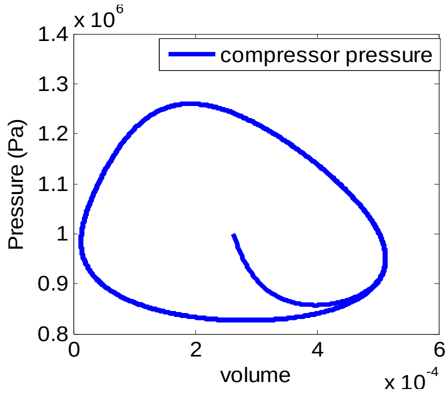
Fig. 3: Pressure variationsof different PTRs with respect to time (A) BPTR, (B) OPTR and (C) DPTR.

Fig. 3 shows pressure variations of compressor, PT and reservoir with respect to time. Figures A, B and C show BPTR, OPTR and DPTR systems respectively. In BPTR, there are no mass flows through any external valves. Due to this, compressor pressure amplitude is highest when compared with other systems. Hence in this case, Pulse Tube pressure is nearly same as that of compressor pressure. In OPTR system, PT and reservoir pressures are smaller compared to those of DPTR. This is because; both pulse tube and reservoir pressure amplitudes decrease when the orifice valve is opened. But in the DPTR system, additional mass flow rates occur through the DI valve, which leads to slightly increased pressure amplitude for both pulse tube and reservoir as can be seen from the below figures.

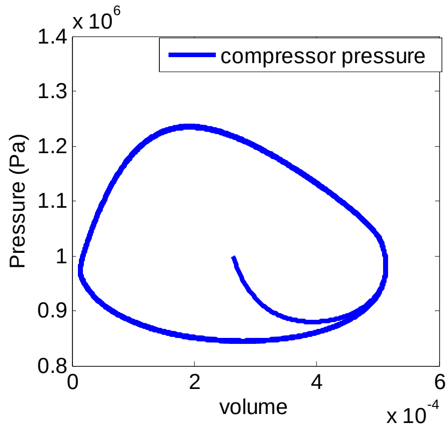
B. PV Diagram for Compressor



(A)



(B)



(C)

Fig. 4: Compressor pressure variationswith respect to volume for different PTRs. (A) BPTR, (B) OPTR and (C) DPTR.

Fig. 4 (A), (B) and (C) plot the compressor pressures with respect to its volume for BPTR, OPTR and DPTR respectively. The PV diagram of OPTR and DPTR appear to be similar and PV work (i.e. the area within the PV curve) is higher when compared to BPTR. It is observed that phase difference between pressure and volume is nearly 90º for DPTR and OPTR, whereas it is approximately 180º in case of BPTR.

C. Typical Mass Flow Rates at Compressor, Cold and Hot Ends.

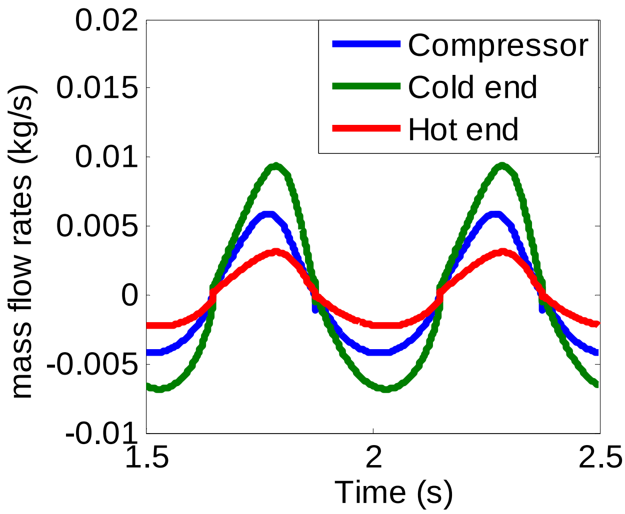


Fig. 5: Mass-flow rates at compressor, cold and hot ends for DPTR.

Fig. 5 plots the mass flow rates of compressor, cold and hot ends for the specific case of DPTR. Massflow rate through cold zone is slightly larger when compared to that of compressor. This is due to density differences of working gas in compressor (which is at ambient temperature) and at cold end. Due to same reason, hot end mass flow rate is lower than that of cold end.

D. Typical Mass Flow Rates of Regenerator, Orifice and DI Valves

Fig. 6 shows mass flow rates through regenerator, orifice and DI valves for DPTR. In this case, mass flow rate through orifice is found to higher when compared to that through the regenerator. This is perhaps due to additional mass flow through DI valve. Also it observed that mass flow rate through DI valve is lesser when compared to thatthrough the orifice valve.

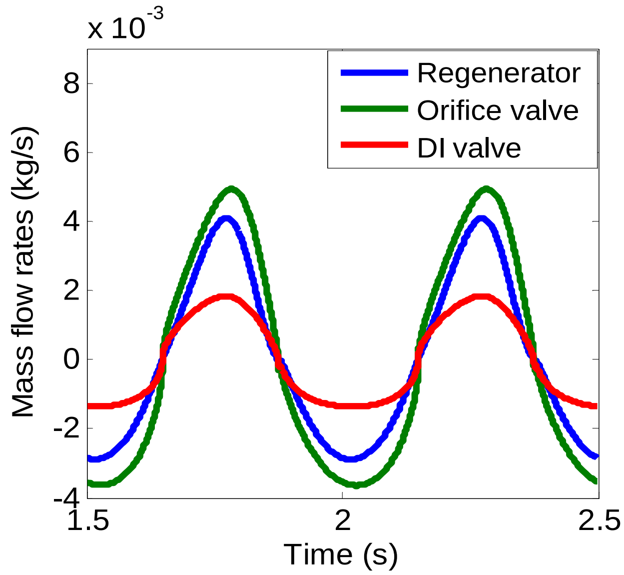


Fig. 6: Mass flow rates through regenerator, orifice and DI valves for DPTR.

E. Refrigeration Powers for a PTR:

Since DPTR is the configuration for achieving maximum refrigeration power, only this configuration is considered for comparison of refrigeration powers between theory and experimental results. Actual dimensions of PT, regenerator and other components have been introduced in the numerical analysis and the refrigeration powers are theoretically estimated by the above model. These are compared with the experimentally measured refrigeration powers for single stage PTR.

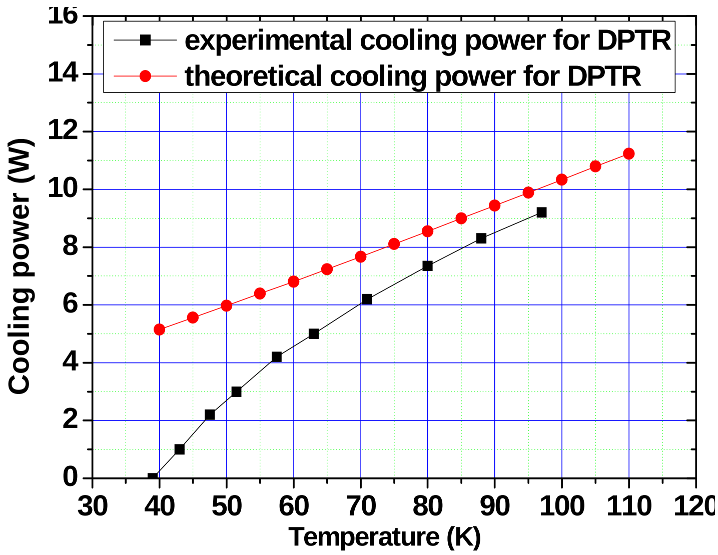


Fig. 7: Comparison between experimental and theoretical cooling powers for DPTR.

Fig. 7 plots the experimentally measured cooling powers with respect to cold endtemperature for DPTR. Also plotted in same figure are theoretically predicted cooling powers by numerical analysis. It is observed that there a reasonably good agreement between the experimentally measured cooling powers and the theoretical predictions.

It is seen that the theoretical predictions of cooling powers are higher than the experimental data. This is because; the theoretical model does not consider the losses occurring in thePTR system. Some of them are: a) shuttle loss, b) axial conduction loss and c) radiation loss etc. Since these losses are higher at lower temperatures, more deviations between the theoretical and experimental values are observed at lower temperatures.

V. CONCLUSION

In this work, a single stage PTR has been theoretically analyzed by a numerical model which assumes the adiabatic flow behavior in the Pulse Tube. This model is able to predict the pressure and massflow rates at different locations of the Pulse Tube Refrigerator in different configurations such as BPTR, OPTR and DPTR. The refrigeration powers produced by the PTR are predicted by the model without the introduction of any losses. Even in this case, a reasonably good agreement is observed between the theory and the experimental data. Hence one should expect that the theoretical predictions will be quite close to those of the experimental values when different losses are taken into the account in the numerical model.

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