Design and Manufacturing of Spur gear tooth: A New Approach Towards Composites

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Abstract -Gear is one of the most reliable power transmission systems in modern industry, operates at various speeds and loads. Breakage of gear tooth is a serious issue. Gear manufactured with alternate material can compensate this problem. With the advent of composite materials, it has been possible to reduce the weight of the spur gear without any reduction in the load carrying capacity. Composites are well suited for spur gear applications due to its high strength to weight ratio, fatigue resistance and hence, less chances of failure. All these have made composites an excellent replacement for the currently used metallic steel as a gear material. The present work is an attempt to provide an exclusive design technique regarding composite spur gear tooth based on an analysis of software affirmation. However, an effort has been taken towards the evaluation of bending stress at the root of the tooth and the total deflection of tooth tip associated with the new construction methodology. The gear tooth is modelled in CATIA V5R18 and the same are analysed under similar conditions using ANSYS (Workbench 16.2) software considering composite and structural-steel as the tooth material. Software based results are presented and compared for the two distinct cases mentioned above.

Key words: ANSYS (Workbench 16.2), CATIA V5R18, Composite, Spur Gear.

1. INTRODUCTION

Gear is basically a toothed wheel. It plays a vital role in mechanical power transmission system. Due to its high degree of reliability and compactness a wide variety of applications has been prevailed starting from wrist watch to heavy industries. It is a positive drive maintaining constant velocity ratio and possesses high efficiency in transmission. Among the various types gears spur gear with involute profile is the most simplest considering the design and manufacturing cost [1]. In this paper static analysis of the spur gear in the existing automobile gear box has been discussed with standard torque specifications. An attempt has been made to evaluate the root bending stress and total deflection of gear tooth made of composite material with a new methodology. The modern power transmission system requires highly efficient gear, and, for that the evaluation of stress and deflection of the tooth is important [2]. Research work on this field revealed that a pair of teeth in action generally fail by two types of stresses; namely bending fatigue due to bending stress at the root of the tooth and the another one is the contact fatigue which is a surface failure. Highest stress occur at two locations i.e. the point at which the force acts and the root of the tooth [3]. A lot of research works have been carried out on this gearing technology considering its geometrical parameters, tooth fillet radius, velocity factors and till some are going on Shubham et al., [4] presented velocity factors approximation. A reduction of bending stress has also been established by introducing the circular root fillet in comparison to the standard root fillet [5]. Study has been carried out on the choice of gear tooth profile and is found that only involute and cycloidal curves satisfy the law of gearing [6]. Involute gears have certain advantages over the cycloidal gears like varying centre distance with constant velocity ratio during matting, constant pressure angle with less wear and finally ease of manufacturing. The only problem with the involute profile is the interference. Although the cycloidal gear is not totally obsolete. It is used in spring driven watches, in some instruments [1]. A number of research studies have been carried out in this context to replace the conventional materials of the gear tooth by composites to enhance its load carrying capability and to reduce the overall weight of an automobile. A composite material can be defined as a combination of two or more materials that results in better properties than those of the individual parent components used alone. In contrast to metallic alloys, each material retains its separate chemical, physical, and mechanical properties. Reinforcement and matrix are the two constituents. The main advantages of composite materials are its high strength and stiffness, combined with low density, when compared with bulk materials, allowing for a weight reduction in the finished part. The reinforcing phase provides the strength and stiffness. In most cases, the reinforcement is harder, stronger, and stiffer than the matrix [7]. AnujNath et al.,[8] in their work, have shown the design and analysis of a composite spur gear made of 50% carbon fibers in epoxy resin matrix and compared to steel gear, 0.5324% lesser stress and 55.619% reduction in total deformation have
been found for the composite gear. Utkarsh. M. Desai et al., [9] have presented composite spur gear with 70% weight reduction without compromising the strength of the tooth. An overall comparison has been drawn between the existing alloy element (Nickel Chrome Steel) and composite material (GF 30 PEEK) gear. V. Siva Prasad et al. In their paper describes design and analysis of spur gear and it is proposed to substitute the metallic gears of sugarcane juice machine with polymer gears to reduce the weight and noise [10]. P.B. Pawara et al., [11] have given a detail comparison of metallic spur gear with the stir casted Al- SiC composite spur gear and an improved hardness, tensile strength has been found with almost 60% weight reduction. Due to the unique advantages, such as light weight, high strength, higher dimensional stability and corrosion resistance the metal matrix composite (MMC) is preferred to manufacture different machines components. However, there is a cost problem when this MMC is compared with the polymer based composite [12, 13]. Dynamic analysis has been presented using MATLAB and FEA software on composite gear and a study has been carried out for natural frequency with the fibre orientation of the composite gear [14]. In some modern machinery such as textile industries involve oil less transmission in those cases composite gear has no alternative because of its oil less lubrication. From the above course of study one can come in a conclusion that surely composite materials are able to provide a better performance and efficiency in the practical applications but, from manufacturing point of view it leads to expensive cost than that of steel. That’s why extending the research work performed by Sushovan Ghosh et al., [15] the present exertion completely dismisses the proposition associated with manufacturing the complete spur gear by composites; but maintains its intense focus to manufacture those parts which are relatively more critical as well as to maintain conventional materials for the other part to achieve an optimum extent towards the manufacturing costs. In the present work, the tip of the gear tooth is modelled separately with composites whereas the root of the same tooth is with conventional steel. Then after the entire system is combined with the proper contact constraints and analysed under similar conditions in ANSYS software.

II. MATHEMATICAL FORMULATIONS

A. Assumption of Lewis equation

The analysis of bending stress in gear tooth was done by Mr. Wilfred Lewis. Gear tooth is considered as a cantilever beam with static normal force F applied at the tip. Assumptions made in the derivation are [16]:
1. The full load is applied to the tip of a single tooth in static condition.
2. The radial component is negligible.
3. The load is distributed uniformly across the full face width.
4. Forces due to tooth sliding friction are negligible.
5. Stress concentration in the tooth fillet is negligible.

![Fig.1. Gear tooth as a cantilever beam [16].](image1)
![Fig.2. Parabolic gear tooth [17].](image2)

The Fig. 1 shows clearly that the gear tooth is stronger throughout than the inscribed constant strength parabola, except for the section at ‘a’ where parabola and tooth profile are tangential to each other [16]. In the above Fig. 2 the following notations are used: F is the Full load, Fr and Ft are the Radial and Tangential component of the full load. h, b and t are the height, face-width and thickness of the tooth at critical section respectively.

III. DESIGN SPECIFICATIONS

For calculating bending stress and total deformation we have taken a standard model for designing the spur gear tooth [17] and different torque specification from the existing vehicle-models of Maruti Suzuki [18, 19, 20]. The following data is given for the design of 20° full depth spur gear made of structural steel transmitting torque at different rpm:

DOI: 10.21817/ijet/2017/v9i3/170903510  Vol 9 No 3 Jun-Jul 2017 1552
TABLE I. Specification of Spur Gear Tooth

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Module (m)</th>
<th>Pitch circle diameter (d)</th>
<th>Nos. Teeth (Z)</th>
<th>Tooth width (b)</th>
<th>Thickness of tooth (T)</th>
<th>Pressure angle (θ)</th>
<th>Lewis form factor (Y)</th>
<th>Torque and speed (N-m @ rpm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>10</td>
<td>180</td>
<td>18</td>
<td>54</td>
<td>15.7</td>
<td>20°</td>
<td>0.308</td>
<td>132@3000 190@2000 225@4000</td>
</tr>
</tbody>
</table>

The designing parameters of the gear are taken from existing automobile gear box model and others parameters can be found from the standard module (m). Structural steel was considered as the tooth material with an elastic modulus E = 2.1*10^5 MPa, tensile yield strength Sy = 250 MPa, Ultimate tensile strength is 460 MPa and Poisson’s ratio=0.3 while performing the analysis in ANSYS (Workbench 16.2) software. In the next case, Carbon fiber steel has been considered as the material for the tip of the gear tooth (fiber lamina or laminate is loaded in a direction parallel to its fibers) whereas, material for the rest of the part i.e. root of the tooth is kept same as structural steel. For Carbon fiber steel, an Elastic Modulus E = 2.64*10^5 MPa and ultimate strength Sut=540 MPa (at dry room temperature) were considered for simulation purpose; determined on the basis on rule of mixtures.

A. Formulations of Bending stress [1]

In the current analysis of bending stress of tooth we consider the Lewis assumption as discussed above in 2.2. From the Fig.2 at point ‘a’

Bending moment \( M_b = F_t \times h \quad \text{(1)} \)

Area moment of inertia \( I = \frac{b \times t^3}{12} \quad \text{(2)} \)

Then the bending stress is given by \( \sigma_b = \frac{6 \times F_t \times h}{b \times t^2} \) from the Eq. (1) and Eq. (2). After the rearranging we have

\[ F_t = b \times \sigma_b \times \left( \frac{t^2}{6 \times m} \right) \quad \text{(3)} \]

Multiplying numerator and denominator by module (m) from the Eq. (3) we have the tangential component of the force given by

\[ F_t = m \times b \times \sigma_b \times Y \quad \text{(4)} \]

\( Y = \frac{t^2}{(6 \times m)} \) is known as Lewis form factor. Equation. (4) Can be rewritten as

\[ F_t = m \times b \times \sigma_b \times Y \quad \text{(5)} \]

When the tangential force increased the stress also increases. When the stress reaches the permissible magnitude of bending stress the corresponding force \( F_t \) is known as Beam strength and denoted by \( S_b \). So replacing \( F_t \) in the Eq. (5) we have

\[ S_b = m \times b \times \sigma_b \times Y \quad \text{(6)} \]

B. Formulations of total deformation [17]

It is observed that the cross section of the gear tooth varies from free end to the fixed end. Lewis has assumed it as a constant strength parabola. Using Castigliano’s Theorem total deformation of the tooth can be found with minor error. For linearly elastic structure, where external forces only cause deformations, the complementary energy is equal to the strain energy. For such structures, the Castigliano’s first theorem may be stated as the first partial derivative of the strain energy of the structure with respect to any particular force gives the displacement of the point of application of that force in the direction of its line of action [21]. The theory applies to both linear and rotational deflection, \( \delta = \frac{4U}{EI} \). It should be clear that Castigliano’s theorem finds the deflection at the point of application of the load in the direction of the load. Here U is the strain energy given by \( \int_0^L \frac{M^2}{2E} \, dx \), where M is the moment due to the load. Consider the parabolic tooth of height h and tooth thickness t. The equation of parabola \( y^2 = 4 \times a \times x \). Consider the Fig 2. We have the following boundary condition at \( x = h, y = u/2 \). After substituting the equation of the parabolic tooth is \( y^2 = \frac{t^2}{4 \times s} \) and \( y^3 = \left( \frac{t}{2} \right)^3 \times \left( \frac{x}{2} \right)^{1.5} \). Putting the value of \( M = F_t \times x, I = \left( \frac{2}{3} \right) \times b \times y^3 \), the strain energy \( U = \int_0^1 \frac{M^2}{2E} \, dx \) will be \( U = \int_0^L \frac{(F_t \times x)^2}{2E} \, dx \) from this we have

\[ U = \frac{8 \times F_t \times x^3}{6 \times b \times t^3} \quad \text{(7)} \]

Again we know deflection is given by \( \delta = \frac{4U}{EI} \). From Eq. (7) we have the
This Eq. (8) is the equation of tooth deflection of spur gear when tangential load \( F_t \) is applied at the tip of the tooth.

### III. SAMPLE CALCULATIONS

From the relation of maximum bending stress (\( \sigma_b \)) and deflection (\( \delta \)) based on the above design specifications from the Table 1, and considering steel as a tooth material the analytical calculation is carried out for the torque of 132 N-m at 3000 rpm is carried out.

#### A. Calculation for Bending stress

Consider torque \( T = 132 \) N-m at 3000 rpm. The tangential load \( F_t \) can be found from the below:

\[
F_t = \frac{2\sigma b}{d}; \quad \text{where} \quad d \quad \text{is the pitch circle diameter.}
\]

\[
F_t = \frac{2 \times 132}{100 \times 10^{-3}} = 1466.67 \text{ N} \cdot \text{m. Number of teeth in both gear and pinion is 18, thickness} \ t = 15.71 \text{ mm. face-width} \ b = 54 \text{ mm. Lewis form factor} \ Y = 0.308 \ [22]. \quad \text{The value of bending stress is given by from Eq. (5)} \quad \text{is} \quad \sigma_b = \frac{F_t}{(m+b+Y)}; \quad \text{Then the theoretical bending stress is given by} \quad \sigma_b = \frac{1466.67}{(10+54+0.308)} = 8.818 \text{ MPa. Ultimate tensile strength of gear material is 460 MPa. Considering factor of safety as 3, then the allowable bending stress is 153.33 MPa > 8.818 MPa. So, the design is safe. As the gear and pinion are identical so there is no question of checking the following relation i.e.strength of gear < strength of pinion.}

#### B. Calculation of deflection

Consider the Eq. (8) and the value of \( h \) can be found from the relation

\[
h = \frac{\delta}{(b+ \delta)} \Rightarrow \delta = \frac{16(1+b)^3}{(10+54+0.308)} = 0.0013 \text{ mm (approx.)}
\]

Subsequently, the root bending stresses and total deflection of the tooth tip are evaluated for the other torque sections and presented in Table II.

#### TABLE II. Calculation of various parameters of Spur Gear Tooth

<table>
<thead>
<tr>
<th>Torque (N m)</th>
<th>Theoretical Deflection(mm)</th>
<th>ANSYS based total deflection(mm)</th>
<th>Theoretical bending stress (MPa)</th>
<th>ANSYS based bending stress(MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>132 @ 3000 rpm</td>
<td>0.0013</td>
<td>0.0011</td>
<td>8.818</td>
<td>8.80</td>
</tr>
<tr>
<td>190 @ 2000 rpm</td>
<td>0.0019</td>
<td>0.0016</td>
<td>12.69</td>
<td>12.276</td>
</tr>
<tr>
<td>225 @ 4000 rpm</td>
<td>0.0022</td>
<td>0.0020</td>
<td>15.03</td>
<td>15.160</td>
</tr>
</tbody>
</table>

During composite analysis when a unidirectional continuous-fiber lamina or laminate is loaded in a direction parallel to its fibers (0° or 11-direction), the longitudinal modulus \( E_{11} \) can be estimated from its constituent properties by using what is known as the rule of mixtures \[21\]:

\[
E_{11} = E_f V_f + E_m V_m;
\]

Where, \( E_f \) is the fiber modulus, \( V_f \) is the fiber volume percentage, \( E_m \) is the matrix modulus, and \( V_m \) is the matrix volume percentage.

The longitudinal tensile strength \( s_{11} \) also can be estimated by the rule of mixtures: \[ s_{11} = s_f V_f + s_m V_m \];

Where, \( s_f \) and \( s_m \) are the ultimate fiber and matrix strengths, respectively.

Considering, \( V_f = 0.6, V_m = 0.4, E_f = 3.0*10^5 \text{MPa, } E_m = 2.1*10^5 \text{MPa, } s_f = 600 \text{ MPa, } s_m = 450 \text{ MPa} \ [22-23] \)

We get,

\[
E_{11} = E_f V_f + E_m V_m = 0.6 \times 3.0 \times 10^5 + 0.4 \times 2.1 \times 10^5 = 2.64 \times 10^5 \text{MPa};
\]

\[
s_{11} = s_f V_f + s_m V_m = 600 \times 0.6 + 450 \times 0.4 = 540 \text{ MPa}.
\]

### IV. MODELLING and SIMULATION

The aim of this analysis is to investigate the stresses in the spur gear tooth within the desirable limits to obtain a practical understanding for the theoretical ideas associated with composite materials. After geometric modelling in CATIAV5R18 software the gear tooth is subjected to static analysis, performed in ANSYS (Workbench 16.2) software.

#### A. Modelling

The computer compatible mathematical description of the geometry of the object is called geometric modelling. CATIA is basically CAD (computer-aided design) software that allows themathematical description of the object to be displayed and manipulated as the image on the monitor of the computer \[24, 25\]. While modelling the spur gear tooth, the root of the tooth and the rest portion are designed separately as two different part bodies shown in Fig. 3 and 4, which are again combined to make a single system in ANSYS software through proper contact constraints (shown in Fig. 5 and 6).
ANSYS is engineering simulation software that predicts with confidence about the performance of the product under the real-world environments incorporating all the existing physical phenomena [18-19]. While performing the part of composite analysis, the composite properties were imposed only in these full-length leaves by incorporating the new value of elastic modulus obtained from the rule of mixtures. The layout of static analysis involves meshing, boundary conditions and loading.

**B. Meshing**

Meshing is basically the division of the entire model into small cell so that at each and every cell the equations are solved. It gives the accurate solution and also improves the quality of solution [26]. Here the element size of 1 mm with medium smoothing is considered for mesh generation. Minimum edge length of the elements is 2.886 mm. Within the solution domain under the Adaptive Mesh Refinement segment, the Max. Refinement Loops is taken as 1 and Refinement Depth as 2. Within the Patch Confirming Method domain the method is taken as Tetrahedrons. For the convergence plot, the maximum allowable change was considered as 4%. The whole geometry is selected for mesh generation and total number of nodes and elements are observed as 51135 and 28947 respectively. Fig.7 shows the meshed geometry of the spur gear tooth.

Fig. 3. Root of the tooth is modelled separately

Fig. 4. Tip of the tooth is modelled separately

Fig. 5. Contact between root and tip of the tooth.

Fig. 6. Complete model of the gear tooth.

Fig. 7. Mesh modelled of the spur gear tooth.
C. Boundary Conditions

Based on the assumptions of Lewis equation, the boundary conditions are set in ANSYS Workbench. The fixed support is used at the root end of the tooth and the force is applied on the face having components in Y and Z directions. The tangential force \( F_t \) having magnitude 1466.67 N has been introduced with component at Y and Z direction as 1378.2 N and 501.6 N respectively. In the following Fig. 8 and 9 the respective boundary and loading conditions are shown.

![Fig. 8. Boundary condition of gear tooth.](image1)

![Fig. 9. Loading characteristics of gear tooth.](image2)

V. RESULTS and DISCUSSIONS

Completing the static analysis for both the cases of conventional Steel and Carbon Fiber composite Steel for given dimensional specifications, the results obtained are summarised below:

A. Reduction in weight:

Apart from the other benefits, the biggest benefit, however, is mass reduction for using composite materials for the tip of the tooth. While, the mass for the spur gear tooth was 0.35664kg before applying composites; for the next case on application of composite on the tip portion of the tooth while base remains as usual metallic the mass is reduced to 0.29075 kg. So, almost 18.48% weight reduction per tooth can be obtained with the new construction method which can provide a great help towards the modern automobile industry which are focussing on weight reduction.

B. Maximum equivalent (von-Mises) stress:

Simulations are done for the three different torque conditions and the results obtained are similar in nature indicating a comprehensible trend towards a slight increasing value of maximum equivalent (von-Mises) stress for composite applications shown in Table III. The results obtained in this connection are as follows shown in the following Fig. 10 to Fig. 15:

![Fig. 10. Maximum equivalent (von-Mises) stresses at a torque of 132 N-m @ 3000 rpm for structural steel.](image3)

![Fig. 11. Maximum equivalent (von-Mises) stresses at a torque of 132 N-m @ 3000 rpm with composite and structural steel.](image4)
A graphical representation has been drawn for the conventional steel and Composite based results with torque on X-axis and stress values on Y-axis, shown in the following Fig. 15. There is a slight increase in stress value for the application of carbon fiber.

![Graph](image)

**Fig. 16. Variation of Maximum equivalent (von-Mises) stresses at different torques with composite and structural steel.**

**TABLE III.** Comparison for steel and composite applications on gear tooth

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Torque @ rpm (Newton - m)</th>
<th>With conventional Steel gear tooth</th>
<th>Composite tip gear tooth with conventional steel root of the gear</th>
<th>Increase in stress (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max. Equivalent (von-Mises) stress (MPa)</td>
<td>132 @ 3000 rpm</td>
<td>8.8071</td>
<td>8.8371</td>
<td>0.030</td>
</tr>
<tr>
<td></td>
<td>190 @ 2000 rpm</td>
<td>12.805</td>
<td>13.276</td>
<td>0.471</td>
</tr>
<tr>
<td></td>
<td>225 @ 4000 rpm</td>
<td>15.164</td>
<td>15.722</td>
<td>0.554</td>
</tr>
</tbody>
</table>
C. Directional deformation:

The software based analysis is carried out for the three different standard torque conditions and the results obtained are identical in nature indicating a comprehensible trend towards the decreasing value of total deformation for composite applications. The results obtained are as follows (depicted on the Fig. 17 to Fig. 22):

Fig. 17. Total deformation at a torque of 132 N-m @ 3000 rpm for structural steel.

Fig. 18. Total deformation at a torque of 132 N-m @ 3000 rpm with composite and structural steel.

Fig. 19. Total deformation at a torque of 190 N-m @ 2000 rpm for structural steel.

Fig. 20. Total deformation at a torque of 190 N-m @ 2000 rpm with composite and structural steel.

Fig. 21. Total deformation at a torque of 225 N-m @ 4000 rpm for structural steel.

Fig. 22. Total deformation at a torque of 225 N-m @ 4000 rpm with composite and structural steel.

A graphical representation has been drawn for the conventional steel and Composite based results with torque on X-axis and total deformation values on Y-axis, shown in the following Fig. 23. There is a significant decrease in total deformation value for the application of carbon fiber. Interestingly at higher torque the percentage reduction is less.
Fig. 23. Variation of total deformation at different torques with composite and structural steel.

The mass reduction and reduction in total deformation per tooth are depicted in the following Table IV

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Torque @ rpm (Newton-m)</th>
<th>With conventional Steel gear tooth</th>
<th>Composite tip gear tooth with conventional steel root of the gear</th>
<th>% reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total deformation (mm)</td>
<td>132 @ 3000 rpm</td>
<td>0.0011444</td>
<td>0.000992</td>
<td>13.32</td>
</tr>
<tr>
<td></td>
<td>190 @ 2000 rpm</td>
<td>0.0016485</td>
<td>0.001574</td>
<td>4.519</td>
</tr>
<tr>
<td></td>
<td>225 @ 4000 rpm</td>
<td>0.0019522</td>
<td>0.001864</td>
<td>4.517</td>
</tr>
<tr>
<td>Mass (kg)</td>
<td></td>
<td>0.35664</td>
<td>0.29075</td>
<td>18.48</td>
</tr>
</tbody>
</table>

VI. CONCLUSION

In this work, the spur gear tooth is modelled in CATIA V5R18 and is analysed in the Static structural domain of ANSYS software. A conclusion can be drawn on the basis of result discussed on the previous sections is that for the standard design specifications the values of maximum stresses at different torque conditions are well within the safe limit. Apart from that, most importantly the new design method has proposed to manufacture the tip of the gear tooth separately with composites in contrast to use of composites for the entire tooth. It has been observed that a substantial decreasing trend toward the deformation values for composite applications with a negligible increase in maximum stress. This agrees well with the previous works so far done in this context. A reduction in mass of more than 18% is the one of the prominent benefits with the new method; along with optimum extent towards the manufacturing costs can be achieved as composites being highly expensive [26-27] than that of steels (almost 2-3 times costlier). Therefore, the new method seems to be beneficial exclusively for modern auto industry as it provides an optimum solution towards weight reduction as well as manufacturing costs. The focus can be given to the joining of metal and composites with different fasteners or suitable adhesive [28].

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