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Numerical evaluation of simplified extreme peak load method for determining the double – K fracture parameters of concrete

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Abstract—**A simplified extreme peak load method for determining the double – K fracture parameters of concrete has been put forth in the recent research. The method was validated based on experimental test results available in the literature wherein, a significant deviation in predicted values of double – K fracture parameters for some specimens can be observed. The researchers attributed this deviation to measuring error in critical crack mouth opening displacement during fracture test which needs further investigation. Hence a systematic numerical investigation on predicted double – K fracture parameters of concrete using simplified extreme peak load method is presented in this study wherein the peak load and critical crack mouth opening displacement are obtained using fictitious crack model for varying specimen size and initial notch-length to depth ratio. For computation, use of numerical data seems to be precise as experimental data involves error in measuring of critical crack mouth opening displacement during fracture test. Present study reveals that the simplified extreme peak load method is applicable to a limited specimen size range between 200 to 300 mm. This method yields error in the predicted values of double-K fracture parameters of concrete for specimen size below 200mm and beyond 300mm.**

Keyword-Three-point bending geometry, Mode-I fracture, Concrete, Double-K fracture parameters, Simplified extreme peak load method, Weight function method.

I. INTRODUCTION

The crack propagation study and fracture parameters of concrete structures are described using nonlinear concrete fracture models such as cohesive crack model (CCM) or fictitious crack model (FCM) [1-14] and crack band model (CBM) [15] based on numerical techniques and two parameter fracture model [16], size effect model (SEM) [17], effective crack model (ECM) [18], K_R-curve method based on cohesive force distribution [19-20], double-*K* fracture model (DKFM) [20-27] and double-*G* fracture model (DGFM) [28] based on modified linear elastic fracture mechanics (LEFM) concept.

In last two decades, the double-K fracture model has attracted attention of researchers and academia around the world due to many advantages. This model uses LEFM principle in its modified form; it can describe the three important stages of crack propagation in concrete viz.: crack initiation, stable crack propagation and unstable fracture in concrete and the fracture parameters can be determined without use of close loop testing machine. This method is characterized by two material parameters: initial cracking toughness K_{IC}^{ini} and unstable fracture toughness K_{IC}^{un}. The initial cracking toughness is defined as the inherent toughness of the materials, which holds for loading at crack initiation when material behaves elastically and micro cracking is concentrated to a small-scale in the absence of main crack growth. The total toughness at the critical condition is termed as unstable toughness which is regarded as one of the material fracture parameters at the onset of the unstable crack propagation. The initial cracking toughness can be considered as a failure criterion in the design process for design of large size concrete structures like dam, nuclear reactor vessels, and liquid retaining structures wherein crack initiation is taken as one of design criteria. The unstable fracture toughness of the material can be considered as one of the design criteria at final failure of concrete structures. Recently, Wu *et al.* [29] applied the double-K fracture model to assess the safety of dam concrete with large size aggregates. The authors used the results of wedge-splitting tests on 300 mm-diameter cylindrical compact tension specimens drilled from Danjiangkou Dam to extrapolate the real fracture parameters that are required to assess the safety of the dam. Conventional experimental method and analytical methods for determining double-K fracture parameters of concrete are based on linear asymptotic superposition assumption. The values of K_{IC}^{ini} and K_{IC}^{un} can be determined using experimental method in which the initial cracking load (P_{ini}), initial crack length (a_o), peak load (P_u) and crack mouth opening displacement at peak load ($CMOD_c$) should be measured whereas in the analytical method, P_u and $CMOD_c$ should be recorded during the testing. Analytically, the double – K fracture parameters can be determined using four analytical methods *i.e.,* Gauss–Chebyshev integral method (GCIM), simplified Green's function method (SGFM), weight function method (WFM) and simplified equivalent cohesive force method (SECFM) [30-31]. In both the experimental and analytical methods, $CMOD_c$ must be measured during the test which requires a sophisticated clip gauge. The correct measurement of $CMOD_c$ may be a difficult task in the laboratory if the clip gauges are not properly attached with the specimen. To avoid this difficulty, the double-K fracture parameters can be determined using only peak load similar to the determination of fracture parameters of two parameter fracture model [32]. This method was recently applied by Ince [33] for determining the fracture parameters of two parameter fracture model for different specimen geometries such as cubical, cylindrical and beam specimens. Ince [34] also proposed a concept for determining the double-K fracture parameters of concrete using weight function method with peak load obtained from experiments for various specimen geometries. Qing and Li [35] presented a traditional extreme peak load method for computing the fracture parameters of double-K method in which numerical integration is needed in the calculation. Using peak load method, Kumar *et al* [36] attempted to determine the fracture parameters of double-K fracture model based on peak load method for determining the two parameter fracture model [32]. Recently, Qing *et al* [35] proposed a simplified extreme peak load method for determining these fracture parameters. The peak load method [36] requires at least three specimens with different sizes, or the same size but different initial notch lengths in tests whereas the traditional extreme peak load method [35] and the simplified extreme peak load method (SEPLM) [37] require only a single specimen to be tested for determining the fracture parameters. Further, the traditional extreme peak load method Qing *et al* [35] requires complicated integration procedure to calculate the critical effective crack length whereas the integration procedure is avoided in the simplified extreme peak load method [37].

Qing et al [37] determined the double-K fracture parameters for the experimental data [38] using simplified extreme peak load method (SEPLM) and compared the results with those obtained using weight function method (WFM) [26]. From the comparison, it has been pointed out that the simplified extreme peak load method yields generally smaller values of K_{IC}^{ini} and K_{IC}^{un} as compared to those obtained using conventional weight function method. This disparity has been attributed to the use of measured value $CMOD_c$ in double-K fracture model based on weight function method whereas the measurement of $CMOD_c$ is not required in simplified extreme peak load method. The measurement error of $CMOD_c$ during fracture test may occur as $CMOD_c$ is generally measured with a sophisticated clip gauge. Also, an improper connection between the gauge and specimens may occur and lead to the measurement error of $CMOD_c$. From the results presented by authors [37], it can be seen that the error in results of K_{IC}^{ini} and K_{IC}^{un} for many specimens is more than ±20% which needs further attention for the validity of simplified extreme peak load method. It has been established from the extensive experimental and numerical studies [20-26, 30] that the values of double – K fracture parameters (K_{IC}^{ini} and K_{IC}^{un}) determined using experimental method and conventional analytical methods (Gauss–Chebyshev integral method, weight function method) are in excellent agreement. Hence, in authors' opinion, the conventional analytical methods (Gauss–Chebyshev integral method, weight function method) can be considered as the standard analytical methods for determining the double – K fracture parameters. Any deviation in the results of K_{IC}^{ini} and K_{IC}^{un} obtained using simplified extreme peak load method as compared with those obtained using conventional weight function method should be thoroughly investigated. It is also true that experimental error in $CMOD_c$ measurement during the fracture test cannot be fully avoided. In view of these facts numerical input data, where chance of experimental error in measuring *CMODc* is avoided for determining the double – K fracture parameters using weight function method and simplified extreme peak load method, have been developed using fictitious crack model (FCM). Hence, standard three-point bend test specimen for size range 100-500mm and varied a_o/D ratios (0.25 to 0.4) has been considered in the present study. For given material properties of concrete, the values of P_u and $CMOD_c$ are derived for these specimens using FCM. The double – K fracture parameters are then determined using WFM and SEPLM and a systematic study including effect of specimen size and a_o/D ratios on the computed fracture parameters is carried out and presented in the subsequent sections. For completeness of the paper, a brief formulation of both the methods i.e., WFM and SEPLM is also presented herein.

II. SPECIMEN GEOMETRY

Three-point bending test is considered in the present study for comparative study. RILEM Technical Committee 50-FMC [39] has recommended the guidelines for determination of fracture energy of cementitous materials using standard three-point bend test on notched beam. This method has been widely used for determination of fracture energy of concrete with certain modification in the experimental setup [40]. In present study, standard three - point bend test (RILEM Technical Committee 50-FMC [39]) is considered. The standard dimensions and loading of three-point bend test (TPBT) is shown in Fig.1. In the figure, the symbols: *B, D* and *S* are the width, depth and span respectively for TPBT with $S/D = 4$.

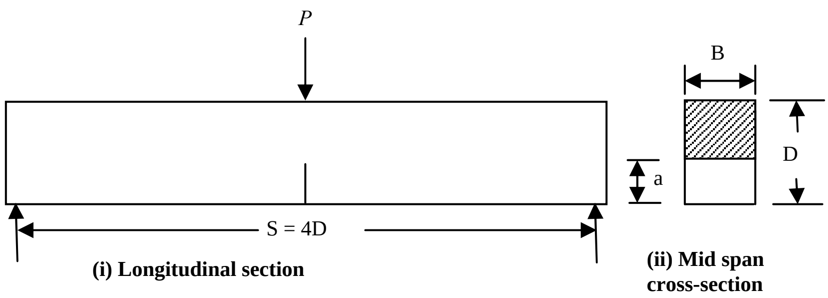

Fig. 1. Standard dimensions and loading of three-point bending test

III. METHODS FOR DETERMINING DOUBLE-K FRACTURE PARAMETERS

A. Weight function method (WFM)

The double-K fracture parameters using weight function method is based on linear asymptotic superposition assumption which uses linear elastic fracture mechanics formulas [26]. Here, WFM with four terms of universal weight function is used for computing the double-K fracture parameters of standard three point bend test (TPBT). The input data required for obtaining the double–K fracture parameters (P_u and $CMOD_c$) are obtained from the developed fictitious crack model. The steps used in WFM are summarized as given below.

For standard TPBT geometry with S/D = 4 using Tada *et al.* [42] formulae, the stress intensity factor is expressed as

$$K_I = \frac{3S\left(2P+W\right)\sqrt{a}}{4BD^2}k(\alpha) \tag{1}$$

$$k(\alpha) = \frac{1.99-\alpha(1-\alpha)(2.15-3.93\alpha+2.7\alpha^2)}{(1+2\alpha)(1-\alpha)^{3/2}} \tag{2}$$

where $k(\alpha)$ is a geometric factor, $\alpha = a/D$, P is the external load and W self weight of the beam.

The value of K_{IC}^{un} is determined from Eq. (1) as $K_I = K_{IC}^{un}$ when, notch length a is equal to effective elastic crack length at peak load a_c and P is equal to P_u.

The value of effective crack extension corresponding to peak load is determined using linear asymptotic superposition assumption using the following LEFM formulae for TPBT geometry, S/D = 4, Tada *et al* [42].

$$CMOD = \frac{6\left(P+\frac{W}{2}\right)Sa}{BD^2E}V(\alpha) \tag{3}$$

$$V(\alpha) = 0.76-2.28\alpha+3.87\alpha^2-2.04\alpha^3+\frac{0.66}{(1-\alpha)^2} \tag{4}$$

In which, a is equal to a_c equivalent-elastic crack length at maximum load, $CMOD = CMOD_c$ and P equals to P_u. The value of a_c can be determined using Eq.(3). Then, according to inverse analytical method, the following relation can be employed to determine the initial cracking toughness of the material.

$$K_{IC}^{ini} = K_{IC}^{un} - K_{IC}^{C} \tag{5}$$

where, K_{IC}^{C} is known as cohesive toughness of the material. Once the value of K_{IC}^{C} is determined, the K_{IC}^{ini} can be obtained using Eq.(5). The stress intensity factor due to cohesive stress K_I^C using weight function method can be obtained as

$$K_I^C = \frac{2}{\sqrt{2\pi a}}g(\alpha) \tag{6}$$

where

$$g(a) = A_1 a\left(2s^{1/2} + M_1 s + \frac{2}{3}M_2 s^{3/2} + \frac{M_3}{2}s^2\right) + A_2 a^2\left[\frac{4}{3}s^{3/2} + \frac{M_1}{2}s^2 + \frac{4}{15}M_2 s^{5/2} + \frac{M_3}{6}\left\{1-(a_o/a)^3 - 3sa_o/a\right\}\right] \tag{7}$$

where, $A_1 = \sigma_s(CTOD_c), A_2 = \dfrac{f_t - \sigma_s(CTOD_c)}{a - a_o}$ and $s = (1 - a_o/a)$. At the critical effective crack extension, a is equal to a_c corresponding to peak loading condition in Eq. (6) which yields $K_{IC}^C = K_I^C$. Also, M_1, M_2 and M_3 are the weight function parameters. Crack tip opening displacement (CTOD) at initial crack-tip becomes its critical value denoted as $CTOD_c$ at peak load and the corresponding value of cohesive stress, $\sigma_s(CTOD_c)$ at the tip of initial notch, is determined using nonlinear softening functions [43] as given in Eq.(8).

$$\sigma(CTOD) = f_t\left\{\left[1+\left(\frac{c_1 CTOD}{w_c}\right)^3\right]\exp\left(\frac{-c_2 CTOD}{w_c}\right) - \frac{CTOD}{w_c}\left(1+c_1^3\right)\exp\left(-c_2\right)\right\} \tag{8}$$

The value of total fracture energy of concrete G_F is expressed as

$$G_F = w_c f_t\left\{\frac{1}{c_2}\left[1+6\left(\frac{c_1}{c_2}\right)^3\right] - \left[1+c_1^3\left(1+\frac{3}{c_2}+\frac{6}{c_2^2}+\frac{6}{c_2^3}\right)\right]\frac{\exp(-c_2)}{c_2} - \left(\frac{1+c_1^3}{2}\right)\exp(-c_2)\right\} \tag{9}$$

in which c_1 and c_2 are the material constants. Also, the w_c is the maximum crack opening displacement at the crack-tip at which the cohesive stress becomes to be zero. In present study c_1, c_2 and w_c are taken as 3, 7 and 167.2 µ-m respectively.

The value of $CTOD$ is computed using the following expression [16].

$$CTOD = CMOD\left\{\left(1-\frac{a_o}{a}\right)^2 + \left(1.081 - 1.149\frac{a}{D}\right)\left[\frac{a_o}{a} - \left(\frac{a_o}{a}\right)^2\right]\right\}^{\frac{1}{2}} \tag{10}$$

In which $CTOD_c = CTOD$ at $P = P_u$, $a = a_c$ and $CMOD = CMOD_c$.

In Eq.(7), M_1, M_2 and M_3 are the weight function parameters of four terms universal weight function which can be represented as a function of a/D ratio. These parameters are expressed in the following form.

$$M_i = \frac{1}{(1-a/D)^{3/2}}\left[a_i + b_i a/D + c_i(a/D)^2 + d_i(a/D)^3 + e_i(a/D)^4 + f_i(a/D)^5\right] \tag{11}$$

for, i = 1 and 3 and

$$M_i = \left[a_i + b_i a/D\right] \text{ for } i = 2 \tag{12}$$

The values of coefficients a_i, b_i, c_i, . . ., f_i are the constant and given in Table 1.

TABLE I Coefficients of Four Terms Weight Function Parameters M_1, M_2 and M_3

i	a_i	b_i	c_i	d_i	e_i	f_i
1	0.0572011	-0.8741603	4.0465668	-7.89441845	7.8549703	-3.18832479
2	0.4935455	4.43649375				
3	0.340417	-3.9534104	16.1903942	-16.0958507	14.6302472	-6.1306504

B. *Simplified extreme peak load method (SEPLM)*

From the load – crack extension (P-a) relationship [37], it is clear that the load increases with crack extension till the peak load and after that it decreases with the crack extension. It is assumed in the SEPLM [37] that the partial derivative of P with respect to a at $P = P_u$ is continuous. This yields that

$$\left[\frac{\partial P}{\partial a}\right]_{a=a_c} = 0 \tag{13}$$

From Eqs(1), (5) and (6), the value of P is expressed as

$$P = \frac{BD^2}{3S\sqrt{a}k(\alpha)}\left[\frac{2}{\sqrt{2\pi a}}g(\alpha) + K_{IC}^{ini}\right] - \frac{W}{2} \tag{14}$$

Also, $\frac{\partial P}{\partial a} = \zeta^{'}(a) + \eta^{'}(a)K_{IC}^{ini}$ (15)

where,

$$\zeta^{'}(a) = \frac{4BD^2}{3\sqrt{2\pi}S}\frac{g^{'}(a)k(\alpha)a - g(a)[k^{'}(\alpha)a + k(\alpha)]}{k^2(\alpha)a^2} \tag{16}$$

$$\eta^{'}(a) = -\frac{2BD^2}{3S}\left[\frac{a^{-1/2}k(\alpha) + a^{1/2}k^{'}(\alpha)}{2ak^2(\alpha)}\right] \tag{17}$$

At $a = a_c$ and $P = P_u$, the value of $K_{IC}{}^{ini}$ can be expressed from Eqs.(5), (1) and (6) as

$$K_{IC}^{ini} = \frac{3S\left(2P_u + W\right)\sqrt{a_c}k(\alpha_c)}{4BD^2} - \frac{2}{\sqrt{2\pi a_c}}g(\alpha_c) \tag{18}$$

The values of $k^{'}(\alpha)$ and $g^{'}(\alpha)$ can be expressed as

$$k^{'}(\alpha) = \left\{\begin{matrix} \left(-2.15 + 12.16\alpha - 19.89\alpha^2 + 10.8\alpha^3\right)(1+2\alpha)(1-\alpha)^{3/2} \\ -\left(1.99 - 2.15\alpha + 6.08\alpha^2 - 6.63\alpha^3 + 2.7\alpha^4\right)\times \\ \left[2(1-\alpha)^{3/2} - \frac{3}{2}(1+2\alpha)(1-\alpha)^{3/2}\right] \end{matrix}\right\} / (1+2\alpha)^2(1-\alpha)^3 \tag{19}$$

$$\begin{aligned} g^{'}(a) = {} & \left(A_1 + A_1^{'}a\right)\left(2s^{1/2} + M_1 s + \frac{2}{3}M_2 s^{3/2} + \frac{M_3}{2}s^2\right) + A_1 a\left(s^{-1/2} + M_1 + M_2 s^{1/2} + M_3 s\right)s^{'} \\ & + A_2 a^2\left[\left(2s^{1/2} + M_1 s + \frac{2}{3}M_2 s^{3/2}\right)s^{'} + \frac{M_3}{2}\left(\frac{a_o^3}{a^4} - s^{'}\frac{a_o}{a} + s\frac{a_o}{a^2}\right)\right] \\ & + \left(2A_2 a + A_2^{'}a^2\right)\left[\frac{4}{3}s^{3/2} + \frac{M_1}{2}s^2 + + \frac{4}{15}M_2 s^{5/2} + \frac{M_3}{6}\left\{1 - (a_o/a)^3 - 3sa_o/a\right\}\right] \end{aligned} \quad 20)$$

In which

$$\begin{aligned} s^{'} &= \frac{a_o}{a^2} \\ A_1^{'} &= \frac{\partial\sigma_s(CTOD)}{\partial a} = \frac{\partial\sigma_s(CTOD)}{\partial CTOD}\frac{\partial CTOD}{\partial a} \\ A_2^{'} &= \frac{-\sigma_s^{'}(CTOD)(a - a_o) - \left[f_t - \sigma_s(CTOD)\right]}{(a - a_o)^2} \end{aligned} \tag{21}$$

Also,

$$\frac{\partial\sigma_s(CTOD)}{\partial CTOD} = f_t\left\{\begin{array}{l}\exp\left(\frac{-c_2 CTOD}{w_c}\right)\left[\left(\frac{c_1 CTOD}{w_c}\right)^2\left(\frac{3c_1}{w_c}\right) - \frac{c_2}{w_c}\left(1+\left(\frac{c_1 CTOD}{w_c}\right)^3\right)\right] \\ -\frac{1}{w_c}\left(1+c_1^3\right)\exp\left(-c_2\right)\end{array}\right\} \tag{22}$$

$$\begin{aligned}\frac{\partial CTOD}{\partial a} &= \frac{6\left(P+\frac{W}{2}\right)S}{BD^2E}\left[0.76-4.56\alpha+11.61\alpha^2-8.16\alpha^3+\frac{0.66}{(1-\alpha)^2}+\frac{1.32\alpha}{(1-\alpha)^3}\right] \\ &\quad\times\left\{s^2+\left(1.081-1.149\alpha\right)\left[\frac{a_o}{a}-\left(\frac{a_o}{a}\right)^2\right]\right\}^{\frac{1}{2}} \\ &\quad+\frac{3\left(P+\frac{W}{2}\right)Sa}{BD^2E}\left[0.76-2.28\alpha+3.87\alpha^2-2.04\alpha^3+\frac{0.66}{(1-\alpha)^2}\right] \\ &\quad\times\left\{s^2+\left(1.081-1.149\alpha\right)\left[\frac{a_o}{a}-\left(\frac{a_o}{a}\right)^2\right]\right\}^{-\frac{1}{2}} \\ &\quad\times\left\{2ss'-\frac{1.149}{D}\left[\frac{a_o}{a}-\left(\frac{a_o}{a}\right)^2\right]-\left(1.081-1.149\alpha\right)\left[\frac{a_o}{a^2}-\frac{2a_o^2}{a^3}\right]\right\}\end{aligned} \tag{23}$$

An equation in term of unknown quantity a_c can be obtained from Eqs. (13), (15) and (18) which can be solved using simple numerical procedure. Then the values of K_{IC}^{ini} and K_{IC}^{un} be determined using Eqs. (18) and (1) respectively for known values of a_c and P_u.

Also for known values of a_c and P_u, the values of $CMOD_c$ and $CTOD_c$ can be determined using the following equations. At critical condition, Eq.(3) can be expressed as

$$CMOD_c = \frac{6\left(P_u+\frac{W}{2}\right)Sa_c}{BD^2E}V\left(\alpha_c\right) \tag{25}$$

Also at peak load condition, Eq.(10) becomes as

$$CTOD_c = CMOD_c\left\{(1-a_o/a)^2+(1.081-1.149a_c/D)[a_o/a_c-(a_o/a_c)^2]\right\}^{1/2} \tag{26}$$

The values of $CMOD_c$ and $CTOD_c$ are determined for known values of a_c and P_u using Eqs.(25) and (26) respectively. In simplified extreme peak load method, the $CMOD_c$ is evaluated for comparison purpose with the similar quantity measured during fracture test (herein obtained from FCM). It is reiterated that the measurement of $CMOD_c$ is necessary during the test for determining the double – K fracture parameters using conventional method *i.e.,* weight function method employing linear asymptotic superposition assumption whereas this value is not required for the simplified extreme peak load method.

IV. FICTITIOUS CRACK MODEL (FCM)

Three material properties such as modulus of elasticity E, uniaxial tensile strength f_t, and fracture energy G_F are required to model FCM. The concrete mix with material properties: $\nu = 0.18$, $f_t = 3.21$MPa, $E = 30$ GPa, and $G_F = 103$N/m along with nonlinear stress-displacement softening relation [42] with $c_1 = 3$ and $c_2 = 7$ are used as the input parameters in the present study. In this method, the governing equation of crack opening displacement (COD) along the potential fracture line is written. The influence coefficients of the COD equation are determined using linear elastic finite element method. Four noded iso-parametric plane elements are used in finite element calculation. The COD vector is partitioned according to the enhanced algorithm [5] and finally, the system of nonlinear simultaneous equation is developed and solved using Newton-Raphson method. For standard TPBT with B = 100 mm having size range D = 100-500 mm, the finite element analysis is carried out for which the half of the specimens are discretized due to symmetry considering 80 numbers of equal isoparametric plane elements along the dimension D. The descretization of the beam is shown in Fig. 2.

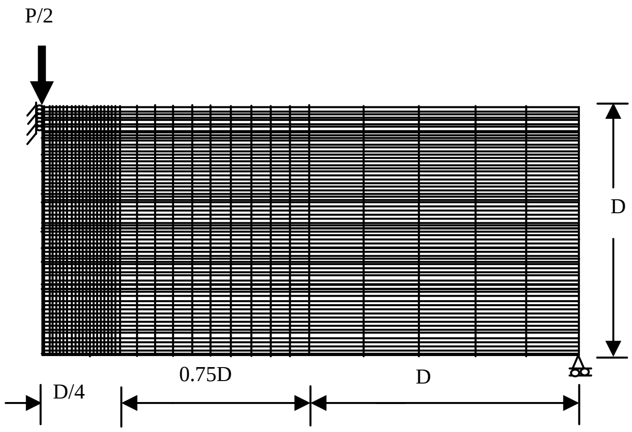

Fig.2. Finite element discretization of three point bend specimen

V. RESULTS AND DISCUSSION

The same material properties viz. modulus of elasticity, uniaxial tensile strength, fracture energy and constants c_1 & c_2 as mentioned in preceding section were considered for computing the double-K fracture parameters using WFM and SEPLM for the TPBT specimen with B = 100 mm. The specimen sizes (D) of 100, 200, 300, 400 and 500mm with different a_o/D ratios as 0.25, 0.30 and 0.40 were used in the study. Values of P_u and $CMOD_c$ for these specimens as derived from FCM are presented in Figs. 3 and 4 respectively. Various fracture parameters of concrete using WFM and SEPLM are then determined and presented in Table 2.

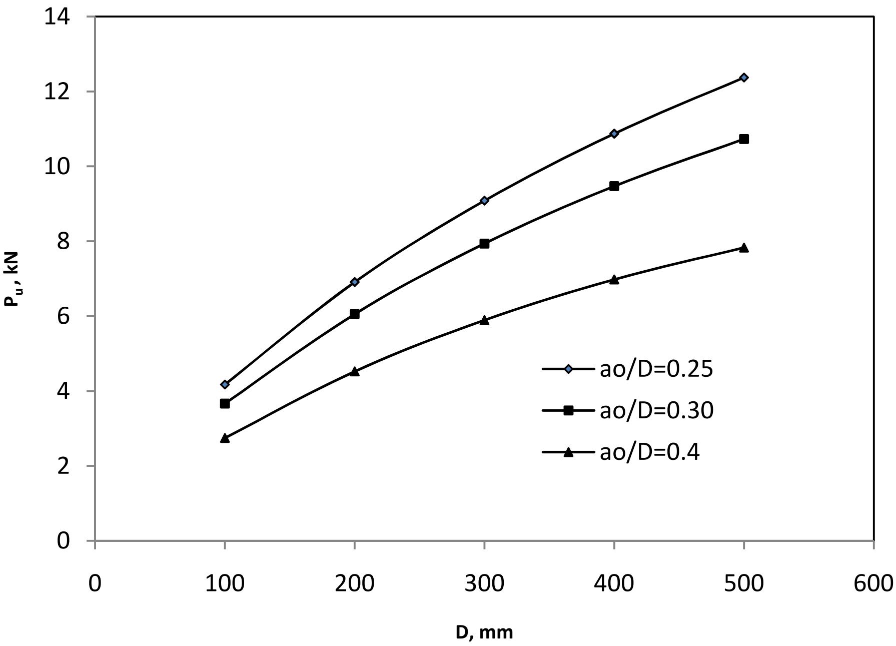

Fig.3. Values of P_u for TPBT specimens as obtained from FCM

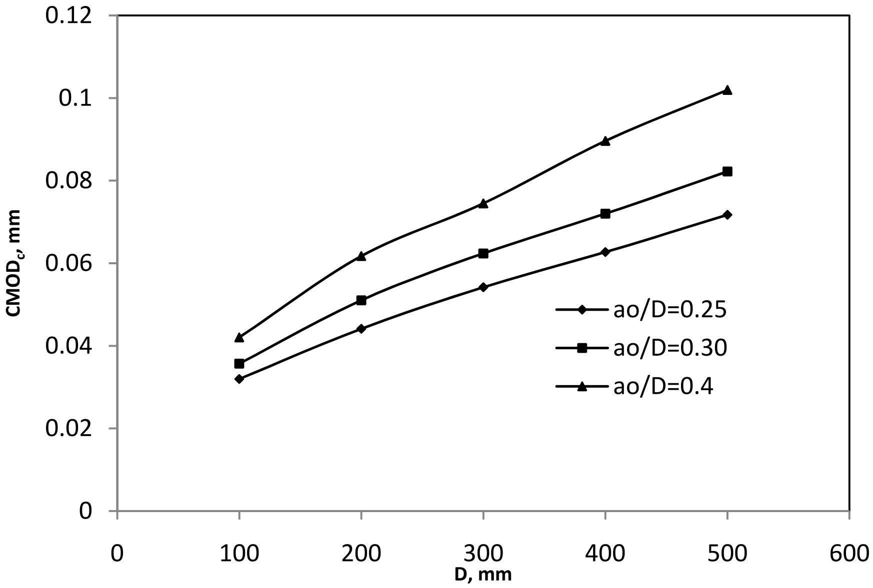

Fig.4. Values of $CMOD_c$ for TPBT specimens as obtained from FCM

TABLE II Double – K Fracture Parameters Using Weight Function Method and Simplified Extreme Peak Load Method

D (mm)	a_o/D	Fracture parameters determined using weight function method				Fracture parameters determined using simplified extreme peak load method			
		a_c/D	$CTOD_c$ (µm)	K_{IC}^{un} (MPa mm$^{1/2}$)	K_{IC}^{ini} (MPa mm$^{1/2}$)	a_c/D	$CTOD_c$ (µm)	K_{IC}^{un} (MPa mm$^{1/2}$)	K_{IC}^{ini} (MPa mm$^{1/2}$)
500	0.25	0.347	33.36	41.75	9.79	0.405	47.60	48.76	5.57
400		0.351	29.54	40.43	10.61	0.392	37.95	45.06	7.94
300		0.362	26.42	39.25	11.11	0.379	28.87	41.11	9.80
200		0.379	22.51	37.53	11.49	0.371	20.58	36.75	10.97
100		0.416	17.64	35.024	11.55	0.368	12.13	30.64	11.17
500	0.30	0.398	34.57	42.10	9.92	0.447	46.12	48.33	6.03
400		0.402	30.69	40.83	10.78	0.434	36.96	44.71	8.35
300		0.413	27.59	39.74	11.32	0.423	28.31	40.85	10.12
200		0.431	23.72	38.16	11.75	0.416	20.31	36.55	11.20
100		0.461	17.76	35.07	11.84	0.413	12.00	30.47	11.30
500	0.40	0.491	34.12	41.74	10.68	0.525	40.66	46.66	7.15
400		0.495	30.55	40.59	11.42	0.516	33.31	43.37	9.17
300		0.498	25.73	38.66	12.11	0.508	26.14	39.83	10.66
200		0.517	22.81	37.50	12.31	0.502	19.09	35.75	11.53
100		0.541	16.64	34.08	12.14	0.500	11.39	29.78	11.42

The ratio of a_c/D obtained from SEPLM to FCM for different sizes of specimens is presented in Fig.5. It is interesting to observe that the value of a_c/D obtained using SEPLM is smaller than those obtained using WFM for small size specimens up to 200mm for all values of a_o/D ratios of 0.25, 0.30 and 0.40. The SEPLM yields higher value of a_c/D for larger size specimens above 300mm for all values of a_o/D ratios of 0.25, 0.30 and 0.40. On an average taken for three values of a_c/D corresponding to a_o/D ratios of 0.25, 0.30 and 0.4, the average ratio of a_c/D obtained from SEPLM to WFM varies between 0.97 to 1.03 for specimen size 200mm and 300mm respectively. These average ratios of a_c/D obtained from SEPLM to WFM for specimen sizes 100mm and 500mm are 0.90 and 1.12 respectively.

The ratio of $CTOD_c$ obtained from SEPLM to WFM for different sizes of specimen is shown in Fig.6. The pattern of this plot is similar to that of Fig.5. The average ratios of $CTOD_c$ obtained using SEPLM to WFM for a_o/D ratios 0.25, 0.30 and 0.40 are 0.87 and 1.05 for specimen size 200mm and 300mm respectively. This average ratio of $CTOD_c$ obtained using SEPLM to WFM is 0.68 for 100mm specimen size and 1.32 for 500 mm specimen size.

Fig.7 represents the variation of ratio of K_{IC}^{un} obtained from SEPLM to WFM with respect to specimen size. The pattern of this curve is similar to those of Figs. 5 and 6. The average ratio of K_{IC}^{un} obtained using SEPLM to WFM for a_o/D ratios 0.25, 0.30 and 0.40 is 0.964 and 1.04 for specimen size 200mm and 300mm respectively. From analysis of Figs 5 to 7, it is obvious that the SEPLM yields almost the same values of fracture parameters for specimen size between 200 to 300mm. This is a peculiar observation i.e., the deviation of predicted results obtained using SEPLM as compared to WFM is more for relatively smaller and larger size specimens than those for the size range of 200-300mm.

The ratio of K_{IC}^{ini} obtained using SEPLM to WFM for different specimen sizes is plotted in Fig.8. From the figure it is observed that SEPLM yields smaller value of K_{IC}^{ini} than that of WFM. For specimen size 200mm, the predicted value of K_{IC}^{ini} by SEPLM is almost same as that obtained using WFM.

The above findings makes it mandatory to investigate further regarding the behavior of predicted results obtained using simplified extreme peak load method as compared with that obtained by conventional analytical weight function method. As the main difference between the two methods SEPLM and WFM uses $CMOD_c$ in WFM where as SEPLM does not require it. In SEPLM for given values of $CTOD_c$ and a_c/D ratio, the value of $CMOD_c$ is determined for comparing it with the measured value of $CMOD_c$ has been used in WFM in the present study. Thus ratio of $CMOD_c$ obtained using SEPLM and measured value as used in WFM is plotted with respect to specimen size in Fig.9.

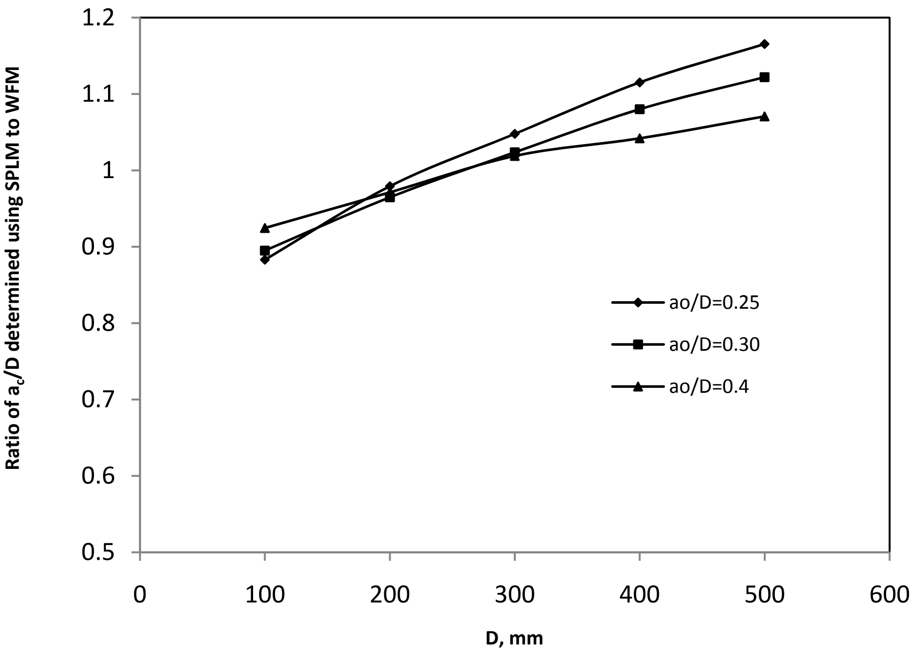

Fig.5. Relationship between the ratio of a_c/D obtained using SPLM to WFM and specimen size

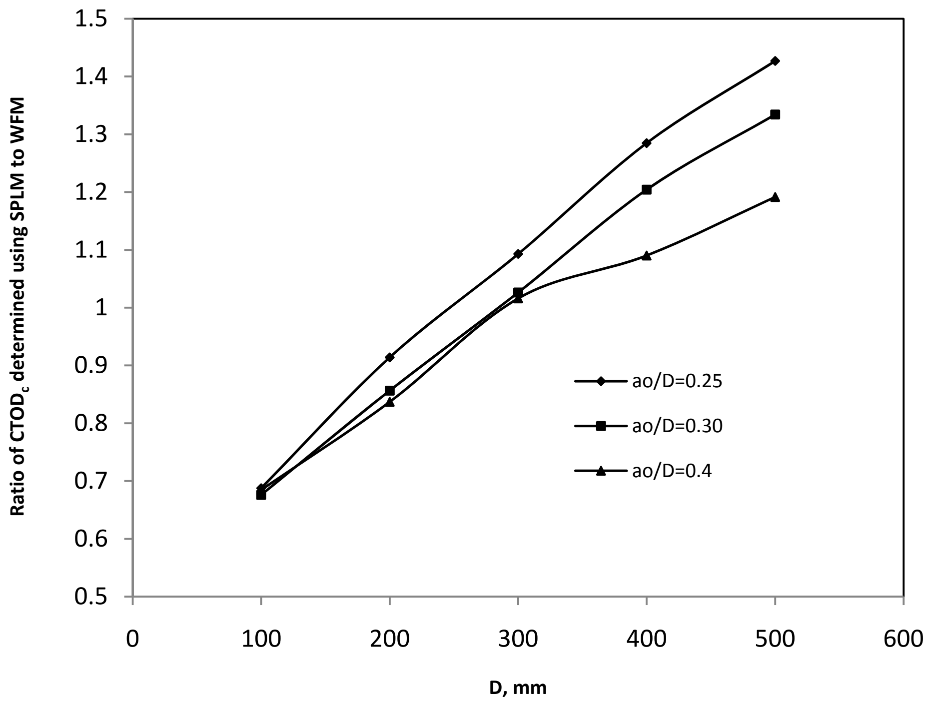

Fig.6. Relationship between the ratio of $CTOD_c$ obtained using SPLM to WFM and specimen size

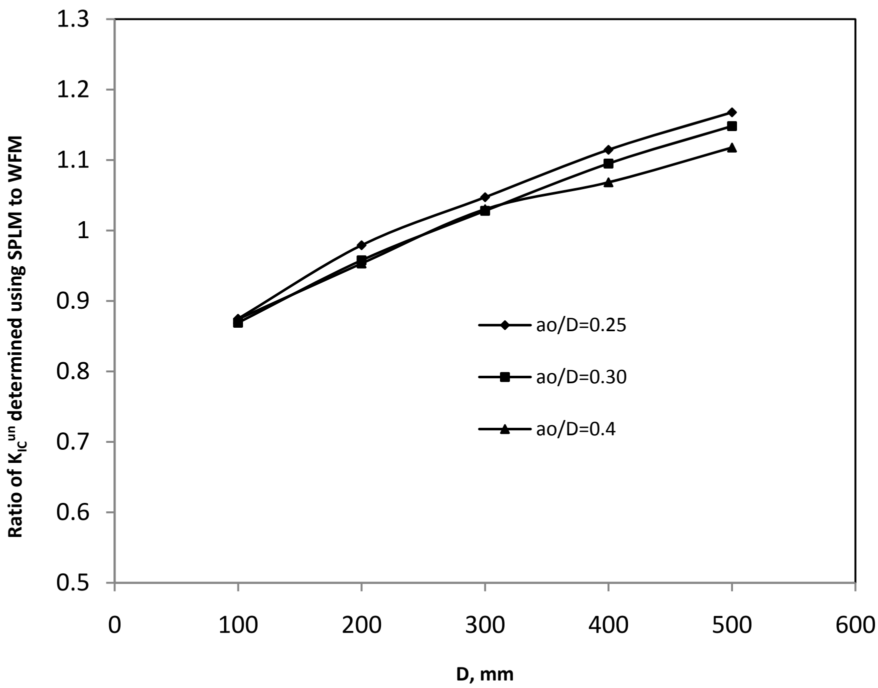

Fig.7. Relationship between the ratio of K_{IC}^{un} obtained using SPLM to WFM and specimen size

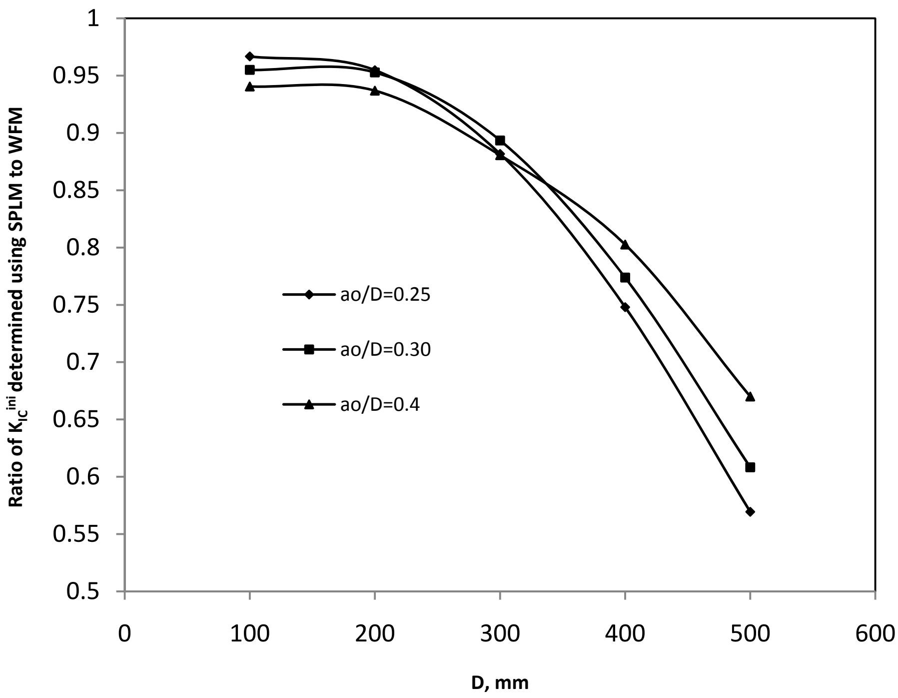

Fig.8. Relationship between the ratio of K_{IC}^{ini} obtained using SPLM to WFM and specimen size

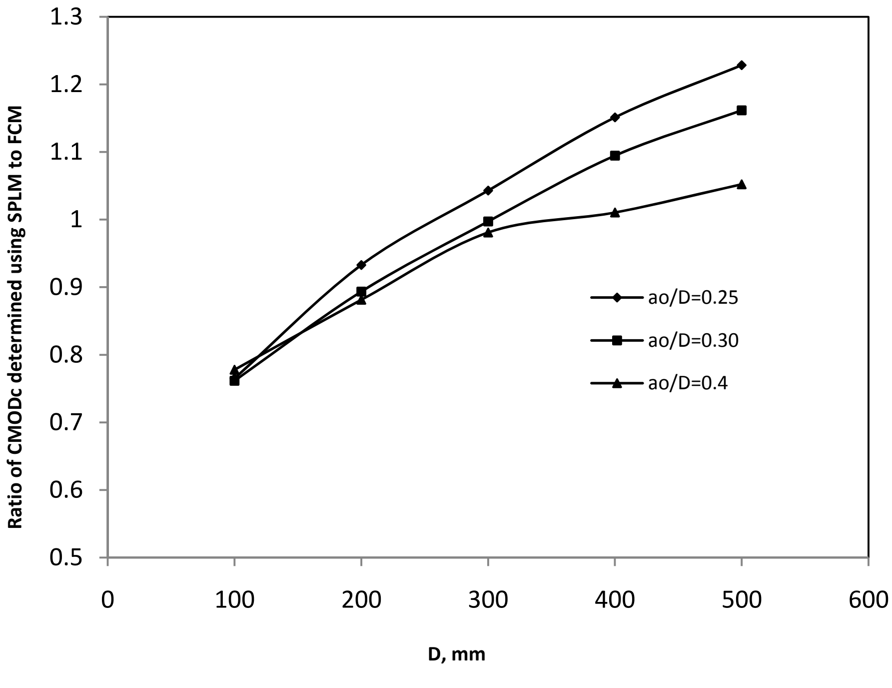

Fig.9. Relationship between the ratio of $CMOD_c$ obtained using SPLM to FCM and specimen size

From Fig.9, it is observed that this ratio slightly depends on a_o/D ratio and it increases with specimen size. It is interesting to note that the ratio of $CMOD_c$ obtained using SEPLM and measured value as used in WFM is almost equal to 1.0 for specimen size of nearly 300mm. This ratio is less than 1.0 for specimen size smaller than that of 300 mm and more than 1.0 for specimen size larger than 300mm.

It can be observed that the range of specimen size used by [37] is 203 -305 mm. In the study, it was concluded that the double – K fracture parameters obtained using SEPLM is slightly smaller than those by WFM. The similar prediction can be observed in the present study for the specimen size range 200-300mm. However, based on the results obtained in present study, the application of SEPLM needs to be further investigated with experimental data for larger size specimens.

The significant deviation in predicted values of double – K fracture parameters for some specimens (B26, C13) by SEPLM is attributed to measuring error in $CMOD_c$ during fracture test as reported in the study of Qing et. al. [37]. It is somewhat true that experimental error may occur in measuring the $CMOD_c$ due to improper connection between the clip gauge and specimen and also due to feeble sensitivity of the gauge used. However, it is well established by Zhang and Xu [30] that the double-K toughness parameters for compact tension wedge splitting specimens for size range 200-1000mm determined from the experimental measurements and existing analytical solution *i.e.,* weight function method are well agreed. Therefore the application of SEPLM for computing double-K fracture parameters of concrete needs to be investigated experimentally for wide range of specimen size.

VI. CONCLUSION

The simplified extreme peak load method does not require the measured value of $CMOD_c$ for computing double – K fracture parameters. This method needs only peak load and also avoids complicated numerical integration while determining the double – K fracture parameters. From the present study, it can be concluded that simplified extreme peak load method yields the best results of double – K fracture parameters for the specimen size range 200-300mm. The deviation of predicted results obtained using simplified extreme peak load method as compared to conventional analytical weight function method is more for relatively smaller and larger sizes specimens than those of size range 200-300mm. This limitation of the predicting capacity of simplified extreme peak load method is attributed to use of only peak load in the formulation, where as both the input parameters: peak load and $CMOD_c$ are required in conventional analytical method for computing the double-K fracture parameters of concrete. Based on the results obtained in present study, it can be concluded that the application of simplified extreme peak load method is limited for specimen size range 200-300mm and this method needs to be further investigated with experimentally measured peak load and $CMOD_c$ for validity of wide range of specimen sizes.

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