Structural Optimization Using the Grouping Method and the 1/3rd Rule Based on Specific Strain Energy

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Abstract— **Structural optimization approaches may be categorized into three major types. One type of approach is topological optimization, which involves many sensitivity analysis variables. This type of approach sometimes results in odd shapes, such as checkerboard patterns. The other types are shape optimization and parametric optimization, which involve certain difficulties in dealing with the selection of proper parameters and require repeated meshing for the purpose of finite element analysis. We propose an efficient method for grouping finite elements to reduce the number of degrees of freedom of the system considerably and to perform the optimization of several groups of elements. If we reject elements using a cutoff criterion based on the specific strain energy for several steps, we may obtain a topologically optimized result for the discrete configuration, without any irregularity. This optimization may have a higher-speed process based on the grouping method. The grouping method divides the elements into three groups on the basis of strain energy—a high-energy group, a low-energy group, and a mid-energy group. By moving the high-energy group to a high-priority group and eliminating the low-energy group, a 1/3rd rule can be used to obtain an optimized design. The 1/3rd rule is fast and effective and provides a way to obtain a realistic result. Several examples were considered to test the optimization efficacy of the grouping technique.**

Keyword- Grouping Method, 1/3rd rule, Strain Energy, Structural Optimization

I. INTRODUCTION

The importance of optimization [1], which can be used to remove unnecessary parts and improve the effectiveness of a design, has grown gradually in engineering design. Structural optimization approaches may be categorized into three major types. One type of approach is topological optimization, which considers many variables in a sensitivity analysis [2]-[4]. This type of approach sometimes results in odd shapes, such as checkerboard patterns. The other two major types of approaches are shape optimization and parametric optimization. These types of approaches have certain difficulties in dealing with the selection of proper parameters and require repeated meshing for finite element analysis. This is a complex and time-consuming execution method, and a single execution involves considerable effort and cost. Therefore, producing objects using this approach involves considerable costs and time expenditures, which decreases its economic efficiency.

Therefore, it is necessary to develop a new and more realistic optimization method. In this paper, we propose an efficient method for grouping finite elements to reduce the number of degrees of freedom of the system and performing optimization for several groups of elements. If we reject elements using a cutoff criterion based on the specific strain energy [5]-[9] for several steps, we can obtain a topologically optimized result with a discrete configuration and without any irregularity. This method exhibits fast calculation times and yields realistic results.

In addition, a 1/3rd rule based on strain energy was developed. The 1/3rd rule divides elements into groups and compares them in terms of their strain energy, retaining the high-energy groups and removing the low-energy groups. The 1/3rd rule increases the understanding of the structure by examining it in terms of the strain energy, and thereby increases the efficiency of the structure. In addition, the results obtained are practical because the operator considers the entire shape at each step in the process.

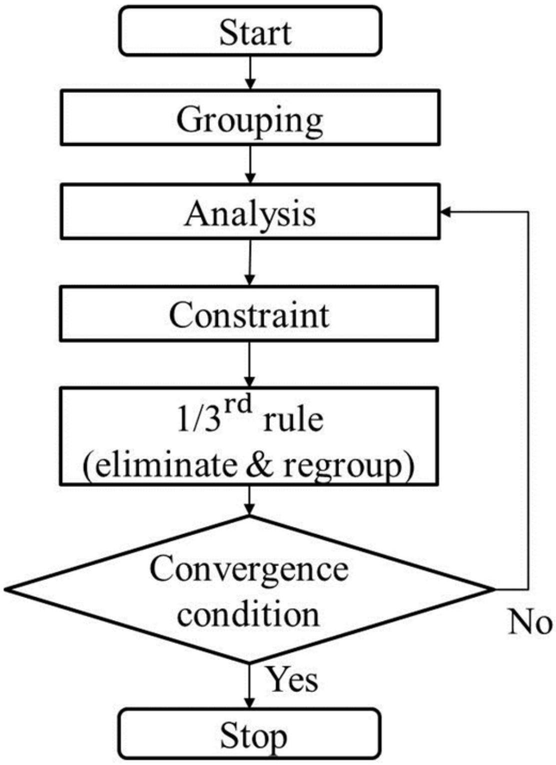


Fig. 1. Flowchart of optimization procedure

Several examples were considered to test the efficiency of the grouping technique and application of the 1/3rd rule to optimization. A cantilever structure was fully optimized using PIAnO [10]. Then, its elements were grouped into several categories and optimized with far less computational effort. We then applied several rejection steps to obtain topologically optimized results.

II. Grouping Method and 1/3rd Rule

In topological optimization, the number of variables is high, and the computation cost is high as a result. Satisfying the strength constraint is another problem that needs to be addressed, as well as addressing the checkerboard pattern problem. In this section, a grouping method is proposed for use in resolving these types of problems to a certain extent. The procedure for applying the grouping method and 1/3rd rule is as follows:

1 The entire domain of a model is classified into several groups.
1-1 Each group is selected manually based on similar levels of strain energy density, based on experience and mechanics.
1-2 Every group is discretized into many finite elements.
2 The finite element model is analyzed after the application of boundary conditions. The strength constraint is satisfied by adjusting element thicknesses.
3 The elements in each group are sorted in order of their strain energy density (SED).
4 The groups are sorted in order of the highest SED of the elements within each group or in the order of the highest average SED.
5 The highest SEDs of each group are compared with the highest SED of the 1st sorted group, and the groups of the highest SED smaller than 1/3rd of the highest SED are eliminated from the model.
6 Elements in the groups with the highest SED smaller than 2/3 of the highest SED are modified, depending upon the level in the group.
6-1 The elements with SEDs in the top 1/3rd are moved to the adjacent group.
6-2 The elements with SEDS in the middle 1/3rd remain in the same group.
6-3 The elements in the lowest 1/3rd are removed from the group.
7 The process returns to step 2 and repeats until there is no group whose highest SED is smaller than 1/3rd of the first highest SED.

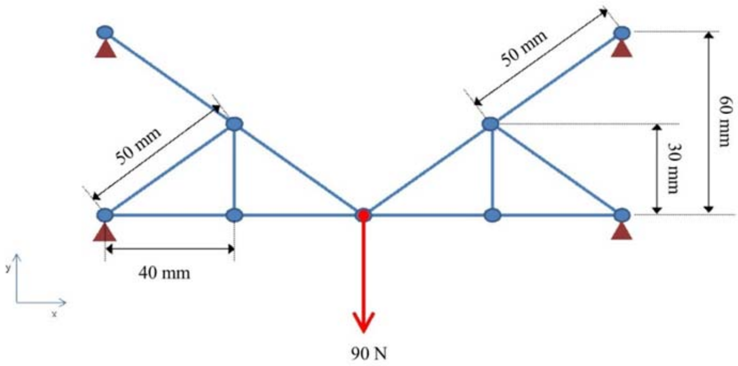


Fig. 2. Truss structure model

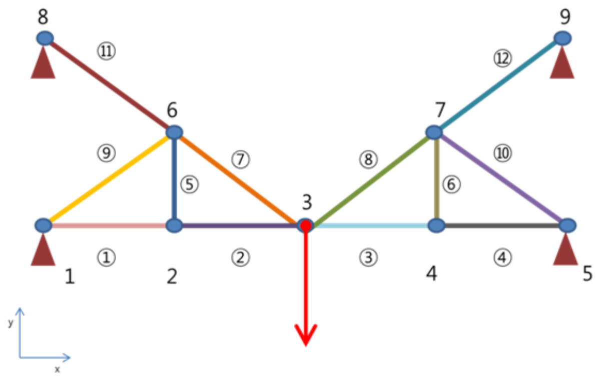


Fig. 3. Numbering of elements and nodes

Through the actions of elimination and modification (elements being, moved removed, or unchanged), the overall model is topologically optimized while the strength constraint is satisfied. The chance of a checkerboard pattern occurring and the necessity for smoothing during post-processing are minimized by the reasonable selection of groups for the model. A flowchart of the procedure is shown in Fig. 1.

III. EXAPMPLES

Three examples—truss, cantilever, and clamped deep beam problems—are presented to demonstrate the effectiveness of the grouping method.

A. Truss

The first example is a truss structure, illustrated in Fig. 2 and Fig. 3. If the results obtained and time required for the calculations for each element are compared with the results obtained using the grouping method, the efficiency of the grouping method is easily understood. The truss structure consists of steel with a Young's modulus of 200 GPa and a Poisson's ratio of 0.3. As mentioned above, based on engineering judgment and experience, the elements under stress in a similar direction are grouped together. If the elements are not grouped, the number of elements is 12. However, as shown in Table 1, if the elements acting in similar directions are grouped together based on symmetry, they are categorized into six groups. In addition, among the six grouped elements, those acting in a similar direction are regrouped. This results in a total of four elements, so four elements are generated. The amount of calculation required is thus reduced to one third of the original amount required. This example is a simple case. Because the amount of calculation required is reduced by grouping the elements, the calculation time is decreased, and similar results are obtained.

TABLE I
Numbering of Non-grouped and Grouped Elements

nodes	1, 2	2, 3	3, 4	4, 5	2, 6	4, 7	3, 6	3, 7	1, 6	5, 7	6, 8	7, 9
12 elements	①	②	③	④	⑤	⑥	⑦	⑧	⑨	⑩	⑪	⑫
6 groups	***1***	***2***	***2***	***1***	***4***	***4***	***5***	***5***	***3***	***3***	***6***	***6***
4 groups	Ⅰ	Ⅰ	Ⅰ	Ⅰ	Ⅲ	Ⅲ	Ⅳ	Ⅳ	Ⅱ	Ⅱ	Ⅳ	Ⅳ

TABLE III
Results of the Calculations with Non-grouped Elements

Element No.	A (mm^2)	L (mm)	volume (mm^3)
1	8.23E-04	40	0.0329
2	7.38E-04	40	0.0295
3	1.01E-03	40	0.0403
4	2.03E-04	30	0.0061
5	7.79E-04	30	0.0234
6	4.29E-05	50	0.0021
7	1.50E+01	50	750.0000
8	1.50E+01	50	750.0000
9	6.53E-04	50	0.0326
10	9.37E-03	50	0.4687
11	1.50E+01	50	750.0000
12	1.50E+01	50	750.0000

TABLE IIII
Results of the Calculation with Six-group Elements

Group No.	G1	G2	G3	G4	G5	G6
A (mm^2)	1.24E-09	2.58E-06	2.12E-09	1.55E-09	1.50E+01	1.50E+01
L (mm)	40	40	50	30	50	50
Volume (mm^3)	4.97E-08	1.03E-04	1.06E-07	4.64E-08	7.50E+02	7.50E+02

TABLE IVV
Results of the Calculation with Four-group Elements

Group No.	G1	G2	G3	G4
A (mm^2)	1.34E-09	1.00E-09	1.00E-09	1.50E+01
L (mm)	40	50	30	50
Volume (mm^3)	5.35E-08	5.00E-08	3.00E-08	7.50E+02

As Tables 2 to 4 show, the results of the calculations are almost the same, but the number of elements required for the calculation is noticeably reduced. Based on this, the calculation time is also reduced. The effect of the grouping method is greater if the target object is more complicated and the number of elements is increased. The results of the truss problem demonstrate the efficiency of the grouping method.

B. *Cantilever*

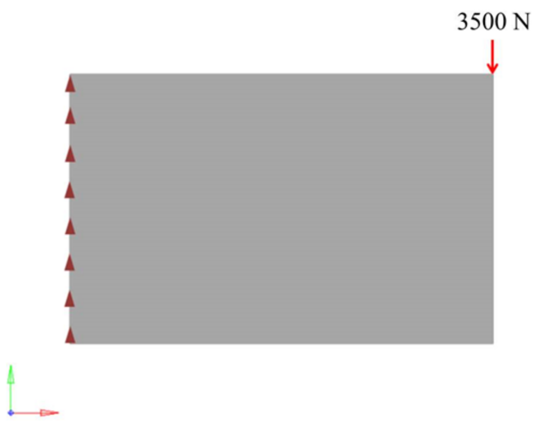


Fig. 4. Boundary conditions and load on cantilever

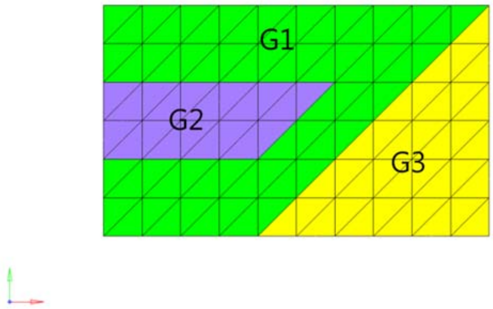
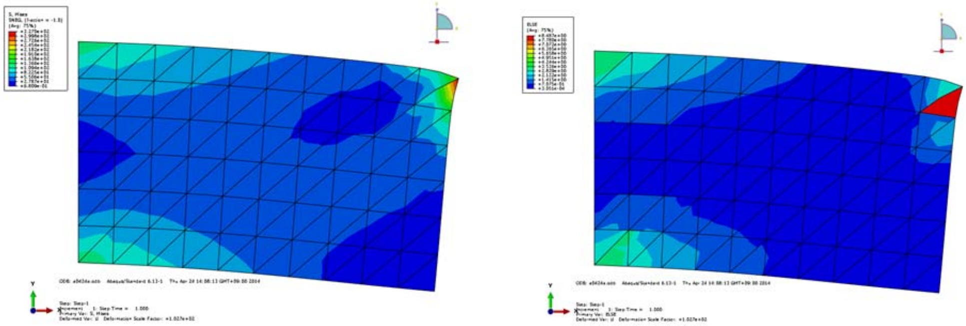


Fig. 5. Grouped elements



(a) von Mises stress (b) Strain energy

Fig. 6. Analysis results

Figure 4 illustrates a cantilever problem that can be solved using the grouping method and the 1/3rd rule. This cantilever has the following properties. The type of steel is AISI 8000 series steel, with a density of 7.8e-9 Mg/mm3 and a Young's modulus of 207 GPa. Its Poisson's ratio is 0.3, and its yield strength is 900 MPa. This cantilever is under a load of 3,500 N, as shown Fig. 4. The objective of the problem is to minimize the weight of the cantilever while satisfying the requirement for a safety factor of three. This example is used to demonstrate that the grouping method and the 1/3rd rule may be the best method for optimizing the design of this type of structure.

The cantilever problem is solved by applying the grouping method to a large bundle of elements rather than individual elements. The elements of the cantilever problem are grouped empirically as shown in Fig. 5. For this shape, the von Mises stress and the strain energy are obtained using Abaqus, as shown in Fig. 6.

By creating a table for the strain energy per unit area (SED) and per element, as shown in Table 5, the sizes and priorities of the element groups can be determined. In considering the entire shape, high-strain-energy elements move to the high level, and low-strain-energy elements are eliminated. Mid-level elements are maintained without any decision being made concerning their placement.

TABLE V
Strain Energy Sheet for the First-step Model

Group 1		Group 2		Group 3	
Element	SED	Element	SED	Element	SED
6	0.2780	73	0.0277	120	0.6789
63	0.2720	69	0.0275	118	0.1544
14	0.1985	76	0.0270	119	0.1046
62	0.1854	77	0.0261	85	0.0733
64	0.1845	72	0.0258	117	0.0519
7	0.1829	80	0.0245	86	0.0455
5	0.1773	78	0.0220	87	0.0453

8	0.1597	81	0.0219	116	0.0431
22	0.1532	74	0.0219	107	0.0352
13	0.1440	75	0.0213	109	0.0320
15	0.1412	83	0.0212	88	0.0318
.	.	65	0.0207	.	.
.	.	79	0.0205	.	.
.	.	71	0.0197	.	.
.	.	82	0.0193	.	.
41	0.0225	70	0.0184	115	0.0205
57	0.0211	84	0.0181	96	0.0170
43	0.0200	68	0.0173	104	0.0150
48	0.0199	67	0.0127	95	0.0140
54	0.0186	66	0.0095	114	0.0110
42	0.0186			103	0.0083
51	0.0180			94	0.0082
49	0.0163			113	0.0058
60	0.0149			102	0.0050
50	0.0127			112	0.0015
55	0.0103			101	0.0014
59	0.0093			111	0.0007
56	0.0089			110	0.0000

According to Table 5, the strain energies of the elements in group 2 are generally low, as are the von Mises stresses. Therefore, group 2 does not seem to be important to the overall shape and is eliminated. The high-strain-energy elements in group 3 move to the high group, and the low-strain-energy elements in group 3 are eliminated. The results of the above procedures are shown in Fig. 7. The optimal thickness that satisfies the constraint is found using PIAnO.

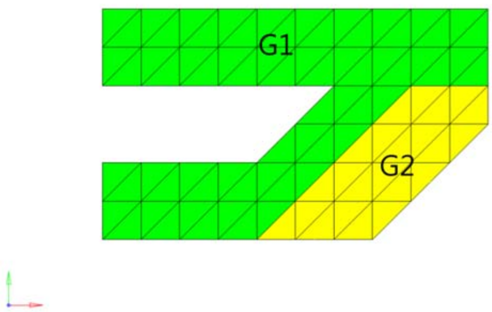
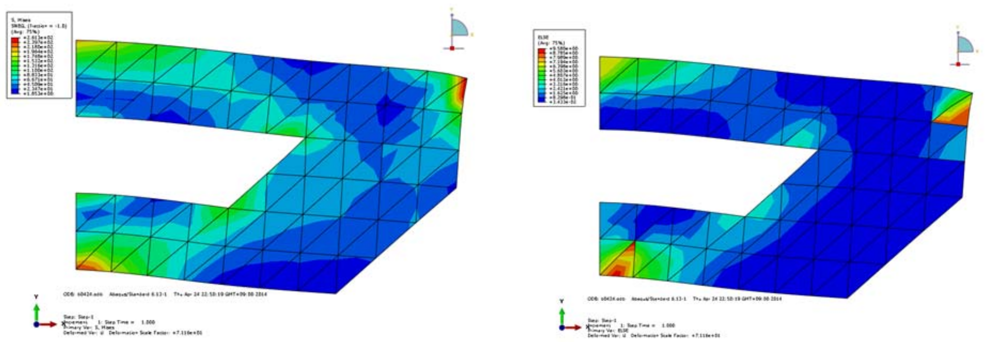


Fig. 7. Results of applying the 1/3rd rule to the first step of the cantilever problem



(a) von Mises stress (b) Strain energy

Fig. 8. Analysis results for regrouped cantilever

Application of the 1/3rd rule to the first step results in division into two groups, as shown in Fig. 7. For this shape, the von Mises stress and the strain energy obtained using Abaqus are shown in Fig. 8.

After the von Mises stresses and SED values for the above results were determined, the SED sheet shown in Table 6 was developed, and the 1/3rd rule was applied.

TABLE VI
Strain Energy Sheet for the Second-step Model

Group 1		Group 2	
Element	SED	Element	SED
12	22.6965	29	2.2643
121	22.2012	17	2.2445
24	16.2038	32	2.2064
120	15.1352	41	2.1345
122	15.0598	20	2.1075
13	14.9302	44	1.9978
11	14.4742	42	1.7998
14	13.0344	54	1.7874
36	12.5100	30	1.7854
23	11.7536	31	1.7394
25	11.5272	56	1.7316
.	.	5	1.6921
.	.	43	1.6724
.	.	19	1.6101
78	1.9857	55	1.5737
66	1.8373	18	1.5043
95	1.7238	68	1.4802
69	1.6339	8	1.4163
79	1.6241	7	1.0366
92	1.5222	6	0.7728
67	1.5191		
82	1.4682		
80	1.3306		

107	1.2186		
81	1.0349		
93	0.8436		
106	0.7626		
94	0.7274		

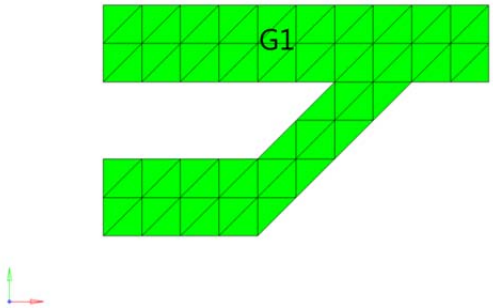
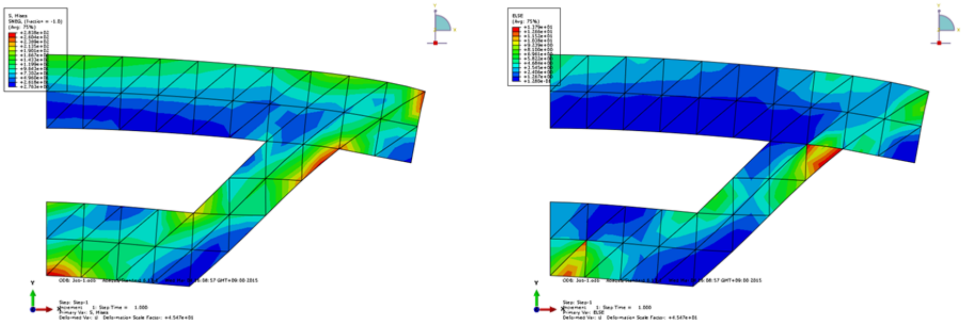


Fig. 9. Final shape of the cantilever model



(a) von Mises stress (b) Strain energy

Fig. 10. Analysis results for final shape of cantilever

According to the 1/3rd rule, group 2 is eliminated. The optimized result that satisfies the constraints is identified using PIAnO. The final shape is shown in Fig. 9. The weight of this structure is 40% less than that before the optimization.

The above shape was analyzed again using Abaqus and optimized once again using PIAnO. The following results were obtained.

In the results shown in Fig. 10, the strain energy is distributed evenly, and the weight has been reduced overall. This shape was optimized once again to calculate the optimal thickness and weight. The results are shown in Table 7.

This result shows that the weight has decreased by approximately 41.39% satisfying the constraint of maximum von Mises stress 300 MPa. Through this cantilever problem, the efficiency of grouping can be reconfirmed.

TABLE VII
Thickness Results Obtained from Optimization

	σ_{max} (MPa)	weight (mg)	thickness (mm)
Step 1	300	1.275e-04	10.8988
Step 2	300	9.964e-05	10.8723
Step 3	300	7.473e-05	11.2724

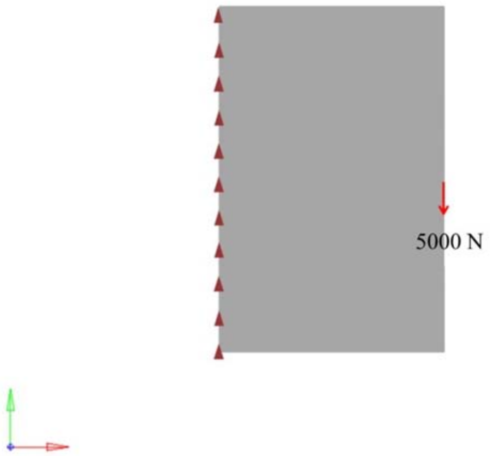


Fig. 11. Boundary condition and load on clamped deep beam

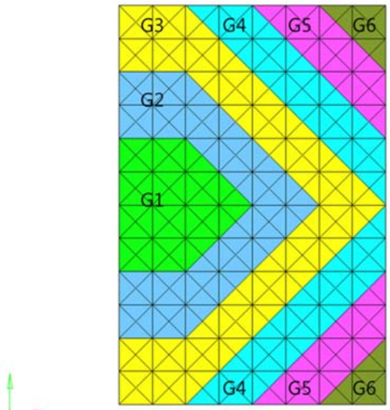
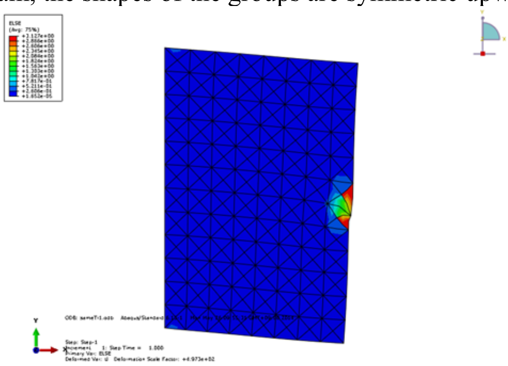


Fig. 12. Grouped elements of clamped deep beam

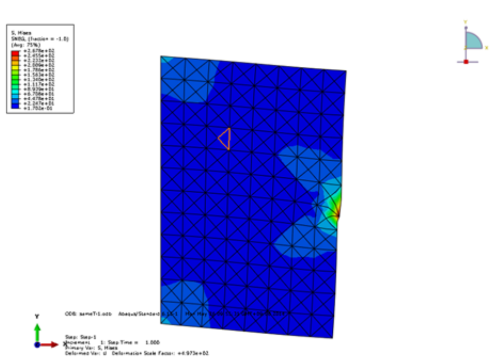
C. *Clamped deep beam*

The next example is the clamped deep beam problem illustrated in Fig. 11. Its vertical length is greater than that of the cantilever, and the location of the load is at the center of the structure. The size of the clamped deep beam is 40 mm × 60 mm, and the left side is fixed. The clamped deep beam is subjected to a vertical downward 5000-N load at the center of the right side. As with the cantilever beam, the objective is to find a shape with an optimum weight while satisfying the constraint of not exceeding the allowable stress of 300 MPa.

The elements for the shape shown in Fig. 11 were assembled using Hypermesh, and the elements were divided into groups empirically, as shown in Fig. 12. Because the load is located at the center of the clamped deep beam, the shapes of the groups are symmetric upward and downward.



(a) von Mises stress



(b) Strain energy

Fig. 13. Analysis results

The von Mises stresses and strain energies for the grouped elements were obtained using Abaqus, as shown in Fig. 13.

Based on the listing of elements and SEDs shown in Table 8, elements were eliminated or moved according to the 1/3rd rule. The results obtained are shown in Fig. 14.

TABLE VIII
Strain Energy Sheet of First-step Model

Group 1		Group 2		Group 3		Group 4		Group 5		Group 6	
Element	SED	Element	SED	Element	SED	Element	SED	Element	SED	Element	SED
113	0.007	58	0.011	360	0.188	723	0.500	731	0.006	740	0.0001
485	0.007	448	0.011	722	0.188	359	0.500	351	0.006	342	0.0001
66	0.007	10	0.011	2	0.046	724	0.084	732	0.005	695	0.0001
440	0.007	400	0.011	408	0.046	358	0.084	350	0.005	291	0.0001
67	0.007	59	0.010	1	0.034	726	0.050	687	0.004	739	0.0001
439	0.007	447	0.010	405	0.034	356	0.050	299	0.004	343	0.0001
116	0.007	105	0.010	357	0.030	727	0.034	688	0.003	696	0.0000
486	0.007	493	0.010	721	0.030	355	0.034	298	0.003	290	0.0000
118	0.006	57	0.010	3	0.027	725	0.029	734	0.003	742	0.0000
484	0.006	445	0.010	407	0.027	353	0.029	348	0.003	340	0.0000
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	741	0.0000
										337	0.0000
										743	0.0000
529	0.006	16	0.006	201	0.009	551	0.005	692	0.001	339	0.0000
433	0.005	394	0.006	589	0.009	147	0.005	294	0.001	744	0.0000
20	0.005	167	0.006	149	0.009	595	0.005	647	0.001	338	0.0000
390	0.005	531	0.006	545	0.009	199	0.005	243	0.001		
23	0.005	213	0.006	152	0.008	596	0.005	648	0.001		
387	0.005	577	0.006	546	0.008	198	0.005	242	0.001		
17	0.005	215	0.005	262	0.007	642	0.005	737	0.001		
389	0.005	579	0.005	628	0.007	248	0.005	341	0.001		
22	0.005	261	0.005	309	0.006	641	0.004	738	0.001		
388	0.005	625	0.005	673	0.006	245	0.004	344	0.001		
24	0.004	216	0.005	263	0.005	597	0.004	694	0.000		
386	0.004	578	0.005	627	0.005	193	0.004	292	0.000		
21	0.004	264	0.004	312	0.004	598	0.004	693	0.000		
385	0.004	626	0.004	674	0.004	196	0.004	289	0.000		

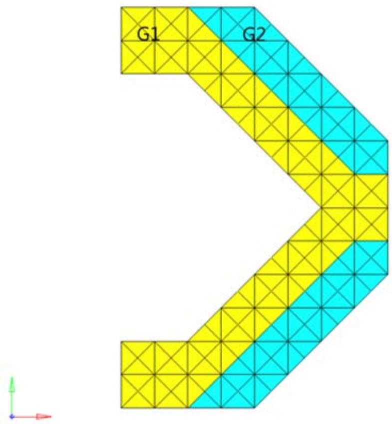
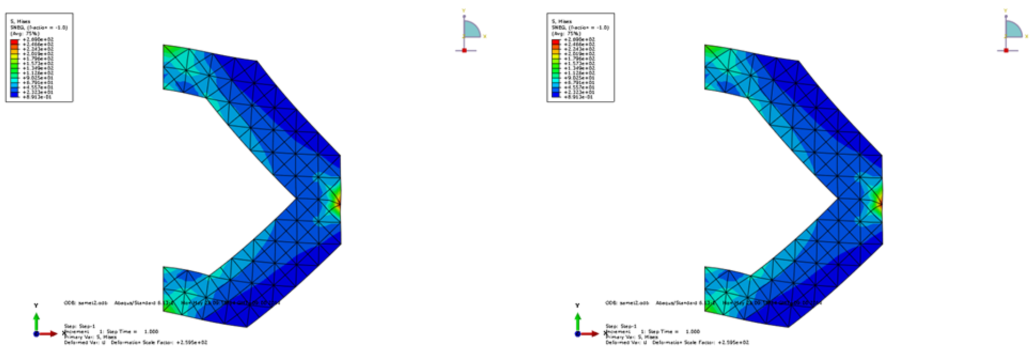


Fig. 14. The results of the 1/3rd rule, first step, for the clamped deep beam



(a) von Mises stress (b) Strain energy

Fig. 15. Analysis results for regrouped clamped deep beam

These results were analyzed again using Abaqus, as shown in Fig. 15. Because considerable portions of the structure still appeared to be unnecessary, we decided to apply the 1/3rd rule one more time.

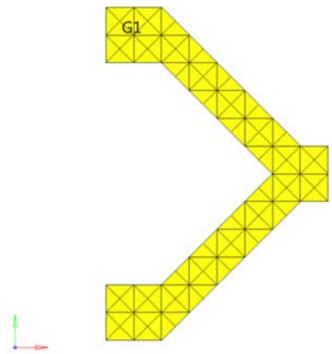
After listing the SEDs and elements, as shown in Table 9, low-strain elements were eliminated, and high-strain elements were moved to the high level, according to the 1/3rd rule.

TABLE IX
Strain Energy Sheet of Second-step Model

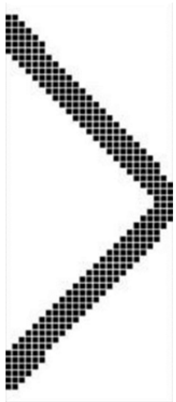
Group 1		Group 2	
Element	SED	Element	SED
359	9.58832	726	0.870392
723	9.58832	725	0.616489
2	7.211136	679	0.573113
408	7.211136	680	0.425903
1	4.261984	635	0.319316
405	4.261984	636	0.30694
360	4.239464	682	0.292934
722	4.239464	591	0.279462
3	3.173456	592	0.264418
407	3.173456	727	0.253897
106	2.637224	681	0.251197

. . . .	. . . .	. . . .	. . . .
204	0.559075	251	0.069972
590	0.559075	148	0.065785
312	0.552746	300	0.05884
674	0.552746	250	0.051003
305	0.545626	145	0.050018
677	0.545626	297	0.038077
152	0.532826	199	0.02597
546	0.532826	198	0.014447
253	0.520713	196	0.013047
633	0.520713	245	0.012496
256	0.516326	146	0.012038
634	0.516326	248	0.011893
149	0.486726	193	0.009099
545	0.486726	147	0.00799

The results obtained are shown in Fig. 16 (a). To illustrate the advantages of the grouping method and the 1/3rd rule, the results obtained using the conventional topology method for a clamped deep beam are shown Fig. 16 (b) [11].

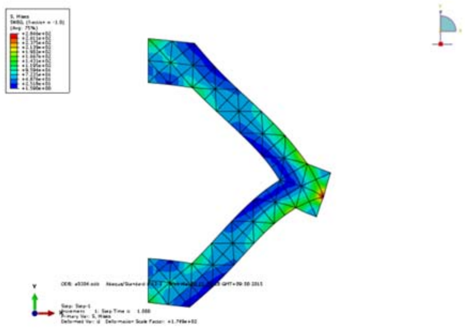


(a) Final shape obtained using the grouping method

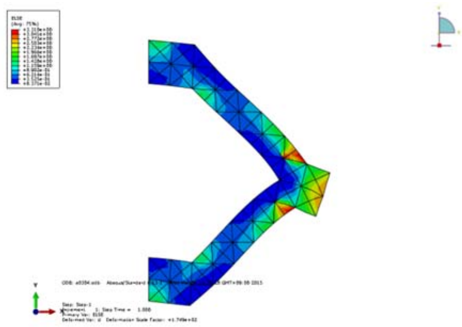


(b) Final shape obtained using a conventional method [11]

Fig. 16. Comparison of grouping method with 1/3rd rule and conventional topology method



(a) von Mises stress



(b) Strain energy

Fig. 17. Analysis results for regrouped clamped deep beam

TABLE X
Thickness Optimization Result

	σ_{max} (MPa)	weight (Mg)	thickness (mm)
Step 1	300	2.012e-04	10.7524
Step 2	300	1.008e-04	10.7756
Step 3	300	6.212e-05	10.9857

The above results were analyzed again using Abaqus and optimized once again using PIAnO. The following results were obtained.

As Fig. 17 shows, the strain energy is evenly distributed, and the weight has been reduced overall. This shape was optimized once again to calculate the optimal thickness and weight. The results are shown in Table 10.

As the results show, the weight was decreased by 69.12% in comparison to the initial design of the beam.

IV. CONCLUSION

In the grouping method developed in this study, engineers intervene directly in the optimization process so that the optimal solutions to realistic problems can be found. The general topology optimization approach satisfies the constraints and tunes itself to the objectives, but in fact, the shape obtained as a result often cannot be produced in reality. In grouping, engineers consider each element directly and can add one or leave one out; therefore, it is more efficient than any other optimization method. In addition, because the optimization is carried out based on the strain energy, the efficiency of the entire shape can be increased considerably. By eliminating unnecessary parts and moving some parts with high strain energy, the strain energy can be distributed evenly, so that an efficient design, without any unnecessary parts, is obtained by considering the contributions of all of the elements. The grouping method and the 1/3rd rule are more efficient than other optimization methods used in production because they make it possible for the engineer to identify and add or subtract each element directly.

The results of this study of the grouping optimization method can be summarized as follows.

1. The grouping method and 1/3rd rule examined in this study can quickly and accurately identify highly realistic and easily manufactured shapes by addressing the problems of the traditional topological optimization method, such as the complexity of the process and the impracticality of some of the results.
2. Analysis in terms of the strain energy per unit area increases the efficiency of the entire shape. By eliminating unnecessary parts and reinforcing high-strain-energy parts, the strain energy is evenly distributed, and the action of the entire shape is made structurally uniform. Thus, the structure can be designed efficiently without unnecessary parts.
3. In conventional topological optimization, a large number of finite element analyses are performed to identify and remove ineffective elements. The proposed grouping method requires several finite element analyses to obtain a reasonable shape in the structural, artistic, manufacturing, and practical senses.

The results obtained show that the grouping optimization method can be used to easily reduce the weight of the designed structure by more than 30%, and avoid unrealistic structural designs and checkerboard patterns. Thus, the grouping optimization method makes it possible to identify the optimal structure in manufacturing practice. Moreover, because element identification is fast in the group optimization and because individual elements are identified in groups, the computational effort required is minimized. In conclusion, because engineers can intervene directly in each step, the grouping optimization method can be used to identify the optimal structural solution that satisfies a variety of conditions at the same time.

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