Multilevel Image Thresholding for Image Segmentation by Optimizing Fuzzy Entropy using Firefly Algorithm

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Abstract - Image thresholding is the process of extracting objects in a scene from the background accompanies for the analysis and interpretation of image which is mostly employed for its advanced simplicity, robustness, less convergence time and accuracy. The main intend of image segmentation is to segregate the foreground from background. As ordinary thresholding method of image segmentation is computationally expensive while extending for multilevel image thresholding, the need for optimization techniques is highly recommended. The so called optimization techniques such as Particle swarm optimization and bat algorithm undergo instability when the particle velocity is maximum and stagnation stage attributable to quick exploration. This paper proposes for the first time the multilevel image thresholding for image segmentation by using Fuzzy entropy maximized by naturally inspired firefly algorithm. A firefly based multilevel image thresholding is established by maximizing Fuzzy entropy where the results are proved better in misclassification, standard deviation, Structural Similarity Index and segmented image quality while comparing with differential evolution, Particle swarm optimization and bat algorithm..

Keywords: Image thresholding; Image segmentation; Fuzzy entropy; Bat algorithm; Firefly algorithm

I. INTRODUCTION

Image thresholding is the process of extracting objects in a scene from the background that helps for analysis and interpretation of image. It is a challenging task for the researchers in image processing to select a preeminent gray level threshold that extracts the object from the background of the gray level image or color image. Selection of threshold is moderately simple in the case where histogram of the image has a deep valley representing background and sharp edges representing objects, but due to the multimodality of the histograms of many image selections of a threshold is difficult task. So researchers proposed many techniques for preeminent gray level threshold. Though there are so many segmentation techniques are in the literature, thresholding is mostly used for its advanced simplicity, robustness, less convergencetime and accuracy. Thresholding approaches are of two types that are nonparametric and parametric. In nonparametric approach thresholding is performed based on class variance as in otsu's technique or based on an entropy criterion, such as Shannon entropy, Fuzzy entropy and Kapur's entropy (Luca and Termini, 1972). If the image is partitioned into two classes, i.e. object and background, then the threshold is called bi-level threshold else multi-level threshold. Thresholding technique has so many real time applications like data, image and video compression, image recognition, pattern recognition, and image understanding and communication (chiranjeevi and jena, 2016). Sezgin and Sankur, (2004) performed comparative study on image thresholding and classified the image thresholding into six categories such as Histogram shape-based methods, Clustering-based methods, Entropy-based methods, object attribute-based methods, spatial methods and local methods. Kapur classifies the image into some classes by calculating threshold which is based on the histogram of the gray level image (Kapur, 1985). Otsu's method classifies the image into some classes by calculating threshold which is based on between-class variance of the pixel intensities of that class (Otsu, 1979). These two methods are under the category of bi-level thresholding and found efficient in case of two thresholds, but for multi-level thresholding, the computational complexity is very high. Entropy may be a Shnnon, fuzzy, between class variations, Kapur's entropy, minimization of the Bayesian error and Birge–Massartthresholding strategy. The disadvantage of these techniques is that convergence time or computational time or CPU time is exponentially increasing with the problem. So alternative to these techniques which minimizes the CPU time for the same problem is evolutionary and swarm-based calculation techniques.

Sathya and Kayalvizhi (2011) applied bacterial foraging optimization algorithm (BF) for optimizing objective functions (Kapur's and Otsu's entropy), so achieved an efficient image segmentation. Further to improve convergence speed and the global searching ability of BF, they modified swarming step and reproduction step, thereby improved the robustness of BF and achieved fast convergence. Sathya and Kayalvizhi (2011) proposed Magnetic Resonance (MR) brain image segmentation by optimizing the multilevel thresholding using amended bacterial foraging (ABF) algorithm. The optimal thresholds are obtained by maximizing the Kapur's or Otsu's entropy with the help of ABF algorithm. The results are compared and proved better in the separation of gray, white and cerebrospinal fluid in MRI image for recognition as well as to diagnosis the disease. The same authors employed some modifications to bacterial foraging (BF) for Segmentation of brain magnetic resonance images (Sathya and Kayalvizhi, 2011). They did adaptive variation of step size of bacteria instead of fixed step size which is followed by ordinary bacterial foraging (Sathya and Kayalvizhi, 2011). Mbuyamba et al. (2016) used Cuckoo Search (CS) algorithm for energy minimization of alternative Active Contour Model (ACM) for global minimum and exhibited that polar coordinates with CS is better than rectangular. Among many optimization techniques are available in the literature, a few are used for bi-level thresholding for ordinary image segmentation, Ye et al. (2015) used fuzzy entropy with bat algorithm (BA) and compared the results with Artificial Bee Colony Algorithm (ABC), Ant Colony (ACO), Particle Swarm Optimization (PSO) and Genetic Algorithm (GA). Agrawal et al. (2013) used Tsallis entropy with CS algorithm and compared the results with BF, PSO and GA. Horng, (2010) used Firefly Algorithm (FA) for multilevel image thresholding. Kapur's and Otsu's entropy methods are simple and effective but computationally expensive when extended to multilevel thresholding because they employed an exhaustive search for optimal thresholds. So Hussein et al. (2016) developed a new strategy which reduces computational time of Kapur's and Otsu's entropy with the help of modified Bees Algorithm (MBA) called the Patch-Levy-based Bees Algorithm (PLBA) and the results showed much faster compared to ordinary Bees Algorithm. Whereas Ashish et al. (2014) used Kapur's entropy with CS and wind driven optimization (WDO) for multilevel thresholding of satellite image segmentation. Bhandari et al. (2015) proposed a Tsallis entropy based multilevel thresholding for colored satellite image segmentation using high dimensional problem optimizer that is Differential Evolution (DE), WDO, PSO and Artifical Bee Colony (ABC). The same authors carried out gray scale satellite image segmentation using modified artificial bee colony (MABC) by optimizing the Kapur's, Otsu and Tsallis entropy and compared the result with ABC, PSO and GA (Bhandari et al. 2015). The drawback of DE is constant tuning parameters (scaling factor (F) and crossover rate (CR)). So Ayala et al. (2015) vary the parameters of DE that follow beta probability distribution function. The beta distribution is flexible for modeling data that are measured in a continuous scale on a truncated interval in range [0, 1]. They compared the beta differential evolution (BDE) based segmentation with fractional-order Darwinian particle swarm optimization (PSO). Li et al. (2015) proposed a modification to PSO that is dynamic-context cooperative quantum-behaved particle swarm in which updating of context vector is dynamic for Otsu's entropy based medical image segmentation. Sun et al. (2016) hybridize the gravitational search algorithm (GS) with genetic algorithm for multi-level thresholding of ordinary images for effective segmentation. They took the advantage and disadvantage of both the algorithms and proper hybridization which has resulted into the best segmentation compared to other methods. Akay (2013) proposed PSO and ABC based image segmentation using Kapur's and Otsu's entropy but PSO performance is further improved with position-velocity model which is based on inherent communication mechanism of cell-like P systems of PSO (Penga et al., 2015). Ouadfel and Ahmed (2016) used social spiders optimization and flower pollination algorithm for multilevel image thresholding by optimizing Kapur's and Otsu's entropy and compared the results with BA and PSO. Saha et al. (2014) proposed Quantum Inspired Genetic Algorithm, Particle Swarm Optimization, Differential Evolution, Ant Colony Optimization, Simulated Annealing and Tabu Search with Otsu method, maximum tsallis entropy thresholding and proved that Quantum Inspired Particle Swarm Optimization is better than others by statistical test and Friedman test measures which reduce the computational complexities.

For the first time in this paper the researchers have applied Firefly algorithm (FA) for image thresholding by optimizing the Fuzzy entropy and compared the results with previous optimization techniques such as DE, PSO and BA. For the performance evolution of proposed firefly algorithm based image thresholding, we considered objective function value, standard deviation, structural similarity index, peak signal to noise ratio, misclassification error and computational complexity. In all performance measuring parameters the proposed algorithm performance is better when compared to other DE, PSO and BA.

II. PROBLEM FORMULATION OF OPTIMUM THRESHOLDING METHODS

Image thresholding is a process of converting a grayscale input image to a black and white image by using optimal thresholds. Thresholding may be a local or global but these methods are computationally expensive, so there is a need of optimization techniques which optimize the objective function results in the reduction of computational time of local or global methods. The optimization techniques find the optimal thresholds by maximizing the objective function such that segmented image clearly distinguishes the background and foreground of image. In this paper, researchers have chosen Fuzzy entropy as objective functions on which optimization techniques works. Let us assume an image that contains L gray levels and the range of these gray levels are $\{0, 1, 2, . . . , (L - 1)\}$. Then probability $P_i = h(i)/N$ $(0 < i < (L - 1))$, where $h(i)$ denotes number of pixels for the corresponding gray-level L and N denotes total number of pixels in the image which is equal to $\sum_{i=0}^{L-1} h(i)$

A. Concept of Fuzzy Entropy

Let $D=\{(i,j):i=0,1,2,.....,M-1;\ j=0,1,2,.......N-1\}$ and $G=\{0,1,2,......,L-1\}$, Where M is width of image, N is height of image and L is number of gray level in image. $I(x,y)$ is the intensity of image at position (x,y) and $D_k = \{(x,y):I(x,y) = k,\ (x,y) = D\},\ k=0,1,2,.....,L-1$. Let us assume two thresholds i.e. T_1, T_2 which divide the domain D of the original image into three regions such as E_d, $E_m$and E_b. E_d region covers the pixels whose intensity value is less than T_1, $E_m$contains the pixels whose intensity is in between T_1, T_2 and E_b covers the pixels whose intensity is greater than T_2. $\Pi_3=\{E_d, E_m, E_b\}$ is an unknown probabilistic partition of D whose probability distribution is given as (Zhao et. al, 2001) $P_d = P(E_d) P_m = P(E_m) P_b = P(E_b)$. μ_d, μ_m and μ_b are the membership functions (μ) of E_d, E_m and E_b respectively and require six parameters like a_1, b_1, c_1, a_2, b_2, c_2. The thresholds T_1 and T_2 values are variable based on the membership functions. For each k=1, 2,…., 255, let

$$D_d=\{(x, y) : I(x, y) \le T_1,\ (x, y) \in D_k\} \quad (1)$$

$$Dm= \{(x, y) : T_1< I(x, y) \le T2,\ (x, y) \in Dk\} \quad (2)$$

$$Db= \{(x, y) : I(x, y) > t2,\ (x, y) \in Dk\} \quad (3)$$

If the conditional probability of E_d, E_m and $E_b$is $p_{d|k}$, $p_{m|k}$ and $p_{b|k}$ respectively under the circumstance that the pixel pertains to D_k with $p_{d|k} + p_{m|k}+p_{b|k} = 1$(k=0, 1, 2,….., 255) then above equations can be rewritten as

$$p_{kd} = p(D_p) = p_k \times p_{d/k} \quad (4)$$

$$p_{km} = p(D_m) = p_k \times p_{m/k} \quad (5)$$

$$p_{kb} = p(D_b) = p_k \times p_{b/k} \quad (6)$$

Let the grade of pixels with gray level value of k belong to the class dark (E_d), dust (E_m) and bright (E_b) be equivalent to their conditional probability $p_{d|k}$, $p_{m|k}$ and $p_{b|k}$ respectively. Then the following equations will hold as:

$$p_d = \sum_{k=0}^{255} p_k * p_{d/k} = \sum_{k=0}^{255} p_k * \mu_d(k) \quad (7)$$

$$p_m = \sum_{k=0}^{255} p_k * p_{m/k} = \sum_{k=0}^{255} p_k * \mu_m(k) \quad (8)$$

$$p_b = \sum_{k=0}^{255} p_k * p_{b/k} = \sum_{k=0}^{255} p_k * \mu_b(k) \quad (9)$$

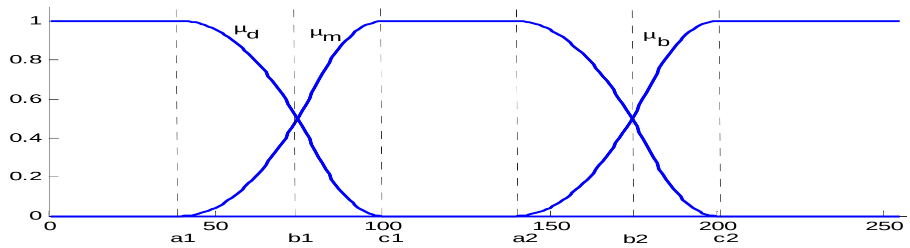

Fig. 1.Membership function graph with a1=40; b1=80; c1=100; a2=140; b2=180; c2=200

The fuzzy membership functions is drawn and shown in Fig. 1. The function Z (k, a_1, b_1, c_1, a_2, b_2, c_2), U(k, a_1, b_1, c_1, a_2, b_2, c_2) and S(k, a_1, b_1, c_1, a_2, b_2, c_2) are assigned as membership functions of class dark μ_d(k), dust μ_m(k) and bright μ_b(k) respectively. Then the membership functions is given as

$$\mu_d(\mathrm{k})=\begin{cases} 1 & k \le a_1 \\ 1-\dfrac{(k-a_1)^2}{(\mathrm{c}_1-\mathrm{a}_1)*(\mathrm{b}_1-\mathrm{a}_1)} & a_1 < k \le b_1 \\ \dfrac{(k-c_1)^2}{(\mathrm{c}_1-\mathrm{a}_1)*(\mathrm{c}_1-b_1)} & b_1 < k \le c_1 \\ 0 & k > c_1 \end{cases} \quad (10)$$

$$\mu_b(\mathrm{k}) = \begin{cases} 0 & k \le a_2 \\ \dfrac{(k-a_2)^2}{(\mathrm{c}_2-\mathrm{a}_2)*(\mathrm{b}_2-\mathrm{a}_2)} & a_2 < k \le b_2 \\ 1-\dfrac{(k-c_2)^2}{(\mathrm{c}_2-\mathrm{a}_2)*(\mathrm{c}_2-b_2)} & b_2 < k \le c_2 \\ 1 & k > c_2 \end{cases} \quad (11)$$

$$\mu_m(\mathrm{k}) = \begin{cases} 0 & k \le a_1 \\ \dfrac{(k-a_1)^2}{(\mathrm{c}_1-\mathrm{a}_1)*(\mathrm{b}_1-\mathrm{a}_1)} & a_1 < k \le b_1 \\ 1-\dfrac{(k-c_1)^2}{(\mathrm{c}_1-\mathrm{a}_1)*(\mathrm{c}_1-b_1)} & b_1 < k \le c_1 \\ 1 & c_1 < k \le a_2 \\ 1-\dfrac{(k-a_2)^2}{(\mathrm{c}_2-\mathrm{a}_2)*(\mathrm{b}_2-\mathrm{a}_2)} & a_2 < k \le b_2 \\ \dfrac{(k-c_2)^2}{(\mathrm{c}_2-\mathrm{a}_2)*(\mathrm{c}_2-b_2)} & b_2 < k \le c_2 \\ 0 & k > c_2 \end{cases} \quad (12)$$

The above said equations are written by assuming $0 \le a_1 < b_1 < c_1 < a_2 < b_2 < c_2 \le 255$. Then, the fuzzy entropy function of each class could be given as (Tao et al., 2007)

$$H_d = -\sum_{k=0}^{255} \frac{p_k * \mu_d(k)}{p_d} * \ln\left(\frac{p_k * \mu_d(k)}{p_d}\right) \quad (13)$$

$$H_m = -\sum_{k=0}^{255} \frac{p_k * \mu_m(k)}{p_m} * \ln\left(\frac{p_k * \mu_m(k)}{p_m}\right) \quad (14)$$

$$H_b = -\sum_{k=0}^{255} \frac{p_k * \mu_b(k)}{p_b} * \ln\left(\frac{p_k * \mu_b(k)}{p_b}\right) \quad (15)$$

The whole fuzzy entropy is calculated through summarizing fuzzy entropy of each class i.e.

$$H\,(a_1, b_1, c_1, a_2, b_2, c_2) = H_d + H_m + H_b \quad (16)$$

The above equation is an objective function which is to be optimized with the optimization techniques. Optimization techniques optimize or maximize H (a_1, b_1, c_1, a_2, b_2, c_2) function by varying a_1, b_1, c_1, a_2, b_2, c_2. Once these values are optimized, then threshold values are calculated with the following equation

$$\mu_d(T_1) = \mu_m(T_1) = 0.5 \text{ and } \mu_m(T_2) = \mu_b(T_2) = 0.5 \quad (17)$$

From Fig. 1 it is observed that T_1 and T_2 are the point of interaction of $\mu_d(k)$, $\mu_m(k)$ and $\mu_b(k)$ curve. From Eqs (10)-(12), the values of T_1 and T_2 calculated with the below equation

$$T_1 = \begin{cases} a_1 + \sqrt{(\mathrm{c}_1-\mathrm{a}_1)*(\mathrm{b}_1-a_1)/2} & (\mathrm{a}_1+\mathrm{c}_1)/2 \le b_1 \le c_1) \\ c_1 - \sqrt{(\mathrm{c}_1-\mathrm{a}_1)*(\mathrm{c}_1-b_1)/2} & \mathrm{a}_1 \le b_1 \le (\mathrm{a}_1+\mathrm{c}_2)/2 \end{cases} \quad (18)$$

$$T_2 = \begin{cases} a_2 + \sqrt{(\mathrm{c}_2-\mathrm{a}_2)*(\mathrm{b}_2-a_2)/2} & (\mathrm{a}_2+\mathrm{c}_2)/2 \le b_2 \le c_2 \\ c_2 - \sqrt{(\mathrm{c}_2-\mathrm{a}_2)*(\mathrm{c}_2-b_2)/2} & \mathrm{a}_2 \le b_2 \le (\mathrm{a}_2+\mathrm{c}_2)/2 \end{cases} \quad (19)$$

As per the requirements of researchers, the two level thresholding can be extended to three or more and can be restricted to single level also. For two thresholds the number of parameters to be optimized is six and as levels of increasing number parameters to be optimized is also increasing, so fuzzy entropy takes much time for convergence. Hence two level image thresholding for image segmentation with the Fuzzy entropy proved to be efficient and effective but for multilevel thresholding, entropy technique consume much convergence time and increase exponential with level of thresholds. The drawback of Fuzzy entropy is convergence time. To improve the performance of these methods further and to reduce the convergence time, researchers used applications of optimization techniques such as differential evolution, Particle swarm optimization, Bat algorithm and Firefly algorithm for image thresholding and henceforth image segmentation. This techniqueis set to maximize the Fuzzy entropy as given in (16).

III. OVERVIEW OF FIREFLY ALGORITHM

Firefly algorithm (FA) was introduced by Yang (2008). FA is inspired by the flashing pattern and characteristics of fireflies where the brightness of a firefly is equal to the objective function value. The lighter firefly (lower fitness value) moves towards brighter firefly (higher fitness value). FA is based on the following idealized behavior of the flashing characteristics of fireflies:

(1) All fireflies are unisex so that one firefly is attracted to other fireflies regardless of their sex.

(2) Attractiveness is proportional to their brightness, thus for any two flashing fireflies, the low brighter one will move towards the high brighter one. The attractiveness is proportional to the brightness and they both decrease as their distance increases. If there is no brighter one than a particular firefly, it will move randomly.

(3) The brightness of a firefly is affected or determined by the landscape of the objective function.

In firefly algorithm, each firefly is assumed as solution to the problem and thereby fitness/brightness (I) is calculated with objective function. In this paper objective function is H(a_1, b_1, c_1, a_2, b_2, c_2) which is to be maximized by optimizing a_1, b_1, c_1, a_2, b_2, c_2 values. So dimensions (D) of the problem are six. Whenever all the firefly fitness values are obtained, firefly whose fitness value is larger among is assigned as brighter firefly. All lighter fireflies (lower fitness value) move towards the brighter firefly by updating their values. Attractiveness (β) is varied exponentially with Cartesian distance (r_{ij}) which is in between brighter firefly i and lighter firefly j. Following is the equation for Cartesian distance between i^{th} firefly and j^{th} firefly at location X_i and X_j respectively.

$$Cartesian\ distance(r_{ij}) = \left\| X_i - X_j \right\| = \sqrt{\sum_{k=1}^{N_c} \sum_{h=1}^{L} (X_{i,k}^h - X_{j,k}^h)^2} \tag{20}$$

Where $X_{i,k}$ is the k^{th} component of the spatial coordinate X_i of i^{th} firefly. Then the attractiveness is given as

$$\beta = \beta_0 e^{-\gamma_{i,j}} \tag{21}$$

Where β is the attractiveness at $r_{i,j}$ = 0 and γ is light absorption coefficient of the medium. With this attractiveness, lighter firefly i moves towards brighter firefly j with the following equation

$$X_{j,k}^h = (1-\beta) X_{i,k}^h + \beta X_{j,k}^h + u_{j,k}^h \quad k=1,2,\ldots..,N, \quad h=1,2,\ldots.D. \tag{22}$$

Where u is a random number that lies between 0 and 1 and is calculated by Eq (23)

$$u_i = (\mathrm{rand}1 - \frac{1}{2}) \tag{23}$$

If there is no brighter firefly in the search space then lighter firefly i move randomly with the Eq (24)

$$X_{i,k}^h = X_{i,k}^h + u_{j,k}^h \quad k=1,2,\ldots..,N, \quad h=1,2,\ldots.D. \tag{24}$$

Where rand1 is a random number lies between 0 and 1

IV. FA-BASED FUZZY ENTROPY METHOD

In this section, the image thresholding for image segmentation by optimizing/maximizing the fuzzy entropy with proposed ordinary firefly algorithm is explained. The proposed method for image thresholding is very simple and easy to implement. The algorithm of firefly for image thresholding with fuzzy entropy is as follows.

Input: Initialize the population (N), maximum number of iterations, level of thresholding (Th) and its corresponding a_1, b_1, c_1, a_2, b_2, c_2 values. Initialize randomization parameter (α), attractiveness (β), absorption coefficient (γ).

Output: The optimized a_1, b_1, c_1, a_2, b_2, c_2 values and its corresponding thresholding values and segmented image.

Initialize all the required parameters and there corresponding dimensions and time t = 0.

Calculate the fitness value or light intensity Ii of each solution Xi (i=1,2,3,….n) using Eq. (16) for Fuzzy.

While (t < Maximum iterations or until termination criteria reached)

for i =1:n all n fireflies

for j = 1:i all n fireflies
if (I_i<I_j)
Then firefly i moves towards j with Eq (23)
Attractiveness (β) varies exponentially with Euclidean distance between firefly i and j
Evaluate new solutions and update light intensity
end if
end for j
end for j
Rank the fireflies according to fitness values and find the current best
end while
Return the optimal solution and the corresponding segmented image with selected thresholds.

V. RESULTS AND DISCUSSION

For the performance evolution which includes robustness, efficiency and convergence of proposed firefly algorithm, researchers selected "Cameraman", "Lena", "Lake", "Goldhill", "Starfish" and "Pirate" as test images. These images are from the image segmentation database whose can be downloaded by anybody through the link (http://www.imageprocessingplace.com/root_files_V3/image_databases.htm) and all are .jpg format images of size 225×225 and corresponding histograms are shown in Fig. 2. In general, perfect threshold can be selected if the histogram of image peaks is tall, narrow, symmetric, and separated by deep valleys. Cameraman, Lake, Goldhill and pirate image histograms peaks are tall, narrow and symmetric, but for Lena and Starfish images histogram peaks are not tall and narrow so it is difficult to segment with ordinary methods. Hence we proposed a firefly algorithm based image thresholding for effective and efficient image segmentation of above said critical images by optimizing Fuzzy entropy. The performance and effectiveness of proposed firefly algorithm is proved better compared to other optimization techniques like DE, PSO and BA.

A. Selection of DE, PSO, BA and FA parameters

The same number of populations and maximum number iterations are employed for all optimization algorithms. The maximum number iterations are 30 and population/solutions are 10 times higher of threshold value (i.e if threshold =2 then population = 10×2). In DE, Weighting Factor (F) value is 0.5 and Crossover probability (CR) is 0.9 since chosen at these values DE gives the best results. The performance of PSO algorithm depends on two tuning parameters such as acceleration constants (C_1 and C_2) and inertia weight factor (W). In general C_1 and C_2 are set as 2; at these values experimentally PSO has given the best fitness values. Whereas inertia weight factor (W) is a random number that lies between 0 and 1.

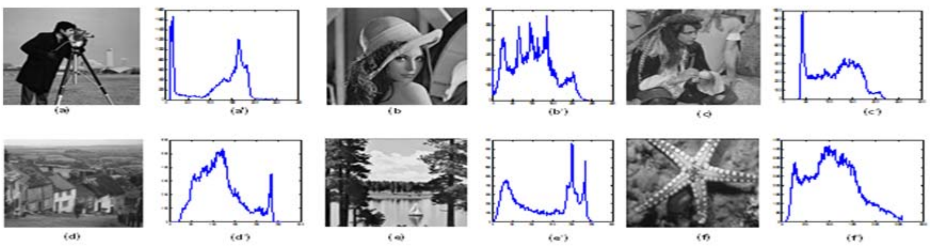

Fig. 2.Test images and corresponding histograms a) Cameraman b) Lena c) Pirate d) Goldhill e) Lake f) Starfish.

The performance of FA depends upon the parameters such as number of solutions (N), dimensions (D), maximum number of iterations (itr), randomization parameter (α), attractiveness (β_0) and absorption coefficient (γ). While applying FA, these control parameters should be carefully chosen for the successful implementation of the algorithm. Successive experiments were conducted for the selection of these parameters and carry out the best values where objective function is found maximum. Table.Ishows the variation of objective function maximum value, mean and standard deviation with respect to the control parameters for Goldhill image with number of thresholds equal to 2. 50 independent experiments are conducted for fixing FA parameter for validation of the algorithm. From Table.I, it is observed that at α = 0.1, β = 0.6, γ = 0.1 and N =100 the objective function value is maximum. These selected parameter values are carried on for all other images. It is observed that beyond N = 100, the value of objective function is slightly improved but with the cost of computational time. The parameters of BA such as Loudness (A = 0.5), Pulse rate (R = 1), Frequency minimum (Qmin = 0), Frequency maximum (Qmax = 30), and Step sizes of random walk (W = 0.001) are initialized.

Table I results over 50 independent runs of tuning FA parameters

Parameter	Max.	Mean	Std.dev.	Others					
α=0.1	**13.99081**	**13.9831**	**0.00609**		γ=0.1	**13.9904**	**13.9786**	**0.01629**	
α=0.2	13.986728	13.972954	0.010783		γ=0.2	13.990414	13.982511	0.005782	
α=0.3	13.988476	13.966132	0.019339		γ=0.3	13.98982	13.98251	0.005235	
α=0.4	13.987238	13.952608	0.024029	β=0.4	γ=0.4	13.990361	13.980398	0.011981	
α=0.5	13.986748	13.94969	0.026852	γ=0.5	γ=0.5	13.989722	13.978113	0.011803	α=0.1
α=0.6	13.988584	13.947874	0.02408	D=2	γ=0.6	13.990156	13.984564	0.004609	β=0.6
α=0.7	13.986616	13.950047	0.025424	N=30	γ=0.7	13.988465	13.977439	0.017647	D=2
α=0.8	13.985751	13.945179	0.027741		γ=0.8	13.989284	13.980822	0.007696	N=30
α=0.9	13.988114	13.952208	0.020767		γ=0.9	13.988399	13.975529	0.022652	
α=1	13.983074	13.949317	0.022222		γ=1	13.989032	13.978542	0.026765	
β=0.1	13.989263	13.975598	0.014005		p=20	13.989072	13.97491	0.022153	α=0.1
β=0.2	13.989782	13.980639	0.014199		p=30	13.989734	13.975585	0.023947	β=0.6
β=0.3	13.989642	13.981046	0.007144		p=50	13.989899	13.981029	0.014042	γ=0.1
β=0.4	13.990054	13.982907	0.007186		p=100	**13.99006**	**13.98551**	**0.00373**	D=2
β=0.5	13.989811	13.97912	0.015729	α=0.1					
β=0.6	**13.99043**	**13.9805**	**0.01483**	γ=0.5					
β=0.7	13.990247	13.974859	0.024922	D=2					
β=0.8	13.988599	13.97978	0.011394	N=30					
β=0.9	13.990187	13.981393	0.006442						
β=1	13.989959	13.978749	0.012731						

B. Quantitative validation

To examine the influence of FA algorithm on multilevel thresholding problem, objectives functions/fitness function is Fuzzy entropy

1)Maximization of fuzzy entropy

In this case, the objective function to be optimized with optimization technique is fuzzy entropy which is said to be popular and better in the performance of FA when compared with the other POS, DE and BA. All the algorithms are optimized to maximize the objective function. Table .II and Table.III show the objective values and corresponding thresholds' values for FA, PSO, BA and PSO. It is observed from Table II that the objective value obtained with FA by using fuzzy entropy is higher than DE, PSO and BA for different images.

Table II Comparison of objective values obtained by various algorithms

		Th = 2	Th = 3	Th = 4	Th = 5
Images	Opt Tech	Fuzzy	Fuzzy	Fuzzy	Fuzzy
Cameraman	DE	13.11179	16.51444	20.2745	23.632996
	PSO	13.24719	16.54862	20.28994	23.077552
	BA	13.27223	16.60881	20.41527	23.284207
	FA	13.34643	17.76107	21.03949	23.524537
Lena	DE	14.12322	17.58576	20.66932	24.0294
	PSO	14.22685	17.91415	21.14748	24.300881
	BA	14.23849	17.93978	21.29851	24.373777
	FA	14.32151	17.95987	21.38698	24.561514
Goldhill	DE	13.44	16.82359	19.89543	22.934799
	PSO	13.53301	17.0209	20.09893	22.983463
	BA	13.53865	17.08749	20.16574	23.235944
	FA	13.56331	17.09854	20.27302	23.360407

Lake	DE	13.94999	17.32295	20.38282	23.426922
	PSO	13.98223	17.63517	20.87337	24.056312
	BA	13.99141	17.71809	20.98016	24.214624
	FA	14.03941	17.69289	20.90727	24.060597
Pirate	DE	13.87994	17.76107	21.03949	24.373777
	PSO	14.00623	17.91472	21.24234	24.560984
	BA	14.01315	17.97402	21.36288	24.766622
	FA	14.03124	17.69289	21.33981	24.754253
Starfish	DE	14.46344	18.43343	21.34769	24.375641
	PSO	14.50729	18.25907	21.67998	24.849605
	BA	14.51284	18.29813	21.74343	25.104794
	FA	14.6113	18.33766	21.79938	25.037815

C. Qualitative results

In this section, researchers concentrated on visual clarity of segmented images with varied threshold values i.e. Th = 2, Th =3, Th =4 and Th =5 by using Fuzzy entropy with PSO, DE, BA and FA algorithms. The segmented images/thresholding images and corresponding thresholds on histogram obtained with BA and FA algorithms at thresholds level 2, 3, 4 and 5 with Fuzzy entropy are shown in from Fig 3 to Fig 8. Among these figures, we observed that segmented image visual quality is better with higher level of threshold (th = 5) in comparison with Th = 4, Th = 3 and Th = 2. Let us look on the visual quality of few segmented images with fuzzy entropy for thesake of effectiveness and robustness test of proposed Firefly algorithm. Visual quality of proposed FA is better to BA. (Ex: Lena image at 2 level threshold as shown in Fig. 6f and Starfish image at 2, 3, 4 and 5- level thresholds as shown in Fig. 8e-h.). Proposed algorithm is better compared to other earlier algorithms in visual quality of image for all other images likewise. The consequence of multilevel thresholding is noticeable from different images. From Fig. 5e, the background in the Lake image is not visibly dissimilar with two level thresholding. But as the number of threshold is extended to 5 (i.e. Fig. 5h), the background becomes recognizable. Similarly in Fig. 8e, the Starfish image mixes up with the background objects. But as the number of threshold is increased to 5 (i.e. Fig. 8h), the Starfish image becomes clearly recognizable.

Table III Optimal threshold values obtained by Fuzzy entropy based evolutionary algorithms.

Images	Opt Tech	Th = 2	Th = 3	Th = 4	Th = 5
Cameraman	DE	92,220	42,134,219	48,94,142,219	44,92,141,191,226
	PSO	139,219	31,142,192	25,59,132,222	45,94,150,194,231
	BA	117,203	45,114,184	44,100,159,204	33,81,125,157,208
	FA	128,181	67,137,202	62,83,152,202	83,129,171,171,173
Lena	DE	59,186	55,117,189	39,99,140,206	37,80,119,159,207
	PSO	71,165	68,154,203	69,139,159,200	27,65,101,145,211
	BA	68,183	68,108,208	59,141,173,215	37,107,147,199,228
	FA	55,181	78,120,170	36,119,149,193	23,61,110,169,172
Goldhill	DE	56,183	65,134,196	45,118,153,206	41,87,126,162,209
	PSO	53,192	60,129,180	32,80,119,200	51,91,118,150,209
	BA	113,128	47,120,183	44,78,130,208	28,86,113,149,184
	FA	99,167	81,141,192	61,129,159,198	47,87,124,161,214
Lake	DE	62,189	61,124,190	42,109,152,209	38,83,127,169,213
	PSO	55,187	54,104,178	53,108,152,196	12,64,120,154,201
	BA	88,186	45,114,184	44,100,159,204	38,88,112,169,203
	FA	54,180	61,107,189	45,116,130,210	48,81,109,137,214
Pirate	DE	77,204	60,125,192	55,107,138,209	88,130,143,173,213
	PSO	74,197	61,132,184	63,107,138,182	58,97,129,154,201

	BA	80,183	53,127,177	51,100,145,199	58,97,129,154,201
	FA	74,200	54,142,179	87,113,162,174	65,119,151,179,216
Starfish	DE	70,197	64,146,210	45,113,159,217	42,85,127,172,217
	PSO	57,189	39,133,221	50,108,150,211	37,87,115,159,221
	BA	128,137	128,150,161	75,147,156,197	90,131,132,155,184
	FA	76,199	44,128,213	56,123,172,218	25,75,108,173,219

D. Comparison of other methods

1) Stability analysis

The optimization technique's outcome is random in nature because randomness is involved in the procedure and the results are not unique for each run. So the algorithm performance is validated by more than one run and with different initial values. An algorithm is said to be robust if its outcome is acceptable (i.e indifferent from one run to another run) under same circumstances. So we run the same algorithm 50 times and considered result at an average of 50 independent runs. The stability of the algorithm is measured with mean and standard deviation. Optimization technique in general can be considered to be better, if its stability factor is higher among all the techniques i.e objective function value should be the same for each run. Mean and standard deviation is calculated by Eq. 25 and Eq. 26

$$mean(\sigma) = \frac{1}{N}\sum_{j}^{N} \mu_j \tag{25}$$

$$std = \sqrt{\frac{1}{N}\sum_{j=1}^{N}(\mu_j - \sigma)^2} \tag{26}$$

Where μ_j is the objective function value/fitness value at j^{th} run and N is the number of runs. Table.IV shows the standard deviation values obtained with Fuzzy entropy by proposed firefly algorithm and other algorithms. An

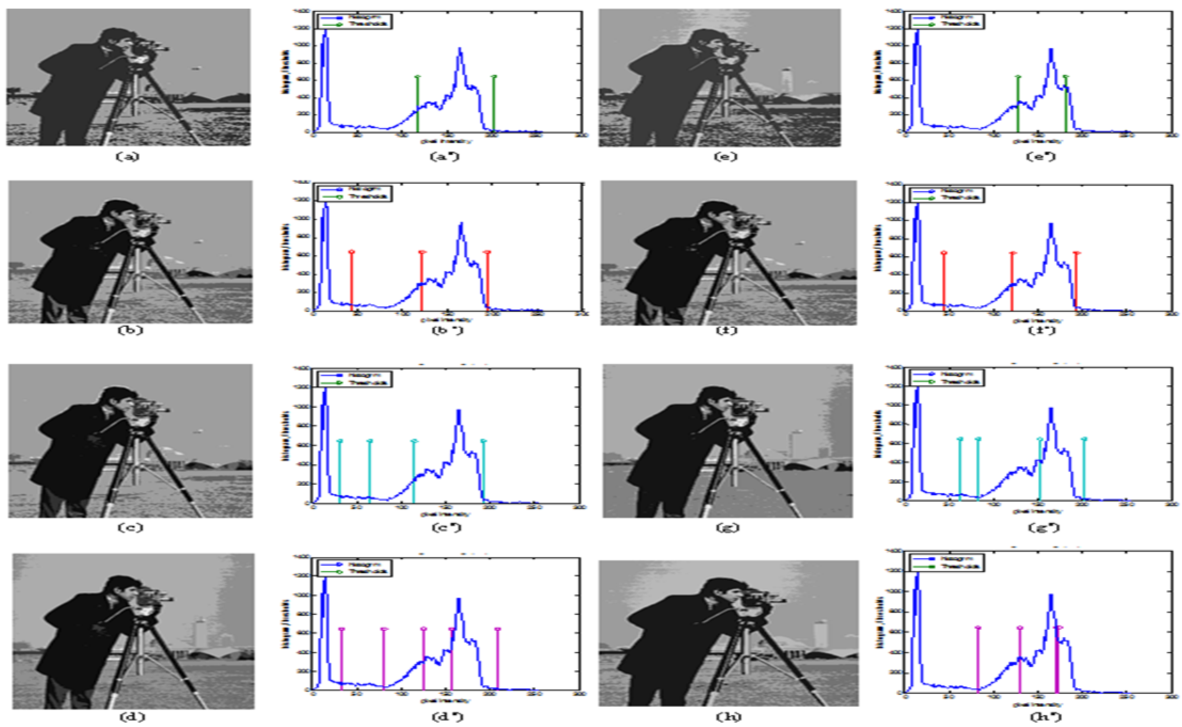

Fig. 3. Segmented images and thresholds on histogram of Cameraman image with various thresholds achieved by BA and FA with Fuzzy entropy. (a)-(d) shows 2-5 level segmented images achieved by BA respectively. (a')-(d') shows 2-5 level thresholds on histogram achieved by BA respectively. (e)-(h) shows 2-5 level segmented images achieved by FA respectively. (e')-(h') shows 2-5 level thresholds on histogram achieved by FA respectively.

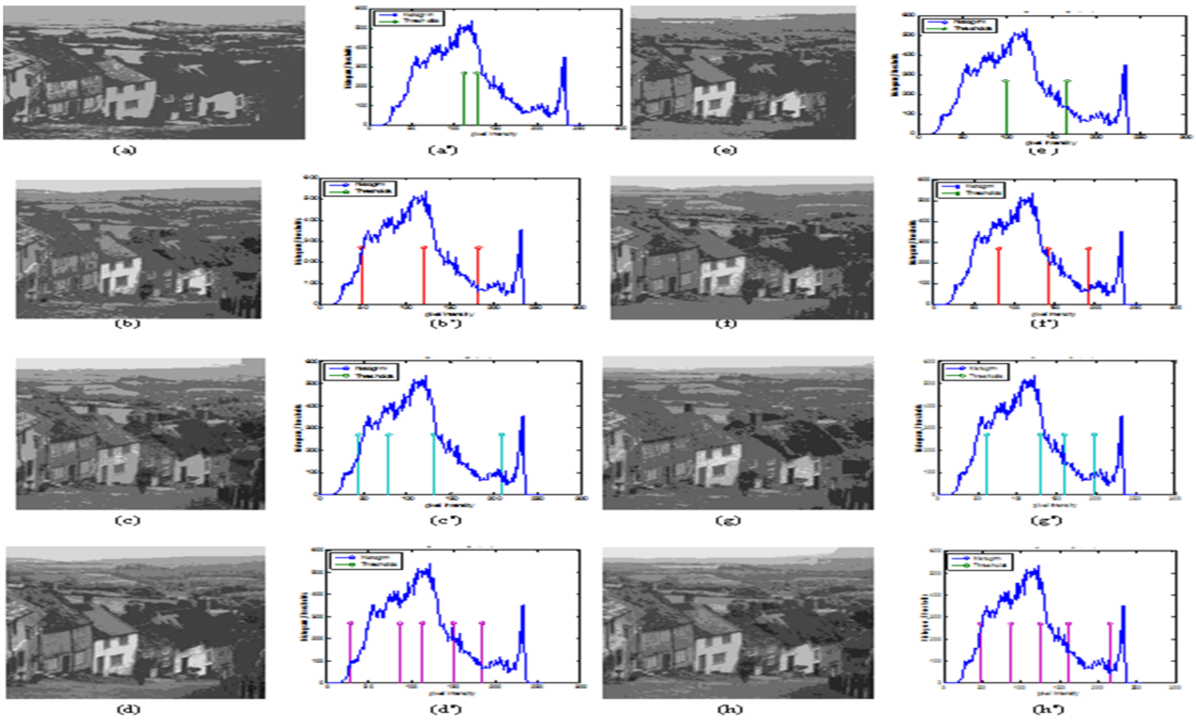

Fig. 4. Segmented images and thresholds on histogram of Goldhill image with various thresholds achieved by BA and FA with Fuzzy entropy. (a)-(d) shows 2-5 level segmented images achieved by BA respectively. (a')-(d') shows 2-5 level thresholds on histogram achieved by BA respectively. (e)-(h) shows 2-5 level segmented images achieved by FA respectively. (e')-(h') shows 2-5 level thresholds on histogram achieved by FA respectively.

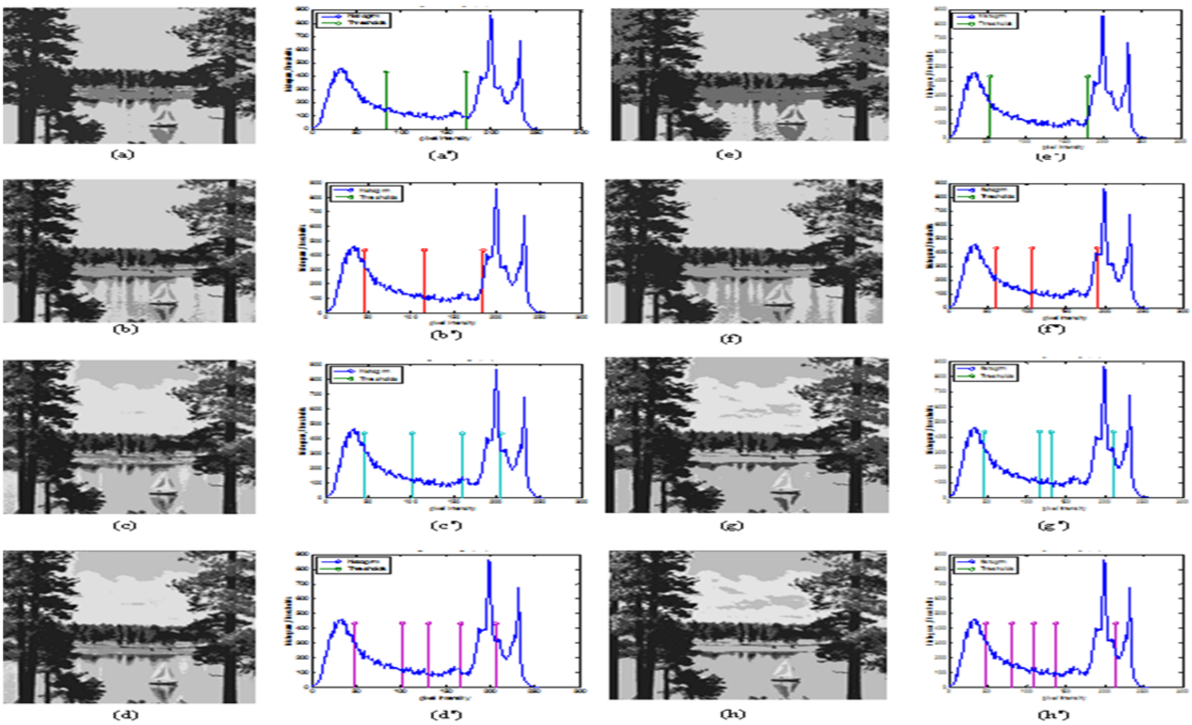

Fig. 5. Segmented images and thresholds on histogram of Lake image with various thresholds achieved by BA and FA with Fuzzy entropy. (a)-(d) shows 2-5 level segmented images achieved by BA respectively. (a')-(d') shows 2-5 level thresholds on histogram achieved by BA respectively. (e)-(h) shows 2-5 level segmented images achieved by FA respectively. (e')-(h') shows 2-5 level thresholds on histogram achieved by FA respectively.

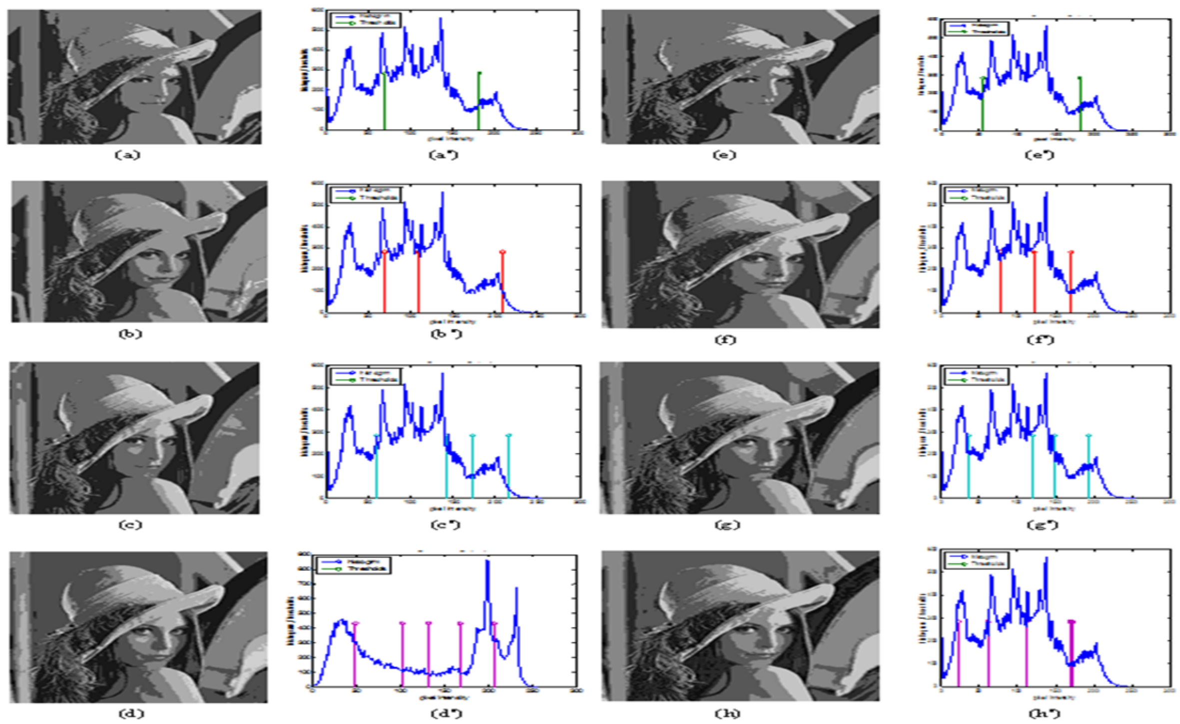

Fig. 6. Segmented images and thresholds on histogram of Lena image with various thresholds achieved by BA and FA with Fuzzy entropy. (a)-(d) shows 2-5 level segmented images achieved by BA respectively. (a')-(d') shows 2-5 level thresholds on histogram achieved by BA respectively. (e)-(h) shows 2-5 level segmented images achieved by FA respectively. (e')-(h') shows 2-5 level thresholds on histogram achieved by FA respectively.

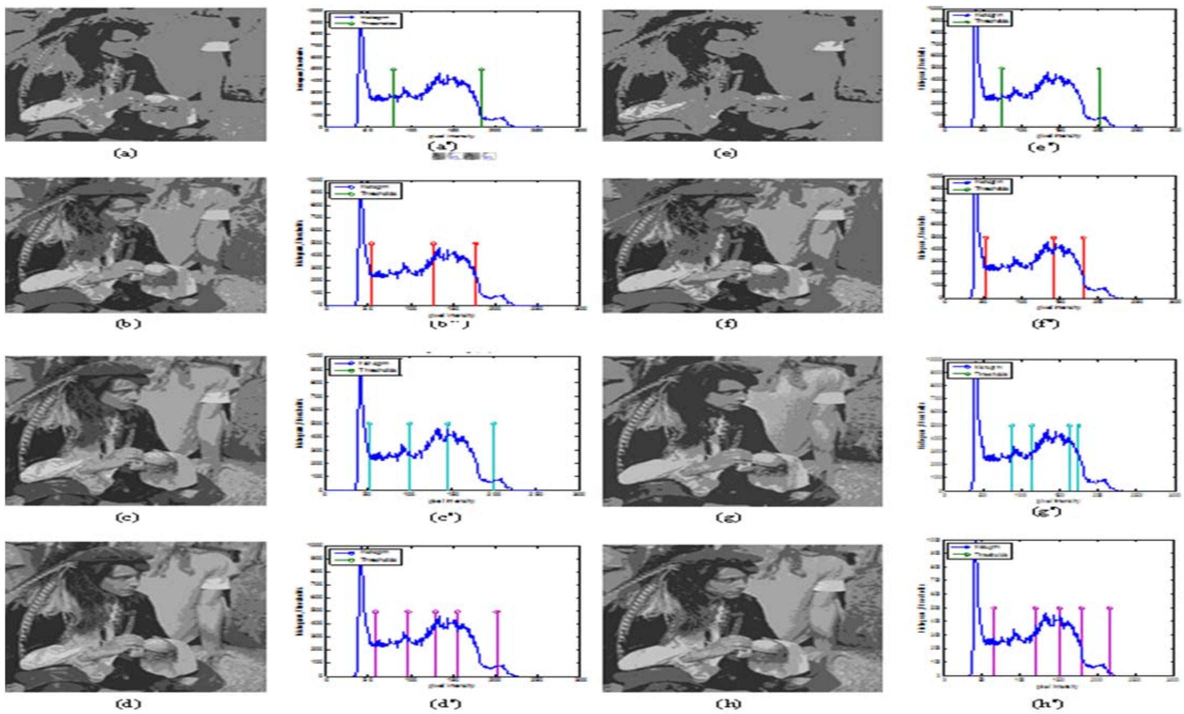

Fig. 7. Segmented images and thresholds on histogram of Pirate image with various thresholds achieved by BA and FA with Fuzzy entropy. (a)-(d) shows 2-5 level segmented images achieved by BA respectively. (a')-(d') shows 2-5 level thresholds on histogram achieved by BA respectively. (e)-(h) shows 2-5 level segmented images achieved by FA respectively. (e')-(h') shows 2-5 level thresholds on histogram achieved by FA respectively.

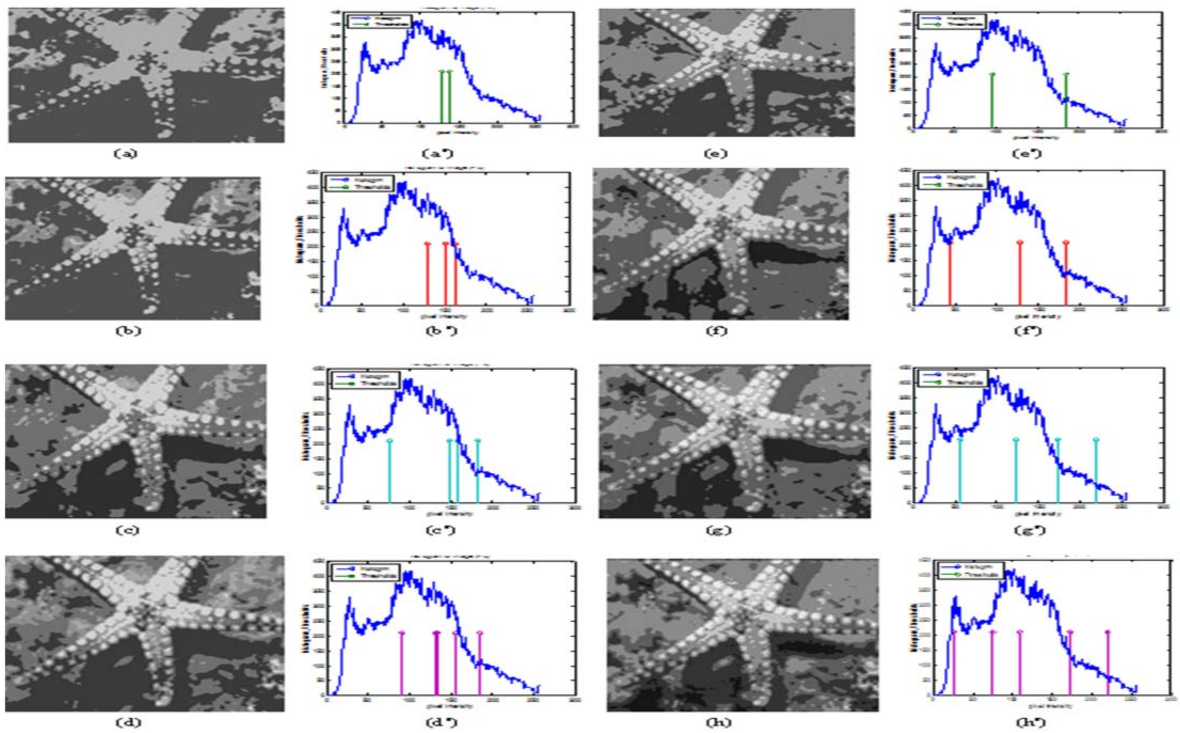

Fig. 8. Segmented images and thresholds on histogram of Starfish image with various thresholds achieved by BA and FA with Fuzzy entropy. (a)-(d) shows 2-5 level segmented images achieved by BA respectively. (a')-(d') shows 2-5 level thresholds on histogram achieved by BA respectively. (e)-(h) shows 2-5 level segmented images achieved by FA respectively. (e')-(h') shows 2-5 level thresholds on histogram achieved by FA respectively.

optimization technique with higher value of standard deviation seems unstable. From Table.IV, it is observed that DE algorithm has lower standard deviation value for all images including Fuzzy entropy; hence DE is stable and better compared with others. The stability of PSO, BA and FA is found almost similar. It is also observed that there is no effect of standard deviation with the increment of thresholding levels for all images.

Table IV Comparison of standard deviation for various algorithms.

		Th = 2	Th = 3	Th = 4	Th = 5
Images	Opt Tech	Fuzzy	Fuzzy	Fuzzy	Fuzzy
Cameraman	DE	1.64E-03	0.01668	**0.01734**	0.00603
	PSO	0.16129	**0.09279**	0.17755	0.19464
	BA	0.18212	**0.15946**	0.21629	0.19268
	FA	0.04025	0.04736	0.11762	0.10343
Lena	DE	1.56E-03	9.82E-03	0.01771	**0.03274**
	PSO	**0.12422**	0.70027	0.19745	**0.19618**
	BA	**0.16829**	0.18905	0.11919	0.23139
	FA	0.02229	0.06475	0.07162	0.13989
Goldhill	DE	8.67E-04	1.01E-02	0.023675	0.038585
	PSO	0.08252	0.173517	0.220505	0.224984
	BA	0.167123	0.187109	0.207755	**0.26189**
	FA	0.023877	0.074544	0.08749	0.148974
Lake	DE	2.67E-04	8.06E-01	1.23E-02	0.034287
	PSO	**0.08092**	0.002326	0.137151	0.234356
	BA	0.103968	0.159464	0.216297	0.125343
	FA	0.012808	0.048909	0.063312	0.109919
Pirate	DE	8.43E-04	7.76E-03	0.019757	0.048521
	PSO	0.110538	0.191169	0.203348	0.286146

	BA	0.167082	0.291656	0.23528	0.429339
	FA	0.021899	0.056602	0.112173	0.115502
Starfish	DE	3.16E-04	1.20E-02	1.93E-02	0.029133
	PSO	0.120118	0.164268	0.179779	0.24098
	BA	**0.20342**	**0.22343**	0.210227	0.215449
	FA	0.01713	0.043457	0.067852	0.113278

2) Computational complexity:

It's a measure of time of convergence of an optimization technique which is variable with respect to the thresholds. The computational complexity of Fuzzy entropy is O (L^m) which rises exponentially with the number of thresholds (Th) and the number of gray levels (L). Convergence time of proposed FA depends on the size of image and maximum number of iterations. Table.VIdraw the convergence time/ computational complexities of PSO, DE, BA and FA with Fuzzy entropy for different thresholds for different images. It is observed that average convergence time of proposed FA is much smaller than other algorithms. All the experiments are performed on Matlab 2009b with Intel core i5 processor capacitated 2 GB RAM

3) Peak Signal to Noise Ratio (PSNR):

PSNR shows dissimilarity between threshold image and input image as a measure of visual difference of two images where the units are decibels (dB). A higher value of PSNR indicates better quality of threshold image or reconstructed image. The equation for PSNR is given in Eq. 27.

$$PSNR \quad = \; 10 \times 10 \log \left(\frac{255^2}{MSE} \right) (\text{dB}) \tag{27}$$

Where (MSE) which is given by in Eq. 28

$$MSE = \frac{1}{M \times N} \sum_{I}^{M} \sum_{J}^{N} \{ f(\text{I, J}) - \overline{f}(\text{I, J}) \}^2 \tag{28}$$

Where M x N is size of image and I & J represent the pixel value of original and decompressed images. In this experiment, researchers have taken N=M a square image. $f(I,J)$ is an original image and $\overline{f}(I,J)$ is a reconstructed image of size 225 by 225. Table.VIshow the PSNR value acquired by different algorithms where the proposed algorithm has achieved higher PSNR value in comparison with DE, PSO and BA. Among all cases as well as in all images, PSNR value is rising with increasing threshold values. FA algorithm provides the utmost value of PSNR value with Th = 5 when compared to DE, PSO and BA. Hence, the excellence of the segmented images gets better with the higher level of thresholds.

Table V Comparison of CPU time (in seconds) for various methods.

		Th = 2	Th = 3	Th = 4	Th = 5
Images	Opt Tech	Fuzzy	Fuzzy	Fuzzy	Fuzzy
Cameraman	DE	17.37899	34.89404	38.9059	45.246832
	PSO	14.72692	22.22853	31.88963	49.214703
	BA	10.92766	17.4504	28.98167	25.063241
	FA	23.86838	30.19158	46.45787	81.331957
Lena	DE	23.10234	28.87228	34.58586	60.67167
	PSO	20.74006	22.81411	29.86393	45.768399
	BA	15.41488	16.99354	32.38648	28.801767
	FA	22.1789	29.5076	44.38185	65.414471
Goldhill	DE	27.35428	29.93418	53.55693	54.623062
	PSO	20.30875	38.3014	46.34719	66.241249
	BA	32.03218	20.00902	45.73632	75.969269
	FA	19.07287	42.90798	87.7485	71.890689
Lake	DE	168.5369	68.65841	42.00675	46.557383
	PSO	23.40779	21.8553	37.19785	45.344876
	BA	22.82106	59.24995	22.59032	40.163425
	FA	28.04954	29.3333	47.70789	65.613619

Pirate	DE	22.49297	27.18165	37.74204	50.643168
	PSO	16.9994	25.46523	36.67485	40.139625
	BA	13.85076	14.88508	25.45828	87.11891
	FA	19.67271	37.68114	48.68866	70.158385
Starfish	DE	26.33895	26.22158	62.34102	56.571056
	PSO	26.68756	24.80893	33.89914	42.17283
	BA	13.06687	40.29626	22.25709	27.008018
	FA	22.89818	29.1351	58.27002	66.192443

4) Misclassification error/Uniformity measure:

It is measure of uniformity in threshold image and is used to compare optimization techniques performance (Sahoo et al., 1988). Misclassification error is measured by Eq. 29

$$M = 1 - 2 * Th * \frac{\sum_{j=0}^{Th} \sum_{i \epsilon R_j} (I_i - \sigma_j)^2}{N * (I_{max} - I_{min})^2} \qquad (29)$$

Where T is the number of thresholds that are used to segment the image, R_j is the jth segmented region, I_i is the intensity level of pixel in that particular segmented area, σ_j is the mean of j^{th} segmented region of image, N is total number of pixels in the image and I_{min}& I_{max} are the maximum and minimum intensity of image respectively. In general misclassification errors lie between 0 & 1 and higher value of misclassification error shows better performance of the algorithm. Hence, the Uniformity measure in thresholding is measured from the difference between maximum value, 1 (better quality of image) and minimum value, 0 (worst quality of image). Table.VII demonstrate misclassification error of proposed and other techniques where the proven proposed method has lesser misclassification error and draws better visual quality.

Table VI Comparison of PSNR values for the methods under evaluation.

Images	Opt Tech	Th = 2 Fuzzy	Th = 3 Fuzzy	Th = 4 Fuzzy	Th = 5 Fuzzy
Cameraman	DE	28.677	29.5452	30.002	30.991
	PSO	28.7008	29.9306	30.5631	31.1215
	BA	28.9523	30.0574	30.6835	31.5316
	FA	29.0196	30.3965	31.0511	32.0426
Lena	DE	28.6276	28.6481	29.1021	29.8851
	PSO	28.7274	29.3717	29.1572	30.1187
	BA	28.742	29.5076	29.5213	30.2771
	FA	28.8438	29.6273	29.745	30.9163
Goldhill	DE	28.8427	28.9637	29.2554	30.6413
	PSO	29.0318	29.1168	29.5548	30.7302
	BA	29.0657	29.4898	30.0733	30.9964
	FA	29.1103	29.5282	30.2903	31.481
Lake	DE	28.8852	29.3218	29.9428	30.6342
	PSO	28.9172	29.6214	30.0652	30.8547
	BA	29.0525	29.7387	30.6819	31.5673
	FA	29.3006	33.3683	31.1231	31.7122
Pirate	DE	28.7004	29.5453	30.3487	30.5687
	PSO	28.7038	29.7589	30.8305	31.0817
	BA	28.7385	29.8286	31.1737	31.2118
	FA	28.7525	29.8614	31.2954	32.2861
Starfish	DE	28.7096	28.6613	29.306	29.7045
	PSO	28.32849	28.9917	29.4915	30.1118
	BA	28.7013	29.0443	29.5343	30.2872
	FA	28.70678	29.2359	29.5676	30.5047

5) Structural Similarity Index (SSIM):

It evaluates the visual similarity between the original image and the reconstructed image/thresholded image and is calculated with below equation

$$SSIM = \frac{(2\mu_I\mu_{\tilde{I}} + C1)(2\sigma_{I\tilde{I}} + C2)}{(\mu_I^2 + \mu_{\tilde{I}}^2 - C1)(\sigma_I^2 + \sigma_{\tilde{I}}^2 - C2)} \quad (30)$$

Where µI and µĨ are the mean value of the original image I and reconstructed image Ĩ, σ_I and $\sigma_{\tilde{I}}$ are the standard deviation of original image I and reconstructed image Ĩ, $\sigma_{I\tilde{I}}$ is the cross-correlation and C1 & C2 are constants which are equal to 0.065. The range of SSIM is -1 to +1 and SSIM value equal to one shows original image and reconstructed image/thresolded image is similar. Algorithm is said to be good if SSIM value is near around positive one. Table.VIII show the SSIM of various methods with Fuzzy entropy and it shows that the proposed method SSIM is higher than other methods.

$$\text{Standard deviation } \sigma_{I\tilde{I}} = \frac{1}{N-1}\sum_{i=1}^{N}(\mathrm{I}_i - \mu_I)(\tilde{\mathrm{I}}_i - \mu_{\tilde{I}}) \quad (31)$$

Table VII Comparative misclassification error for various thresholding methods.

		Th = 2	Th = 3	Th = 4	Th = 5
Images	Opt Tech	Fuzzy	Fuzzy	Fuzzy	Fuzzy
Cameraman	DE	0.965365	0.977768	0.927555	0.9075964
	PSO	0.951125	0.975786	0.924084	0.9449547
	BA	0.936811	0.972633	0.899591	0.9138578
	FA	0.916367	0.936172	0.882454	0.878416
Lena	DE	0.968633	0.95537	0.939388	0.8932978
	PSO	0.959245	0.952794	0.935731	0.8073844
	BA	0.951017	0.949908	0.917558	0.7287992
	FA	0.948264	0.946852	0.902922	0.7233069
Goldhill	DE	0.971564	0.950803	0.895224	0.6886802
	PSO	0.947282	0.938885	0.853442	0.6833011
	BA	0.938959	0.92963	0.748672	0.6637794
	FA	0.930716	0.909869	0.723765	0.6179214
Lake	DE	1.369244	0.948094	0.892369	0.7512455
	PSO	0.96305	0.971748	0.889823	0.6881701
	BA	0.954991	0.936172	0.88658	0.6263704
	FA	0.953434	0.931674	0.882454	0.5644232
Pirate	DE	0.946298	0.951736	0.918854	0.9060872
	PSO	0.933165	0.95061	0.903981	0.905192
	BA	0.931228	0.944304	0.900143	0.871334
	FA	0.930858	0.933864	0.896563	0.8350455
Starfish	DE	0.95527	0.974187	0.928185	0.8560859
	PSO	0.952507	0.953383	0.927381	0.7894524
	BA	0.94859	0.939	0.910614	0.7713202
	FA	0.943572	0.915473	0.902217	0.7304659

Table VIII Comparison of structural similarity index (SSIM) for various algorithms.

		Th = 2	Th = 3	Th = 4	Th = 5
Images	Opt Tech	Fuzzy	Fuzzy	Fuzzy	Fuzzy
Cameraman	DE	0.637394	0.799621	0.784806	0.8200447
	PSO	0.649298	0.816236	0.843798	0.8252173
	BA	0.698512	0.816741	0.847386	0.8736756
	FA	0.745811	0.820292	0.84779	0.8562776
Lena	DE	0.656616	0.747751	0.770816	0.8210519
	PSO	0.663094	0.711089	0.734453	0.7670472
	BA	0.663156	0.745134	0.745062	0.7895847
	FA	0.695085	0.700278	0.732765	0.7979711
Goldhill	DE	0.441218	0.644457	0.670154	0.7810997
	PSO	0.474546	0.667122	0.689939	0.7909164
	BA	0.606807	0.668688	0.727309	0.7915057
	FA	0.645513	0.694545	0.732217	0.8096297
Lake	DE	0.688739	0.805535	0.835122	0.8434764
	PSO	0.672471	0.987456	0.824017	0.8463463
	BA	0.719987	0.799621	0.847386	0.8756467
	FA	0.710761	0.795692	0.816913	0.8773202
Pirate	DE	0.562101	0.662256	0.763526	0.7811629
	PSO	0.565558	0.704156	0.784691	0.8029126
	BA	0.566939	0.716693	0.785478	0.8158027
	FA	0.611312	0.720464	0.819659	0.8431184
Starfish	DE	0.493903	0.541492	0.653287	0.7067376
	PSO	0.494132	0.565434	0.691855	0.7283583
	BA	0.517186	0.620772	0.696124	0.7667584
	FA	0.525031	0.765436	0.719282	0.7724659

VI. CONCLUSIONS

A firefly algorithm based multilevel image thresholding for image segmentation has been productively proposed with desired output. Firefly algorithm maximizes the Fuzzy entropy for the efficient and effective image thresholding. The proposed algorithm is tested on natural images to show the merits of algorithm. Results of the proposed method are compared with other optimization techniques such as DE, PSO and BA with Fuzzy entropy. It is observed that proposed algorithm has higher/maximum fitness value compared to DE, PSO and BA. The PSNR value shows higher values with proposed algorithm than DE, PSO and BA and thereby draws better quality of the segmented image with proposed method. It can be concluded that proposed algorithm outperform the DE, PSO and BA in all performance measuring parameters. Further in future scope the firefly algorithm convergence time and efficiency is improved by modifying the algorithm process.

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