

Method to Calculate the Stress in Complex Welded Joints

Hernán HernándezHerrera^{#1} Rafael Goytisoló Espinosa^{#2} Juan José Cabello Eras^{#3}.
Alexis Sagastume Gutiérrez^{#4}

^{#1} Faculty of Engineering, Universidad de la Costa. Barranquilla, Colombia
¹hhernand16@cuc.edu.co.

^{#2} Faculty of Engineering, Universidad de Cienfuegos. Cienfuegos, Cuba.
²ragoyti@ucf.edu.cu

^{#3} Faculty of Engineering, Universidad de la Costa. Barranquilla, Colombia
³jcabello2@cuc.edu.co.

^{#4} Faculty of Engineering, Universidad de la Costa. Barranquilla, Colombia
⁴asagastu1@cuc.edu.co.

Abstract-This study presents a method to calculate the stress on the different components of complex welded joints to calculate the welded seam. The method is applied to a complex welded joint on an I shaped beam, welded and reinforced with plates, in an overlap joint. The proposed method is validated by the Finite Element Method. Results show differences lower than 5% between the proposed method and the Finite Element simulation, proving the applicability of the method to any complex welded joint.

Keywords: calculation, stress, finite element method, modelation, complex welded joints.

I. INTRODUCCIÓN.

Welding technologies stand as one of the most used technologies in the manufacturing of metal structures [1]. However, regardless of its widespread use, there are few studies discussing methods for calculating and designing welded joints. Shigley emphasized the difficulty for engineers because of the limited scope and depth of the existing methods to calculate welded joints [2].

The methods discussed in literature of welded joints are too general, focusing typical configurations, failing to discuss more complex welded joints. Some authors [2], [3], [4] discussed more complex welded joints using rather different models, in some cases resulting in contradictory.

The influence of technological parameters like the process development, the joint geometry and welding materials on the strength of welded joints was studied [5], [6], [7], [8], [9]. The main factors influencing weld joints failure are residual stress, local temperature effects and initial crack formations [10],[11], [12]. To evaluate the performance of cracks and forecast the lifespan of welded joints, fracture mechanics have been studied [13], [14], [15], [16], [17], [18].

Several researchers have discussed the mechanical strength related with the geometry of complex welded joints, for 2 I shaped beams fillet welded under bending [19], open profile fillet welding under torsion [20], in both cases results were validated with Finite Elements Analyze models, also used to determinate the stress concentration factor in welded joints of different geometries [21], [22].

The analytical methods used to design welding joints have remain unchanged for decades. Since the simplifications used in analytical methods are well known, the use of high values of safety factors is rather frequent [23], causing the over dimensioning of welded joints. This study aims at developing a method to evaluate the distribution of the mechanical stress in the different parts of the welded seam in complex welded joints to more accurately calculate the stress on welded joints.

II. MATERIAL AND METHODS.

The simplest of the welded joints is the fillet weld seam affected by transversal loads as shown in Fig. 1.

The stress and the stress concentrations in this joint are lower in the transverse seams than in the longitudinal ones [1], [2], [24]. However, for complex welded joints is rather difficult to define how loads affect the different elements of the weld seam.

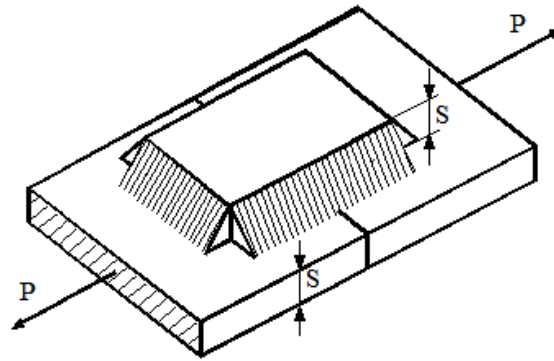


Fig. 1. Longitudinal and transverse fillet welding joint.

The basis of the proposed methods is the rigidity analysis [25]. Fig. 2 shows two I shaped beams joined with welded plates in the lateral, upper and lower surfaces. This configuration has a wide range of applications to cover spans larger than the available beam longitudes [26]. A normal load (NT), a transverse shear load (QT) and a bending moment (MFT) is applied to the welded joint.

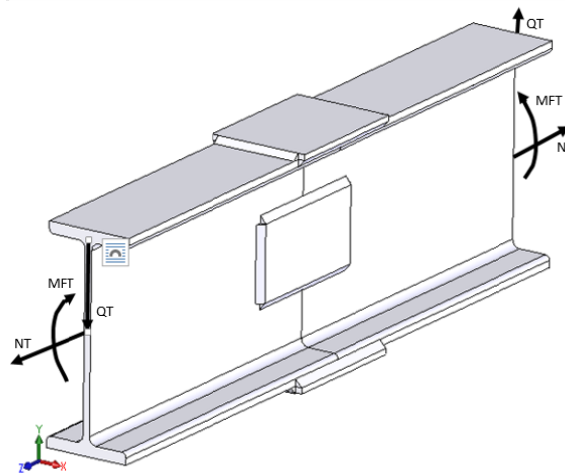


Fig. 2. Complex joint scheme to analyze.

The external loads are distributed between the three elements of the welded joint see Fig. 3. However, there are no procedures or methods to define which component is supported by each of the welded joints elements in order to calculate the stresses of each welded seam.

The stress value of each element of the analyzed welded joint is determined by applying the principle of superposition and distribution of internal forces, analyzing it for:

Axial force: NT, **Shear force:** QT, **Bending moment:** MFT

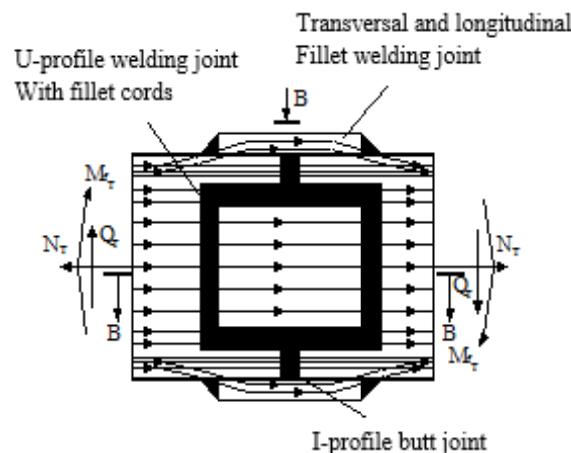


Fig. 3. Forces flow distribution between joint elements.

A. Axial Force.

The force N_T equals the three force elements of the joint:

$$N_T = N_1 + N_2 + N_3 \tag{1}$$

Where:

N_T - Total axial force acting on joint.

N_1 - Axial force supporting joint 1.

N_2 - Axial force supporting the top and bottom plates.

N_3 - Axial force supporting lateral plates.

Under the axial force (N_T), the welded joint deforms a magnitude ε . Each element of the welded joint supports a fraction of the total load, which is directly proportional to the rigidity of its corresponding section:

$$\varepsilon = \frac{\Delta l}{l} = \frac{N}{E \cdot A} \tag{2}$$

and $\varepsilon_1 = \varepsilon_2 = \varepsilon_3$

$$\frac{N_1}{E \cdot A_1} = \frac{N_2}{E \cdot A_2} = \frac{N_3}{E \cdot A_3} \tag{3}$$

Replacing (3) into (1) and expressing $N_T = f(N_1)$, it gives:

$$N_T = N_1 + \frac{A_2}{A_1} N_1 + \frac{A_3}{A_1} N_1 \tag{4}$$

$$N_1 = \frac{N_T}{1 + \frac{A_2}{A_1} + \frac{A_3}{A_1}} = \frac{N_T \cdot A_1}{A_1 + A_2 + A_3} \tag{4}$$

Similarly, expressions for N_2 and N_3 can be obtained:

$$N_2 = \frac{N_T \cdot A_2}{A_1 + A_2 + A_3} \tag{5}$$

$$N_3 = \frac{N_T \cdot A_3}{A_1 + A_2 + A_3} \tag{6}$$

In the case under assessment:

A_1 – Cross sectional area of the I shaped beam.

A_2 – Cross sectional area of both, the two upper and lower plates

A_3 – Cross sectional area of the two lateral plates.

Fig. 4 shows the different dimensions considered in this study.

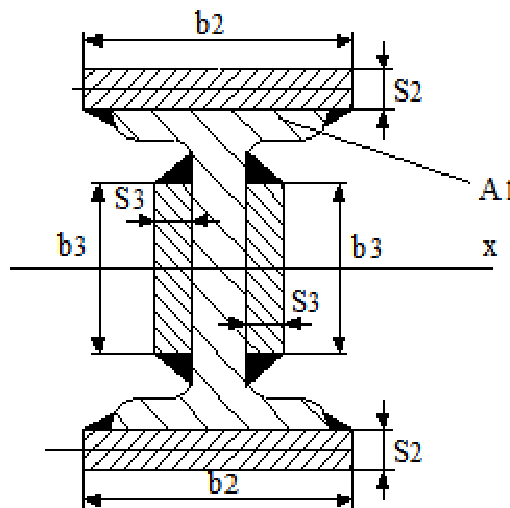


Fig. 4. Dimensions of welded joint elements.

B. Shear Force.

Similar to N_T , the shear force (Q_T) is:

$$Q_T = Q_1 + Q_2 + Q_3 \tag{7}$$

Where:

Q_T -Total shear force acting on joint.

Q_1 - Shear force supporting joint 1.

Q_2 - Shear force supporting both, the top and bottom plates.

Q_3 -Shear force supporting the lateral plates.

Since the distortion caused in the section is the same for the three elements of the joint, the force acting on each element will be proportional to their shear rigidity.

$$\frac{Q_1}{G \cdot A_1} = \frac{Q_2}{G \cdot A_2} = \frac{Q_3}{G \cdot A_3} \tag{8}$$

Similarly to the axial forces:

$$Q_1 = \frac{Q_T \cdot A_1}{A_1 + A_2 + A_3} \tag{9}$$

$$Q_2 = \frac{Q_T \cdot A_2}{A_1 + A_2 + A_3} \tag{10}$$

$$Q_3 = \frac{Q_T \cdot A_3}{A_1 + A_2 + A_3} \tag{11}$$

C. Bending Moment

The value of the total bending moment supporting each elements of the welded joint depends on the flexural rigidity. Similar to N_T and Q_T :

$$M_{F_T} = M_1 + M_2 + M_3 \tag{12}$$

From the differential equation of the elastic line:

$$M_1 = E \cdot I_{x_1} \cdot y_1'' \tag{13}$$

$$M_2 = E \cdot I_{x_2} \cdot y_2'' \tag{14}$$

$$M_3 = E \cdot I_{x_3} \cdot y_3'' \tag{15}$$

Since $y_1 = y_2 = y_3$, where $y_1' = y_2' = y_3'$ and $y_1'' = y_2'' = y_3''$ then.

$$\frac{M_1}{E \cdot I_{x_1}} = \frac{M_2}{E \cdot I_{x_2}} = \frac{M_3}{E \cdot I_{x_3}} \tag{16}$$

Combining equations 16 and 12:

$$M_1 = \frac{M_{f_T} \cdot I_{x_1}}{I_{x_1} + I_{x_2} + I_{x_3}} \tag{17}$$

$$M_2 = \frac{M_{f_T} \cdot I_{x_2}}{I_{x_1} + I_{x_2} + I_{x_3}} \tag{18}$$

$$M_3 = \frac{M_{f_T} \cdot I_{x_3}}{I_{x_1} + I_{x_2} + I_{x_3}} \tag{19}$$

Where:

I_{x_1} –Inertia moment of the Ishaped beam related to its centroid axis X.

I_{x_2} – Inertia moment of the upper and lower plates in relation to the axis X (see Fig. 5).

$$I_{x_2} = 2b_2 \cdot S_2 \left[\frac{3h^2 + 6h \cdot S_2 + 4S_2^2}{12} \right] \tag{20}$$

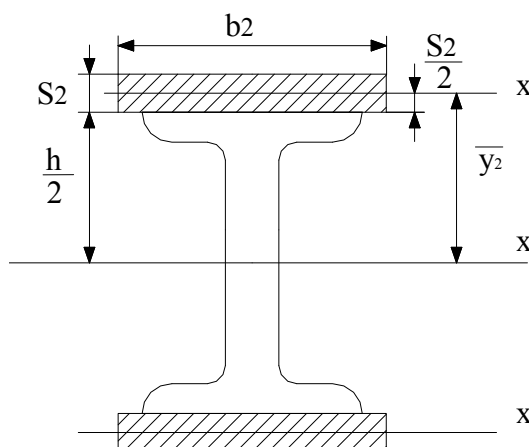


Fig. 5. Basic dimensions of the upper and lower plates.

I_{x3} – Inertia moment of the lateral plates in relation to the axis X (see Fig. 6).

$$I_{x3} = 2 \cdot \frac{1}{12} S_3 b_3^3 = \frac{S_3 \cdot b_3^3}{6} \quad (21)$$

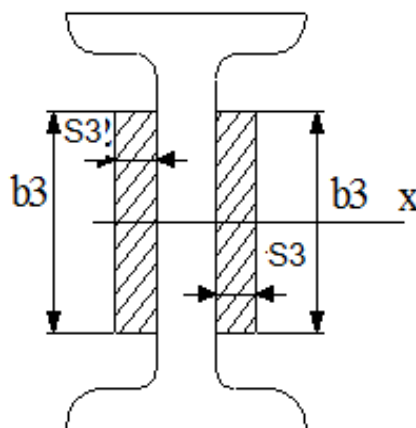


Fig. 6. Basic dimensions of the lateral plates.

D. Calculation of the stress caused by the MFT.

From the above equations, the bending moment for the three elements of the welded joint is calculated,

E. Stress caused by the bending moment M_1 in the joint of the I-shaped beam.

The bending moment causes traction and compression normal stress, proportional to the coordinate. They are calculated from the Navier equation

$$\sigma_y = \frac{M_1 \cdot y}{I_x} \quad (22)$$

The maximum stress is calculated as:

$$\sigma_{Mf \max} = \frac{M_1 \cdot y \max}{I_x} = \frac{M_1}{W_x} \quad (23)$$

F. Calculation of the stress on the top and bottom plates resulting from the moment M_2 .

The top and bottom plates welded joint can be considered as a fillet welding joint under the action of different loads, as shown in Fig. 7.

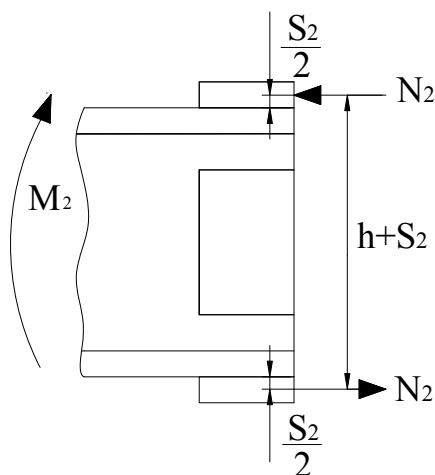


Figure 7. Stress on the welded joint fillet of the top and bottom plates.

Forces N_2 acting on these plates is calculated as:

$$M_2 = N_2 \cdot (h + S_2)$$

$$N_2 = \frac{M_2}{h + S_2}$$

$$\sigma_2 = \frac{N_2}{A_2} \tag{24}$$

G. Stress caused by the bending moment M_1 in the lateral plates.

Lateral plates are affected by the bending moment M_3 , which is calculated with the Navier equation:

$$\sigma_3 = \frac{M_3 \cdot y_{\max}}{2 \cdot Ix_3}$$

H. Simulation of the stress with the Finite Element Method.

To validate the proposed equations a complex welded joint under pure bending moment (see Fig.2), which is the most complex welded joint discussed in this study. First the proposed methodology is applied. Afterwards, a simulation based on the Finite Element Method is developed. The moments applied are shown in table I.

TABLE I Moments applied to the complex welded joint.

Case	Moment value (N·mm)
1	$1,0 \cdot 10^9$
2	$1,5 \cdot 10^9$
3	$2,0 \cdot 10^9$
4	$2,5 \cdot 10^9$
5	$3,0 \cdot 10^9$
6	$4,0 \cdot 10^9$

An IPN 300 profile (European Standard Beams) is selected as the I shaped beam. The top, bottom and lateral plates are square with S_2 and S_3 thickness of 8 mm, the length of the lateral plates (b_3) is 200 mm and the length of the top and bottom plates (b_2) is of 145 mm.

I. Calculation of the inertia moments of the welded joint components.

Inertia Moments.

Ix_1 –moment of in relation to its centroid axis X.

For an IPN 300 beam $Ix_1 = 98.000.000 \text{ mm}^4$

Inertia moment of the upper and lower plates in relation to the axis X.

$$Ix_2 = 2b_2 \cdot S_2 \left[\frac{3h^2 + 6h \cdot S_2 + 4S_2^2}{12} \right]$$

$$I_{x_2} = 55033493 \text{ mm}^4$$

Inertia moment of lateral plates in relation to the axis X.

$$I_{x_3} = 2 \cdot \frac{1}{12} S_3 b_3^3 = \frac{S_3 \cdot b_3^3}{6}$$

$$I_{x_3} = 10.666.666,67 \text{ mm}^4$$

III. RESULTS AND DISCUSSION.

A. Total moment applied to the joint (M_T).

$$M_T = 2 \cdot 10^9 \text{ N mm}$$

Calculation of Moment (M_1, M_2, M_3).

$$M_1 = \frac{M_T \cdot I_{x_1}}{I_{x_1} + I_{x_2} + I_{x_3}} = 1\,197\,310\,993 \text{ N mm} \quad M_2 = \frac{M_T \cdot I_{x_2}}{I_{x_1} + I_{x_2} + I_{x_3}} = 672\,369\,450 \text{ N mm}$$

$$M_3 = \frac{M_T \cdot I_{x_3}}{I_{x_1} + I_{x_2} + I_{x_3}} = 130\,319\,555 \text{ N mm}$$

B. Calculation of the stress in the welded joint elements.

Force(N_2).

$$M_2 = N_2 \cdot (h + S_2)$$

$$N_2 = \frac{M_2}{h + S_2} = 2183\,017.74 \text{ N}$$

Stress (σ_1).

$$\sigma_1 = \frac{M_1 \cdot y_{\max}}{I_{x_1}} = 1\,832.61 \text{ MPa}$$

Stress in the top and bottom plates(σ_2).

$$\sigma_2 = \frac{N_2}{A_2} = 1\,783,91 \text{ MPa}$$

Stress in the lateral plates(σ_3).

$$\sigma_3 = \frac{M_3 \cdot y_{\max}}{2 \cdot I_{x_3}} = 610 \text{ MPa}$$

C. Stress obtained by Finite Element Method.

Fig. 8 shows the results obtained from the Finite Element Method simulation for the conditions used in the analytical assessment.

The welded joint is simulated in Cosmos Design Star 4.0 software, a parabolic tetrahedral finite element, which give the best results because it can more easily generate a contour configuration [27],[28], [29].For each case a convergence between the previous and the current model is set to a 5% difference between the stress, ensuring that the stress valuesis independent of the mesh size[30].

For the third case the simulationare result in σ_1 equal to 1.854 MPa, σ_2 equal to 1.809 y and σ_3 equal to 644 MPa.All values are between 2 and 5 % difference to those obtained analytically. These results validate the proposed method for its application in complex welded joints.

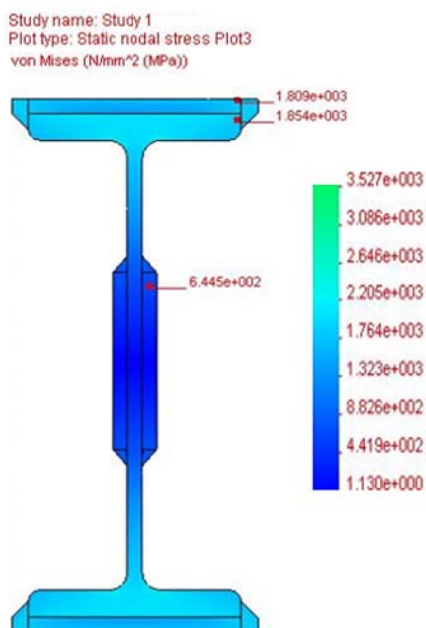


Fig. 8. Results obtained in the Finite Element Method for a bending moment of $2 \cdot 10^9$ N·mm

Table II shows the stress resulting from the analytic method and from the simulation.

TABLE II Comparison between analytical and Finite Element Method results.

Moment N·mm	σ_1 Analytic	σ_1 MEF	σ_2 Analytic	σ_2 MEF	σ_3 Analytic	σ_3 MEF
$1 \cdot 10^9$	916	943	870	882	305	324
$1,5 \cdot 10^9$	1374	1401	1337	1347	458	472
$2 \cdot 10^9$	1832	1854	1783	1809	632	644
$2,5 \cdot 10^9$	2290	2328	2229	2239	764	778
$3 \cdot 10^9$	2748	2782	2675	2697	916	938
$4 \cdot 10^9$	3605	3698	3567	3575	1222	1248

With the stress of each element of the complex welded joint it is possible to define the dimensions of the welded seam [19], [23], which allows to make an adequate design of the welded joint.

IV. CONCLUSION.

The method proposed in this study allows to calculate the stress in the different components of complex welded joints, which in turn allows to more accurately calculate the dimensions of the welded seam. The method proposed can be applied to any complex welded joint.

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AUTHOR PROFILE

Hernan Hernández Herrera is working as professor at Universidad de la Costa. His interest areas are applied mechanics and energy efficiency.

Rafael Goytisol Espinosa is working as professor at Universidad de Cienfuegos. His interest areas is applied mechanics.

Juan Jose Cabello Eras is working as professor at Universidad de la Costa. His interest areas are applied mechanics and energy efficiency.

Alexis Sagastume Gutiérrez is working as professor at Universidad de la Costa. His interest areas are energy efficiency and thermal engineering.