Dynamic Modeling of a Synchronous Generator Using T-S Fuzzy Approach

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***Abstract*— The dynamic behavior of power systems is affected by the interactions between linear and nonlinear components. To analyze those complicated power systems, the linear approaches have been widely used so far. Especially, a synchronous generator has been designed by using linear models and traditional techniques. However, due to its wide operating range, complex dynamics, transient performances, and its nonlinearities, it cannot be accurately modeled as linear methods based on small-signal analysis. This paper describes an application of the Takaki-Sugeno (T-S) fuzzy method to model the synchronous generator in a single-machine infinite bus (SMIB) system. The T-S fuzzy model can provide a highly nonlinear functional relation with a comparatively small number of fuzzy rules. The simulation results show that the proposed the T-S fuzzy modeling captures all dynamic characteristics for the synchronous generator, which are exactly same as those by the conventional nonlinear modeling methods.**

***Keywords*—** dynamic modeling; power system; synchronous generator; T-S fuzzy.

I. INTRODUCTION

A synchronous generator in a power system is a nonlinear fast-acting multiple-input multiple-output (MIMO) device. Due to its wide operating range, complex dynamics, nonlinearity, and the changing system configuration, the entire system cannot be accurately represented by a fixed model, which is then used for the design of conventional linear system/controllers.

Otherwise, by using the "IF-THEN rule", the Takagi-Sugeno (T-S) fuzzy modeling makes it possible to analyze a nonlinear system by approximating the system as a linear input-output system in certain range [1]-[6]. This range can be expanded over the range defined by other linearization techniques without losing generality when the system operates in wide range of operating points [7].

Also, the stability of the system can be analyzed by using a linear matrix inequality (LMI) method based on the Lyapunov condition, which can be formulated by the T-S fuzzy modeling. It has two advantages; one is the convenience of analysis through linearization, and the other is the accuracy of analyzing nonlinear systems. Moreover, this LMI technique on the T-S fuzzy modeling can prove the powerful capability to design the robust controller of any nonlinear dynamic systems.

In this paper, the T-S fuzzy modeling method is firstly applied to model an inverted pendulum system. Then, it is used to model the synchronous generator, which can be described by the fourth-order nonlinear differential equations, in a single-machine infinite bus (SMIB) power system. By applying the large (three-phase short circuit) and small (± 5% step changes in the reference voltage of exciter) disturbances to the SMIB system, its effectiveness is evaluated to show the same dynamic behaviors as given in the exact nonlinear model of the synchronous generator.

II. T-S FUZZY MODELING

A. *Takagi-Sugeno Fuzzy Model*

There is needed new model which is different from linear model to analyze more accurate when large accidents occur. In this paper, the T-S fuzzy method is used to analyze nonlinear system. The T-S fuzzy system can be represented according to Fig. 1 structure. It is hard to analyze nonlinear system which has a linear and a nonlinear part. The nonlinear system is converted to T-S fuzzy model to analyze easily.

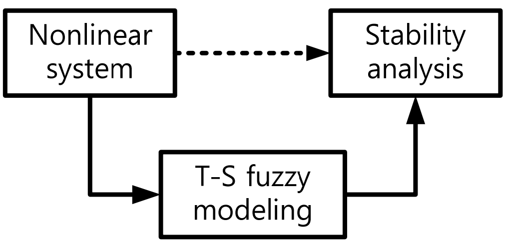


Fig. 1. Approach based on the T-S fuzzy modeling

The i^{th} rule of the affine T-S fuzzy models are following forms, where continuous fuzzy system (CFS), respectively.

Model Rule *i*

IF z_1(t) is M_{i1} and ⋯ and z_p(t) is M_{ip},

THEN $\dot{x} = \mathrm{A}_i x + \mathrm{C}_i$, $i = 1, 2, \cdots, r$

Here, M_{ij} is the fuzzy set and r is the number of model rules; $x(t) \in \mathrm{R}^n$ is the state vector, $\mathrm{A}_i \in \mathrm{R}^{nxn}$, $\mathrm{C}_i \in \mathrm{R}^{nxm}$; $z_1(t)$, ⋯ , $z_p(t)$ are known premise variable that may be functions of the state variables, external disturbances, and/or time. Each linear equation $\mathrm{A}_i x + \mathrm{C}_i$ is represented in (1) [7].

$$\dot{x} = \frac{\sum_{i=1}^{r} \omega_i \{\mathrm{A}_i x + \mathrm{C}_i\}}{\sum_{i=1}^{r} \omega_i} = \sum_{i=1}^{r} h_i(x) \{\mathrm{A}_i x + \mathrm{C}_i\} \tag{1}$$

Where

$$\omega_i(x) = \prod_{j=1}^{n} M_{ij}(x_j) , \; h_i(x) = \frac{\omega_i(x)}{\sum_{i=1}^{r} \omega_i(x)}$$

$$h_i(x) \geq 0 , \; \sum_{i=1}^{r} h_i(x) = 1 \tag{2}$$

B. Design Example: Inverted Pendulum

In this section, an illustrative example is provided to demonstrate the validity of the suggested stability analysis and synthesis method [8].

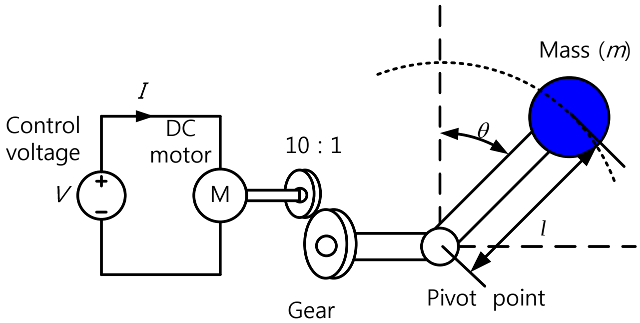


Fig. 2. Inverted pendulum controlled by a DC motor

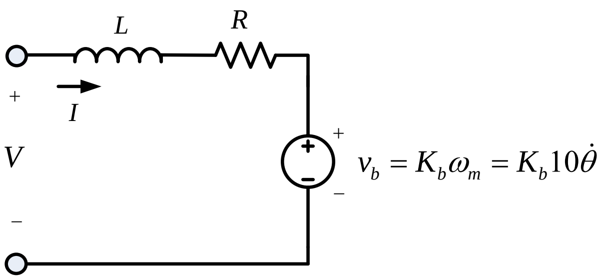


Fig. 3. Model of an armature-controlled DC motor

The plant to be controlled is an electro-mechanical system as shown in Fig. 2. Motor's inertia is negligible when compared with that of pendulum. The equivalent circuit of this system is illustrated in Fig. 3. The torque is supplied by motor, that is,

$$T_m = K_m I \; . \tag{3}$$

the torque of pendulum is delivered by gear (10:1), that is,

$$T_p = 10 T_m = 10 K_m I \; . \tag{4}$$

By using Kirchhoff's loop rule to get equation

$$V = L\dot{I} + RI + K_b 10\dot{\theta} \ . \tag{5}$$

The kinetic torque applied pendulum is sum of torque of kinetic energy and torque of gravity.

$$T_p = -l^2 m\dot{\theta} + lmg\sin\theta \tag{6}$$

where, (-) sign means that the kinetic torque is in opposition to torque of gravity. The dynamic equation is given by

$$\dot{x} = \mathrm{A}x + \mathrm{B}u \qquad x = [x_1 \quad x_2 \quad x_3]^T = [\theta \quad \dot{\theta} \quad I]^T$$

$$\mathrm{A} = \begin{bmatrix} 0 & 1 & 0 \\ K_1 \dfrac{\sin x_1}{x_1} & 0 & K_2 \\ 0 & K_3 & K_4 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ K_5 \end{bmatrix} \tag{7}$$

where the variable and the parameters are as follows [9].

$$K_1 = \frac{g}{l},\ K_2 = \frac{10K_m}{l^2 m},\ K_3 = -\frac{10K_b}{L},\ K_4 = -\frac{R}{L},\ K_5 = \frac{1}{L}$$

x_1 (θ, the angle measured with respect to the vertical axis);

x_2 ($\dot{\theta}$, the time derivative of x_1);

u (the control voltage) = 0;

m (the mass of the pendulum) = 1 kg;

l (the distance of the center of mass m from the pivot point) = 1 m;

g (gravitational constant) = 9.8 m/s^2 ;

R (equivalent resistance) = 1 Ω;

L (equivalent inductance) = 100 mH;

K_b (constant value of generator) = 0.1 Vs/rad;

K_m (constant value of mass) = 0.1 Nm/A;

In this paper, a controller doesn't be considered. So, the control input u = 0. It is straightforward to compute the T-S fuzzy system. In this example, the nonlinear term is only $\dfrac{\sin x_1}{x_1}$. The system matrix A is separated into 2 matrix. IF-THEN rules are follows,

Inverted Pendulum Model Rule

IF $x_1(t)$ is M_1, THEN $\dot{x} = \mathrm{A}_1 x$

IF $x_1(t)$ is M_2, THEN $\dot{x} = \mathrm{A}_2 x$

$$\mathrm{A}_1 = \begin{bmatrix} 0 & 1 & 0 \\ K_1 M_1 & 0 & K_2 \\ 0 & K_3 & K_4 \end{bmatrix},\ \mathrm{A}_2 = \begin{bmatrix} 0 & 1 & 0 \\ K_1 M_2 & 0 & K_2 \\ 0 & K_3 & K_4 \end{bmatrix} \tag{8}$$

Nonlinear system equation (7) is the same as the T-S fuzzy equation (8) when it is satisfied follows by (1)-(2).

$$\dot{x} = \sum_{i=1}^{2} h_i(x)\mathrm{A}_i x = h_1 \mathrm{A}_1 x + h_2 \mathrm{A}_2 x\ ,\ h_1 + h_2 = 1 \tag{9}$$

Equation (10) can be rewritten by (8) and (9).

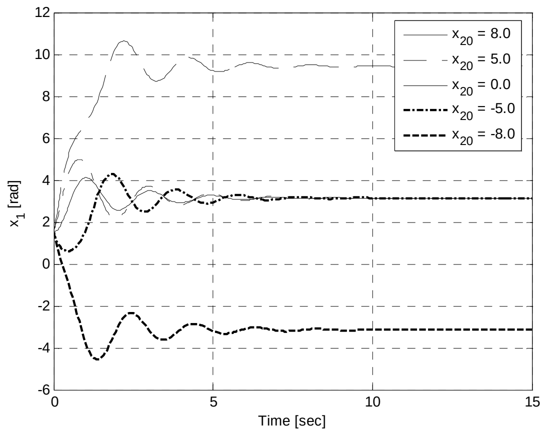
$$h_1 K_1 M_1 + h_2 K_1 M_2 = K_1 \frac{\sin x_1}{x_1}$$

$$h_1 M_1 + (1 - h_1) M_2 = \frac{\sin x_1}{x_1}$$

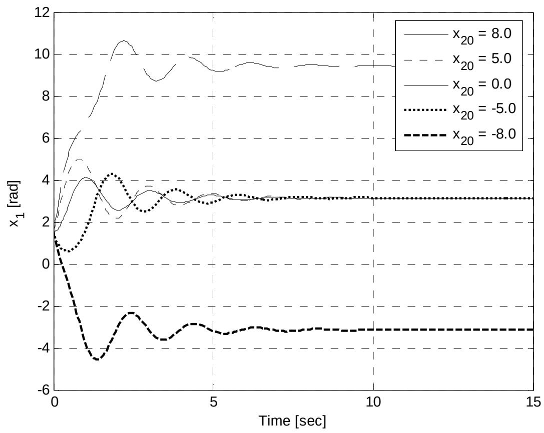
$$h_1(M_1 - M_2) + M_2 = \frac{\sin x_1}{x_1}$$

$$h_1 = \frac{\frac{\sin x_1}{x_1} - M_2}{M_1 - M_2} \quad (10)$$

Equation (10) is always satisfied regardless of values of M_1 and M_2. Here, $M_1 + M_2 = 1$, $M_1 = 1$ and $M_2 = 0$ are assumed.



(a)



(b)

Fig. 4. Pendulum position (u=0, x_1=1.5, x_3=10) (a) Nonlinear model (b) T-S fuzzy model

Fig. 4 shows the inverted pendulum pivot angle θ as a time and various initial angle speed (x_{20}). Fig. 4 (a) is the nonlinear model in (7) where the system has different stable points according to various initial angle speeds. Fig. 4 (b) is the T-S fuzzy model in (8), where its trace is the same as nonlinear model. This example provides the validity of the T-S fuzzy modeling.

III. SYNCHRONOUS MACHINE MODELING

A. Synchronous Modeling

The SMIB system is shown in Fig. 5. The system is one of the fundamental systems and can be expanded multi-machine system easily [10]. In this paper, fourth-order synchronous generator model is described by using the T-S fuzzy modeling method. The state variables are *d-axis* voltage, *q-axis* voltage, rotor angle and rotor speed [11], [12]. The synchronous generator can be expressed by using *d-q axis* in the phasor diagram of Fig. 6 in steady state [13], [14].

These mathematical models are given in (11)-(13). Armature and transmission resistance are negligible compare with reactance. Using Park's transform, *d-q axis* voltage equations are follows.

$$T'_{do}\frac{dE'_q}{dt} = -E'_q - (X_d - X'_d)I_d + E_{fd}$$

$$T'_{qo}\frac{dE'_d}{dt} = -E'_d - (X_q - X'_q)I_q \qquad (11)$$

The rotor angle and rotor speed are described by.

$$\frac{d\delta}{dt} = \omega - \omega_s$$

$$\frac{2H}{\omega_s}\frac{d\omega}{dt} = T_M - E'_d I_d - E'_q I_q - (X'_q - X'_d)I_d I_q - T_{FW} \qquad (12)$$

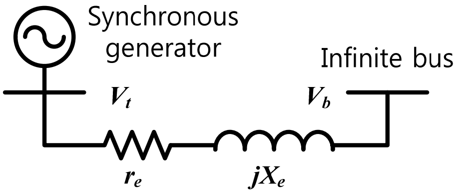


Fig. 5. Single-machine infinite bus system

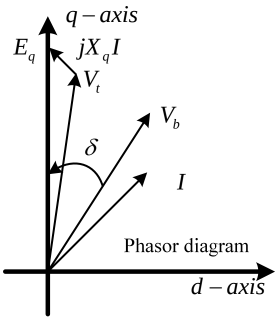


Fig. 6. Steady state phasor diagram of the SMIB system

In (11) and (12), (I_d , I_q) are the *d-q axis* components of current in the armature windings, ($X_d / X_q , X'_d / X'_q$) is the *d-q axis* components of (steady, transient) reactance in the armature windings, E_{fd} is the field voltage, T'_{do} and T'_{qo} are time constants of *d-q axis* transient. In (12), δ is the rotor angle with reference to the infinite bus, ω_s is the synchronous speed in steady state, T_m is turbine output shaft torque, T_{FW} is frictional and windage torque, and H is the polar moment of inertia. By using voltage equation in (11), and swing equation in (12), the fourth-order set of differential equation can be found for the dynamic model of the generator.

The current equations for the transmission system (in Fig. 5) are given in (13), which are derived from the methods of circuit and Park's transformation theory

$$I_d = \frac{E'_q + V_b\cos\delta}{X'_d - X_e},\ I_q = \frac{-E'_d + V_b\sin\delta}{X'_q - X_e} \qquad (13)$$

Where I_d and I_q are the *d-q axis* components of transmission current, E'_d and E'_q are the *d-q axis* components of transient voltage, V_b is the infinite bus voltage, X_e is the component of transmission line reactance.

B. T-S Fuzzy to the SMIB System

For the T-S fuzzy modeling, equation (14), (15) are given in below.

$$\dot{x} = \mathrm{A}x + \mathrm{C},\ x = \begin{bmatrix} E'_q & E'_d & \delta & \omega \end{bmatrix}^T$$

$$\mathrm{A} = \begin{bmatrix} \frac{1}{T_{do}^{'}}(-1-\frac{X_d - X_d^{'}}{X_d^{'}+X_e}) & 0 & \frac{1}{T_{do}^{'}}(\frac{V_b(X_d - X_d^{'})}{X_d^{'}+X_e})\frac{\cos\delta}{\delta} & 0 \\ 0 & \frac{1}{T_{qo}^{'}}(-1-\frac{X_q - X_q^{'}}{X_q^{'}+X_e}) & \frac{1}{T_{qo}^{'}}(\frac{V_b(X_q - X_q^{'})}{X_q^{'}+X_e})\frac{\sin\delta}{\delta} & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{\omega_s}{2H}(\frac{V_b \sin\delta}{X_q^{'}+X_e}) & \frac{\omega_s}{2H}(\frac{V_b \cos\delta}{X_d^{'}+X_e}) & 0 & 0 \end{bmatrix}$$

$$\mathrm{C} = \begin{bmatrix} \frac{E_{fd}}{T_{do}^{'}} & 0 & -\omega_s & \frac{\omega_s T_m}{2H} \end{bmatrix} \quad (14)$$

The SMIB system given in (14) has 4 nonlinear terms ($\frac{\cos\delta}{\delta}$, $\frac{\sin\delta}{\delta}$, $\sin\delta$, $\cos\delta$). Matrix A is divided into 16 linear matrix by the T-S fuzzy modeling. The input-output form of the fuzzy system of (14) is represented as

$$\dot{x} = \sum_{i=1}^{16} h_i(x)\{\mathrm{A}_i x + \mathrm{C}\},\ 0 \le h_i(x) \le 1,\ \sum_{i=1}^{16} h_i(x) = 1 \quad (15)$$

Max and Min values of nonlinear terms described in Table 1 are inserted the position of separated matrix by the T-S fuzzy rule. Each nonlinear terms have 2 values and represented follow IF-THEN rule.

TABLE I. FUZZY MODEL PARAMETERS OF THE SMIB SYSTEM

Nonlinear terms	**Inserted values**		**Membership function**	
	Max	***Min***	***Max***	***Min***
$\frac{\cos\delta}{\delta}$	1	0	$M_1^1 = \frac{\cos\delta}{\delta}$	$M_1^2 = 1 - M_1^1$
$\frac{\sin\delta}{\delta}$	1	0	$M_2^1 = \frac{\sin\delta}{\delta}$	$M_2^2 = 1 - M_2^1$
$\sin\delta$	1	0	$M_3^1 = \sin\delta$	$M_3^2 = 1 - M_3^1$
$\cos\delta$	1	0	$M_4^1 = \cos\delta$	$M_4^2 = 1 - M_4^1$

The SMIB System Model Rule

IF x_1 is "Max", x_2 is "Max", x_3 is "Max" and x_4 is "Max", THEN $\dot{x} = \mathrm{A}_1 x + \mathrm{C}$, $h_1 = M_1^1 \times M_2^1 \times M_3^1 \times M_4^1$

IF x_1 is "Max", x_2 is "Max", x_3 is "Max" and x_4 is "Min", THEN $\dot{x} = \mathrm{A}_2 x + \mathrm{C}$, $h_2 = M_1^1 \times M_2^1 \times M_3^1 \times M_4^2$

$\vdots$

IF x_1 is "Min", x_2 is "Min", x_3 is "Min" and x_4 is "Min", THEN $\dot{x} = \mathrm{A}_{16} x + \mathrm{C}$, $h_{16} = M_1^2 \times M_2^2 \times M_3^2 \times M_4^2$

For example, to find A_1, nonlinear term $\cos\delta/\delta$ is replaced by 1, $\sin\delta/\delta$ is replaced by 1, $\sin\delta$ is replaced by 1, and $\cos\delta$ is replaced by 1.

$$\mathrm{A}_1 = \begin{bmatrix} \frac{1}{T_{do}^{'}}(-1-\frac{X_d - X_d^{'}}{X_d^{'}+X_e}) & 0 & \frac{1}{T_{do}^{'}}(\frac{V_b(X_d - X_d^{'})}{X_d^{'}+X_e}) & 0 \\ 0 & \frac{1}{T_{qo}^{'}}(-1-\frac{X_q - X_q^{'}}{X_q^{'}+X_e}) & \frac{1}{T_{qo}^{'}}(\frac{V_b(X_q - X_q^{'})}{X_q^{'}+X_e}) & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{\omega_s}{2H}(\frac{V_b}{X_q^{'}+X_e}) & \frac{\omega_s}{2H}(\frac{V_b}{X_d^{'}+X_e}) & 0 & 0 \end{bmatrix}$$

The 16 linear matrixes can be found above method. Rotor angle (δ) of synchronous generator is bounded $0 < \delta \le \pi/2$ by (15) to satisfy the T-S fuzzy modeling method.

C. Simulation Results

The T-S fuzzy model must have dynamic characteristics the same as nonlinear model, to confirm the validation of the T-S fuzzy modeling. Simulation parameters are given follows.

f_e (fundamental frequency) = 60 Hz;

ω_s (steady state rotor speed) = 120π rad/s;

X_d (steady state d-axis reactance) = 1.8 Ω;

$X_d^{'}$ (transient d-axis reactance) = 0.3 Ω;

X_q (steady state q-axis reactance) = 1.7 Ω;

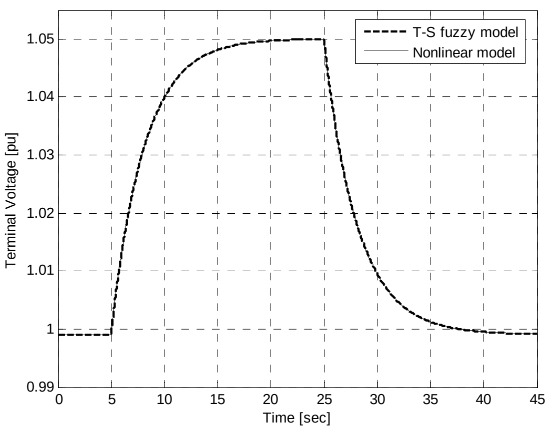
$X_q^{'}$ (transient q-axis reactance) = 0.4 Ω;

X_e (transmission line reactance) = 0.4 Ω;

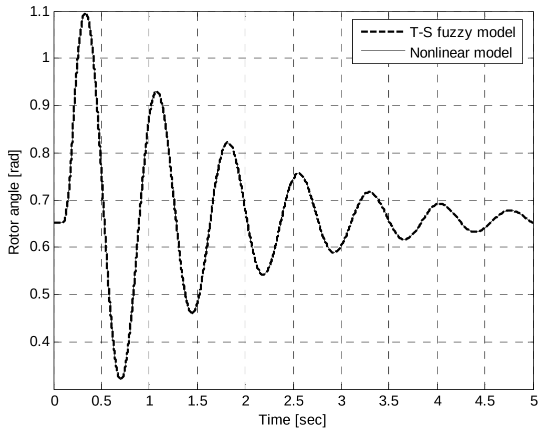
$T_{do}^{'}$ (transient d-axis time constant) = 8;

$T_{qo}^{'}$ (transient q-axis time constant) = 0.4;

Two different types of disturbances, namely, a ± 5% step changes in the reference voltage of exciter and a 100 ms three-phase short circuit at the infinite bus in Fig. 5, are carried out to evaluate the effectiveness of the T-S fuzzy modeling for the synchronous generator.



(a)



(b)

Fig. 7. Performance evaluation of the T-S fuzzy modeling: (a) Terminal voltage response when 5(%) step-change is applied to the excitation system (b) Rotor angle response when 100 ms to the infinite bus three-phase short circuit fault is applied

Fig. 7 (a) shows terminal voltage response of synchronous generator when the exitation voltage changes. The terminal voltage increases as the exitation voltage increases at 5 sec. When the exitation voltage decreases to initial value at 25 sec, the terminal voltage decreases, too. The solid line which indicates the nonlinear model shows same pattern with the dotted line that represents the T-S fuzzy model. Fig. 7 (b) shows the simulation result when three-phase short circuit fault occurs for 0.1 sec. The damping effect can be observed because of system unstability when accident occurs. In this case, the same routines are observed in both models; nonlinear model (solid line) and the T-S fuzzy model (dotted model). The effectiveness of the T-S fuzzy model can be checked through these simulation.

IV. CONCLUSIONS

This paper proposed the new modeling of synchronous generator by applying the T-S fuzzy method in order to analyze stability of the nonlinear SMIB system. Conventional linear model cannot represent the characteristics of synchronous generator which has wide operating range, complicated dynamic characteristics, and nonlinearity. The T-S fuzzy model shows more accuracy than the linear model because it considers nonlinear characteristics. It will be more easy to analyzing system stability and designing controller by using the T-S fuzzy model because nonlinear terms can be ignored when applying Lyapunov condition.

Acknowledgment

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