

# A Novel Method for Direct Extraction of Base and Collector Delay Times in HBT

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**Abstract**— A technique for determination of the delay times  $\tau_C$  and  $\tau_B$ , forward transit time  $\tau_F$  and fitting parameter  $m$  of heterojunction bipolar transistors HBT is presented. The procedure is accurate than conventional methods because the new model based on the magnitude of common-base current gain  $\alpha(\omega)$  is linear in the all-frequencies range. By utilizing the Z parameters of InGaP/GaAs HBT, excellent agreement is observed between measured  $\alpha_{mes}(\omega)$  and modeled  $\alpha(\omega)$  in the frequency range 0-50 GHz.

**Keyword**-HBT, Delay times, microwave, Z parameters, current gain.

## I. INTRODUCTION

Heterojunction bipolar transistors HBTs are the suitable devices for the analog, digital and power high-frequency applications including mixed-signal ICs for digital radar and advanced communication systems [1-3]. The base in HBT is highly doped to reduce the resistance and consequently increase the operating frequency. The material in base is made narrow to obtain the high gain from the large band gap in the emitter.

The extraction of HBT parameters is important step for assessing the process technology and optimizing the device design. The one of the crucial parameters of HBT is forward transit time,  $\tau_F$ , considered as key factor affecting microwave performance. Generally, the value of  $\tau_F$  is obtained by extrapolating the emitter-collector delay time  $\tau_{EC} = 1/(2\pi f_T)$  versus  $1/I_C$  curve [4].

The value of the transition frequency  $f_T$  is calculated by extrapolating the current gain  $h_{21}$  curve versus frequency with slope -20 dB/decade to locate  $f_T$  at unity gain frequency ( $h_{21} = 1$ ) [5-6]. This slope is distorted at higher frequencies, because of the extra poles and zeros induced by the various combinations of equivalent circuit parameters. To minimize this effect, the  $\tau_{EC}$  and  $\tau_F$  are determined by fitting methods based on “first-order” approximation models for low frequency measured data [7-8].

In this study, an analytic method based on a new linear model related to common-base current gain  $\alpha(\omega)$  is proposed to characterize the forward transit time,  $\tau_F$ , by extracting values of its components, base transit time,  $\tau_B$ , and collector transit time,  $\tau_C$ . This approach is valid for all measured frequencies data obtained from Z parameters of HBT equivalent circuit.

## II. Z PARAMETERS MODEL

Fig. 1 shows the HBT small-signal equivalent circuit based on T-model configuration in which parasitic pad capacitances have been de-embedded undercut off operation. This model is usually employed in extraction methods because it is closely related to the original derivation of the common base Y parameters of bipolar transistors [9]. After analysis of the electrical circuit equations of Z-parameters are expressed as following:

$$Z_{11} = \frac{R_{Bi}[(1-\alpha)Z_{BCi} + Z_{BCx}]}{Z} + Z_B + Z_E \quad (1)$$

$$Z_{12} = \frac{(1-\alpha)R_{Bi}Z_{BCi}}{Z} + Z_E \quad (2)$$

$$Z_{21} = \frac{Z_{BCi}[(1-\alpha)R_{Bi} - \alpha Z_{BCx}]}{Z} + Z_E \quad (3)$$

$$Z_{22} = \frac{(1-\alpha)Z_{BCi}(R_{Bi} + Z_{BCx})}{Z} + Z_C + Z_E \quad (4)$$

where

$$Z = R_{Bi} + Z_{BCi} + Z_{BCx} \quad (5)$$

and

$$Z_B = R_B + jL_B\omega \quad (6)$$

$$Z_C = R_C + jL_C\omega$$

$$Z_E = R_E + jL_E\omega + \frac{R_{BE}}{1 + jR_{BE}C_{BE}\omega} \quad (7)$$

$$Z_{BCi} = \frac{R_{BCi}}{1 + jC_{BCi}R_{BCi}\omega} \tag{8}$$

$$Z_{BCx} = \frac{R_{BCx}}{1 + jC_{BCx}R_{BCx}\omega} \tag{9}$$

$L_B, R_B, L_C, R_C, R_E, L_E$  are the extrinsic elements which are bias-independent. The intrinsic model varying with bias is represented by active source  $\alpha I_E$ , BE Junction  $R_{BE}, C_{BE}$  and the elements  $R_{Bi}, R_{BCi}, R_{BCx}, C_{BCi}, C_{BCx}$  characterizing distribution effect of the base-collector junction.

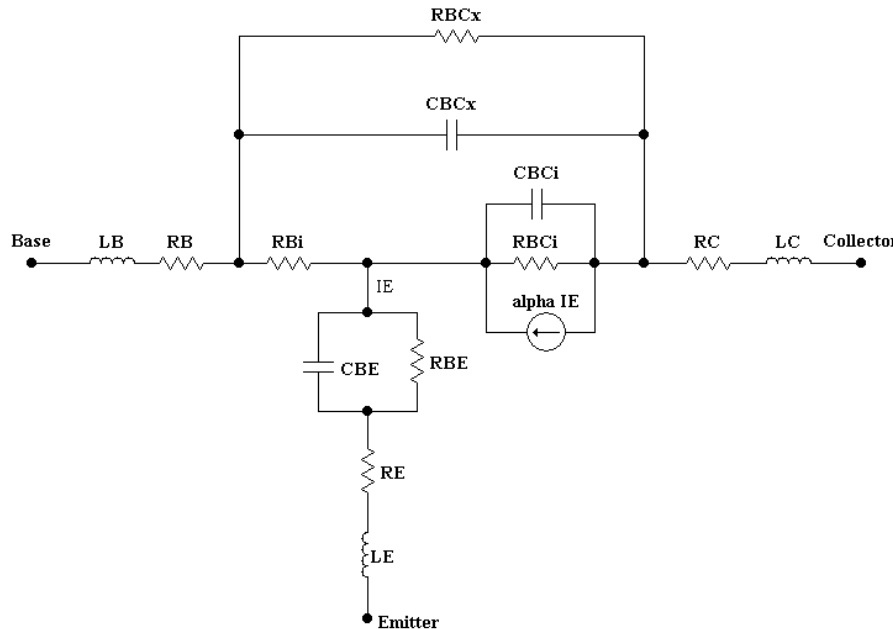


Fig. 1. T-Topology small-signal equivalent of a typical HBT

The experimental Z-parameters of HBT are calculated from the S-parameters measured by a vector network analyzer as the relationship between transmitted and reflected power waves. The numerical optimization methods of the Z models may result in non physical of the components and not unique values of solutions which are largely dependent on the initial values of the optimization process [10]. For this reason alternative methods based on analytic and empirical fitting procedures have been developed for the extraction of the HBT-model components related to frequency behavior (LF, MF and HF) [11-21].

### III. EXTRACTION PROCEDURE

The delay times,  $\tau_B$ , and,  $\tau_C$  are calculated by fitting experimental common-base current gain  $\alpha_{mes}$  data to  $\alpha$  model. After a series of transformation of  $Z_{12}, Z_{21}$  and  $Z_{22}$  equations (2-4), measured  $\alpha_{mes}$  is determined as :

$$\alpha_{mes} = \frac{Z_{12} - Z_{21}}{Z_{22} - Z_{21} - Z_C} \tag{10}$$

The expression of  $\alpha$  model is obtained by solving the time-dependent diffusion equation in the base [22-25]:

$$\alpha = \alpha_0 \frac{\exp[-j\omega((1-m)\tau_B + \tau_C)]}{1 + jR_{BE}C_{BE}\omega} \times \frac{\sin \omega\tau_C}{\omega\tau_C} \tag{11}$$

where  $\alpha_0$  denotes common-base DC current gain and  $m$  denotes a fitting parameter from a transistor's high-frequency characteristics [25-26].

All fitting methods reported in literature are strongly related on the first order approximated models, which are only valid at low frequency ranges and under certain assumptions [7-8]. So, the measured data at high frequency ranges are not used in fitting procedure and one cannot ensure that calculated parameter values can be verifying assumptions and approximations.

Our approach is focused on developing from equation of  $\alpha(\omega)$  a new linear model in order to obtain  $\tau_B, \tau_C$  and  $m$  parameters by a linear optimization algorithm without a need to use initial values considered as a main source of divergence and not physical solutions.

The squared magnitude and phase of  $\alpha$  can be expressed as:

$$|\alpha|^2 = \frac{\alpha_0^2}{1 + (R_{BE}C_{BE}\omega)^2} \times \left(\frac{\sin \omega\tau_C}{\omega\tau_C}\right)^2 \tag{12}$$

$$\varphi = -\omega((1-m)\tau_B + \tau_C) - \tan^{-1}(R_{BE}C_{BE}\omega) \quad (13)$$

Considering a second order limited Taylor development

$$\left(\frac{\sin \omega\tau_C}{\omega\tau_C}\right)^2 = 1 - \frac{(\omega\tau_C)^2}{3} \quad (14)$$

Then,

$$|\alpha|^2 = |\alpha|_{fit}^2 = \alpha_0^2 \frac{1 - \frac{(\omega\tau_C)^2}{3}}{1 + (R_{BE}C_{BE}\omega)^2} \quad (15)$$

In order to generate linear model one proposes to define new function  $\alpha_x$  determined from transformation of  $|\alpha|_{fit}^2$  by:

$$\alpha_x = \frac{1}{1 - \frac{|\alpha|_{fit}^2}{\alpha_0^2}} \quad (16)$$

After manipulation one can arrive at the linear function expressed as

$$\alpha_x(\omega_x) = A\omega_x + B \quad (17)$$

with variable:

$$\omega_x = \frac{1}{\omega^2} \quad (18)$$

where

$$A = \frac{1}{\frac{\tau_C^2}{3} + (R_{BE}C_{BE})^2} \quad (19)$$

$$B = \frac{(R_{BE}C_{BE})^2}{\frac{\tau_C^2}{3} + (R_{BE}C_{BE})^2} \quad (20)$$

A fitted linear regression model of  $\alpha_x(\omega_x)$  data is used to determine values of  $A$  and  $B$ . Then, the value of the collector delay time  $\tau_C$  is extracted as follows:

$$\tau_C = \sqrt{3 \frac{1-B}{A}} \quad (21)$$

To extract base delay time  $\tau_B$ , a combination of two functions is employed. First is time constant  $\tau_{BE} \equiv R_{BE}C_{BE}$  and the second is variation of phase  $\varphi$ .

The time constant  $\tau_{BE}$  is calculated as:

$$\tau_{BE} = R_{BE}C_{BE} = \sqrt{\frac{B}{A}} \quad (22)$$

where the capacitance  $C_{BE}$  is divided into two components:

$$C_{BE} = C_{jBE} + \frac{m\tau_B}{R_{BE}} \quad (23)$$

$C_{jBE}$  is emitter–base depletion capacitance

and

$$\frac{m\tau_B}{R_{BE}} \quad (24)$$

is transition or storage capacitance.

Consequently, the time constant  $\tau_{BE}$  can be expressed as:

$$\tau_{BE} = R_{BE}C_{jBE} + m\tau_B \quad (25)$$

Because  $R_{BE}$  is a function follows the inverse of collector current  $I_C$ :

$$R_{BE} = \frac{nKT}{qI_C} \quad (26)$$

the constant time  $\tau_{BE0} \equiv m \tau_B$  is evaluated from the y-intercept by using fitted linear regression model of  $\tau_{BE}$  curve versus  $1/I_C$ . Values of  $\tau_c, m \tau_B, \tau_{BE} = R_{BE} C_{BE}$  are used in  $\varphi$  phase function of common-base current gain  $\alpha(\omega)$  to extract the value of the base delay time  $\tau_B$ . Re-ranging  $\varphi$  equation (13) gives:

$$(\tau_B \omega)_{mes} = -\varphi_{mes} + \omega(m\tau_B - \tau_c) - \tan^{-1}(R_{BE} C_{BE} \omega) \tag{27}$$

Therefore, the value of  $\tau_B$  is evaluated from the linear fitting of calculated and measured data at various frequencies in the right hand side of this equation. Once  $\tau_B$  is know, value of fitting parameter  $m$  can be found by

$$m = \tau_{BE0} / \tau_B \tag{28}$$

#### IV. RESULTS AND DISCUSSION

Different analytical approaches ignore the term  $\sin \omega \tau_c / \omega \tau_c$  during parameter determination algorithms of HBT [8]. Effect of sinusoidal term, on variation of common base current gain,  $\alpha(\omega)$ , can be seen in figure 2. A coincidence between two curves is only observed in low frequency. In the present study, this term is taken into account and is replaced by its second order limited Taylor development provided by equation (14).

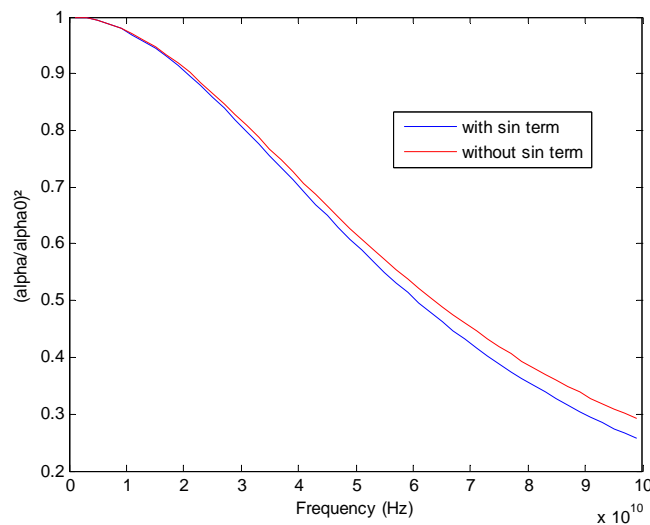


Fig. 2. Variation of  $|\alpha|^2 / \alpha_0^2$  versus frequency with and without the term  $(\sin \omega \tau_c / \omega \tau_c)^2$  in 100 GHz range at simulated parameters  $R_{BE} = 5 \Omega, C_{BE} = 500 \text{ pF}$  and  $\tau_c = 1 \text{ ps}$ .

As shown in fig.3, good agreement can be observed between model of equation (12) and its limited development of equation (15) in the range of 100 GHz frequency corresponding to that used in HP8510 Network Analyzer employed in HBT characterization.

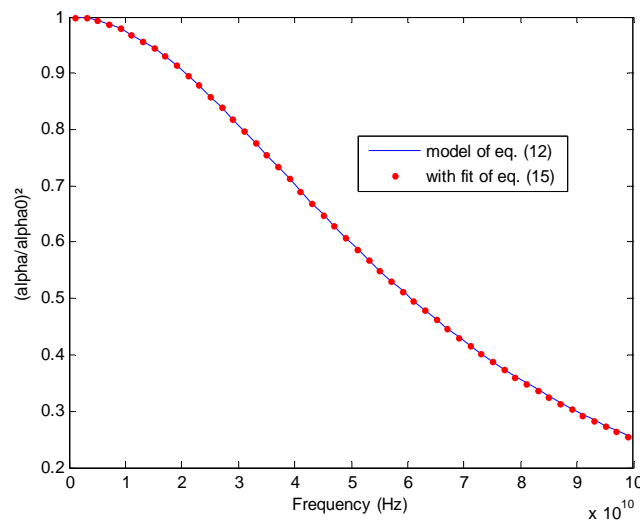


Fig. 3. Comparison between the function of  $|\alpha|^2 / \alpha_0^2$  and its limited development  $|\alpha|^2_{fit} / \alpha_0^2$  in 100 GHz range at simulated parameters  $R_{BE} = 5 \Omega, C_{BE} = 500 \text{ pF}$  and  $\tau_c = 1 \text{ ps}$ .

In order to verify the accuracy of the proposed method presented, Z parameters are used to extract the T-Model of InGaP/GaAs HBT [27]. Table 1 gives the small-signal model parameter's based on T-equivalent circuit.

TABLE I. Model Parameters at  $I_C = 10$  mA for small-signal equivalent HBT (fig.1)

<b>Extrinsic Model</b>	<b>Base</b>	$R_B$	<b>6.18 <math>\Omega</math></b>
		$L_B$	<b>31.6 pH</b>
	<b>Emitter</b>	$R_E$	<b>7.98 <math>\Omega</math></b>
		$L_E$	<b>34.9 pH</b>
	<b>Collector</b>	$R_C$	<b>13.72 <math>\Omega</math></b>
		$L_C$	<b>28.6 pH</b>
<b>Intrinsic Model</b>	<b>BE Junction</b>	$R_{BE}$	<b>5.21 <math>\Omega</math></b>
		$C_{BE}$	<b>549 fF</b>
	<b>Active current <math>\alpha I_E</math></b>	$\alpha_0$	<b>0.9842</b>
	<b>Distribution effect of the BC junction</b>	$R_{Bi}$	<b>8.40 <math>\Omega</math></b>
		$R_{BCi}$	<b>126.36 k<math>\Omega</math></b>
		$C_{BCi}$	<b>47.848 fF</b>
		$R_{BCx}$	<b>67.742 k<math>\Omega</math></b>
		$C_{BCx}$	<b>89.252 fF</b>

Using equation (10) and  $Z_{12}$ ,  $Z_{21}$ ,  $Z_{22}$ ,  $Z_C$  data, one can determine experimental common-base current gain  $\alpha_{mes}$ . The validity of proposed technique is verified by observing, in figure 4, a good linearity of the  $\alpha_x$  versus  $\omega_x = 1/\omega^2$ . Note that  $\alpha_0$  is equal to 0.9842 and is obtained by taking the value of magnitude of  $\alpha$  at DC- low frequency. The value of collector delay time  $\tau_c$  is 0.8146 ps evaluated from equation (21) corresponding to the linear fitting of  $\alpha_x(\omega_x)$ .

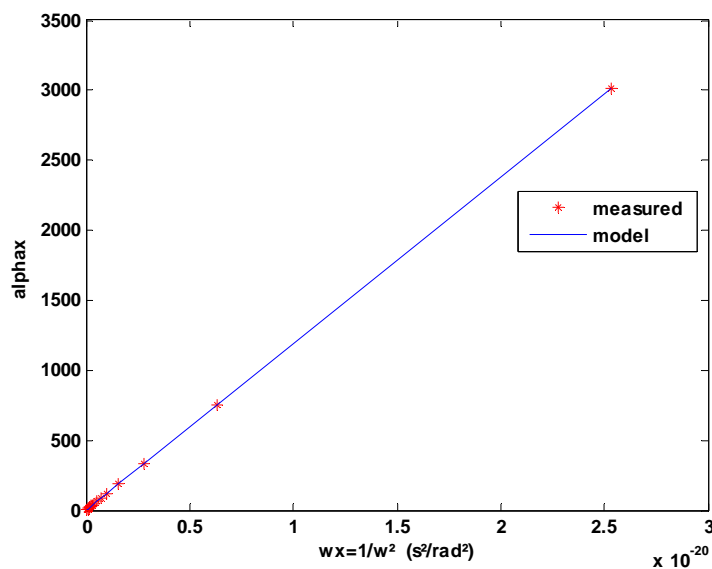


Fig. 4. a. Plot of function proposed  $\alpha_x$  versus  $\omega_x = 1/\omega^2$  in 50 GHz range at  $I_C = 10$  mA. Fitting line gives  $A$  and  $B$ .  $A$  is gradient of curve.

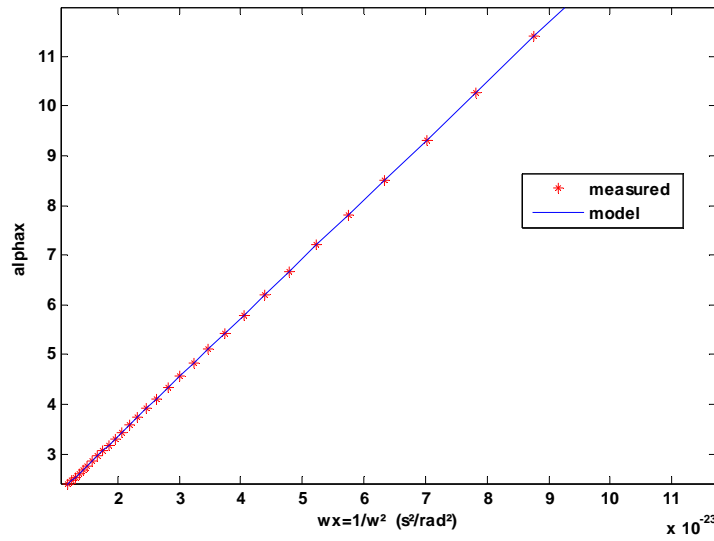


Fig. 4. b. Plot of function proposed  $\alpha_x$  versus  $\omega_x=1/\omega^2$  at low frequency  $I_C=10$  mA. Extrapolated y-intercepts of fitting line gives  $B$ .

The values of  $\tau_{BE}$  evaluated from equation (25) at various current levels are plotted versus  $1/I_C$  in the aim to calculate the value of  $m\tau_B$ . Variation of  $\tau_{BE}$  is plotted as a function of  $1/I_C$  in figure 5. The y-intercept of plot gives  $m\tau_B = 0.885$  ps.

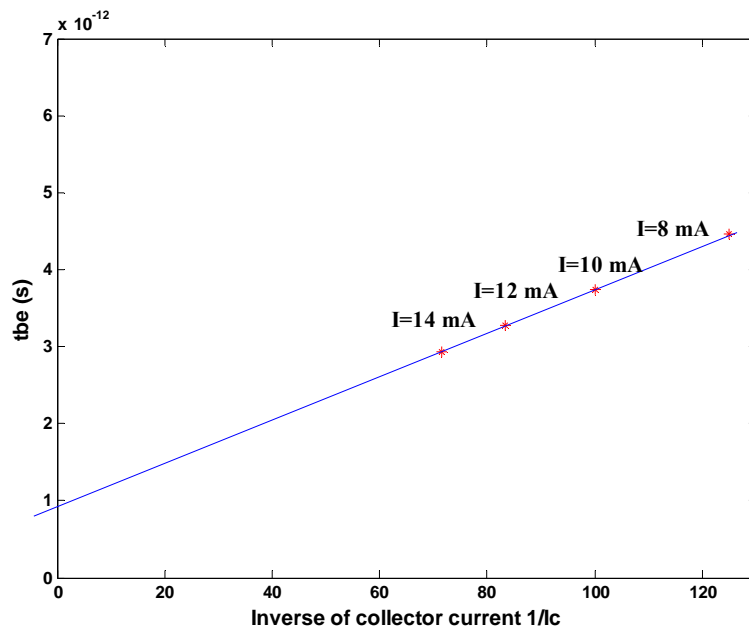


Fig. 5. Variation of constant time  $\tau_{BE}$  with  $1/I_C$ . Extrapolated y-intercepts of fitting line gives  $m\tau_B$ .

After finding  $\tau_C$  and  $m\tau_B$ , the plot of  $\tau_B\omega$  determined by equation (27) is shown in figure 6. Linear fitting gives  $\tau_B = 1.5054$  ps and so  $m = 0.5879$ . Finally value forward transit time is  $\tau_F = \tau_B + \tau_C = 2.3197$  ps.

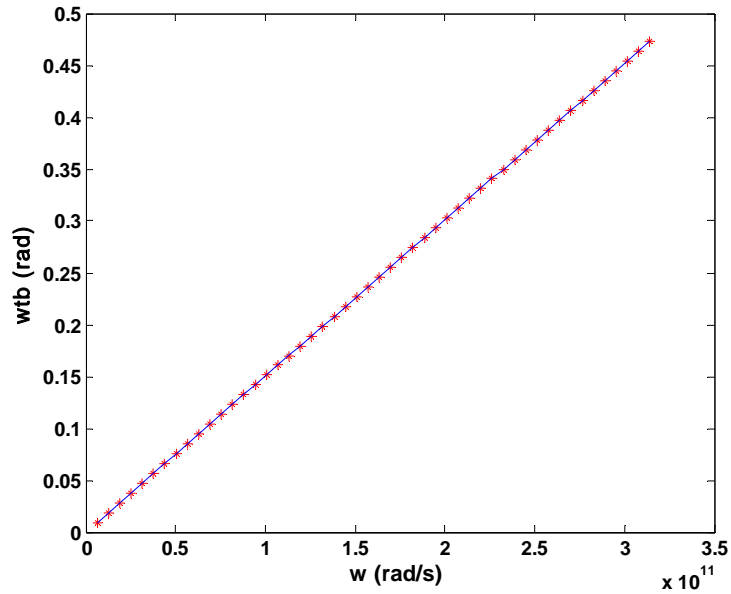


Fig. 6. Plot of measured  $\tau_b\omega$  data versus frequency. The gradient of the fitting line is equal to  $\tau_b$ .

Now, one compares  $|\alpha_{mes}|^2$  with our model and that usually reported in literature [8] :

$$|\alpha|^2_{fit2} = \alpha_0^2 \left( 1 - \frac{(\omega\tau_c)^2}{3} - (R_{BE}C_{BE}\omega)^2 \right)$$

The curves of  $|\alpha|^2_{fit2}$ ,  $|\alpha|^2_{fit}$ , and  $|\alpha_{mes}|^2$  versus  $\omega^2$  are shown in figure 7. The calculated  $|\alpha|^2_{fit}$  shows a good fit to the  $|\alpha_{mes}|^2$  data fit. However, in  $|\alpha|^2_{fit2}$ , linear behavior and fit can be only observed in low frequency range under assumption  $(R_{BE}C_{BE}\omega)^2 \ll 1$ .

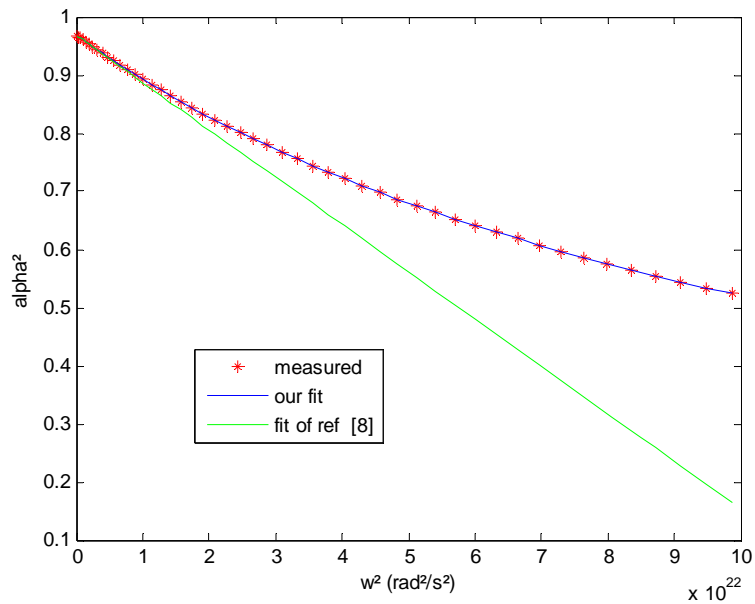


Fig. 7. Comparison between  $|\alpha_{mes}|^2$ ,  $|\alpha|^2_{fit}$  and  $|\alpha|^2$  in 50 GHz range

Our technique is analytic and is applicable for all frequencies to extract delay times and  $m$  parameter. However, in  $|\alpha|^2_{fit2}$ , one must graphically select linear frequency zone in  $|\alpha_{mes}|^2$  to apply procedure extraction.

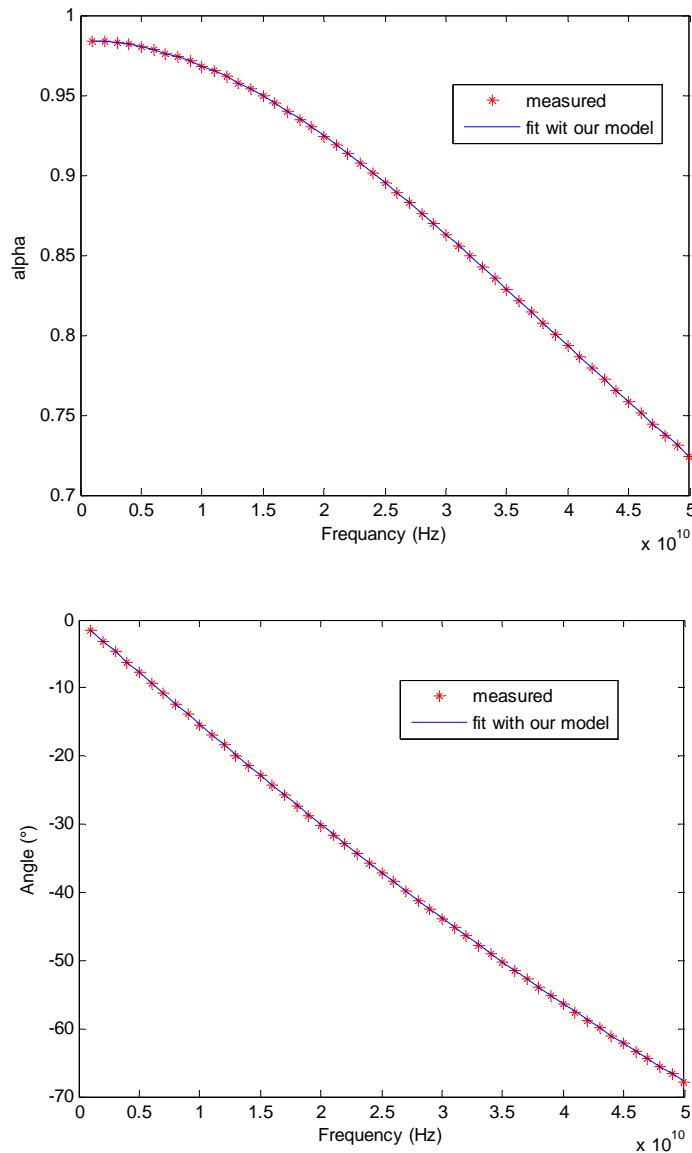


Fig. 8. Comparison between calculated magnitudes and phases of  $\alpha$  and measured  $\alpha$  in 50 GHz range.

Finally one can observe in figure 8, the modeled values  $\alpha$  correlate well with measured data  $\alpha_{mes}$ . Then the excellent fit confirms the validity of the model including  $\sin(\omega\tau_c/\omega\tau_c)$  term and verifies the proposed method applied for all range frequency.

## V. CONCLUSION

This study presents T-model for small-signal HBT equivalent circuit. The measurement of common-base current gain  $\alpha$  is directly extracted from Z-parameters. The values of delay times ( $\tau_B$  and  $\tau_C$ ), forward transit  $\tau_F$  and fitting parameter  $m$  in the formulation of common-base current gain  $\alpha$  are directly extracted by using linear optimization based on new model applied in all frequency ranges thus making the proposed algorithm more precise than  $f_T$  extraction and previous fitting methods. It is also clear that the method leads to excellent agreement between measured and calculated HBT common-mode current gain.



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