Modern Estimation technique for Undersea Active Target tracking

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Abstract: Pseudo Linear Estimator (PLE) developed for active target tracking in underwater environments. The PLE in sequential mode is considered here for this application. The results of PLE are compared with that of EKF. The results of Monte-Carlo simulation are presented for two selected scenarios. In PLE, there is no need to initialize target state vector and its covariance matrix with prior (approximate) knowledge and hence its performance is found to be better than that of EKF.

Keywords: estimation, sonar, Extended Kalman filter, target tracking, simulation

I. INTRODUCTION

Active sonar of the own-ship generates corrupt target measurements. The own-ship processes these measurements and estimates target motion parameters - viz., range, course, bearing and speed. There are many methods available [1]-[6] to achieve this task. Lindgren and Gong [7], Aidala [8], Aidala & Nardone [9] and Nardone, Lindgren & Gong [10] developed Pseudo Linear Estimator (PLE) in batch processing. S. K. Rao [11] developed PLE in sequential processing mode. In this paper, this work is extended to active sonar applications. All covariance matrix elements are represented recursively in terms of the measurement equation. They are called Recursive Sums and are maintained throughout the algorithm. Computation is made simpler by computing only the incremental values for every new measurement. These incremental values are used to update the Recursive Sums in covariance matrix. Few Recursive Sums are to be updated with new bearing & range measurement and hence computational load does not increase with additional measurements.

In general, it is perceived that PLE is a classical estimator and so it is not useful for tracking a manoeuvring target. Hence EKF is used for tracking a manoeuvring target. With little modification in the algorithm, PLE can also be extended to track a manoeuvring target as is presented in this paper. The latest measurements (of fixed number) in sliding window are used to track a manoeuvring target. The length of the measurements is decided by the accuracy of the results obtained in Monte-Carlo simulation. The performance of the algorithm is evaluated for several geometries in Monte-Carlo simulation. For the purpose of illustration, the results of two typical scenarios are presented. In simulation, the performance of PLE is found to be better when compared to EKF.

II. MATHEMATICAL MODELLING OF PSEUDO LINEAR ESTIMATOR

Let the target state vector be $X_t(k)$, given by $X_t(k) = [\dot{x}_t(k) \quad \dot{y}_t(k) \quad x_t(k) \quad y_t(k)]^T$ where $\dot{x}_t(k)$ and $\dot{y}_t(k)$ are target velocity components, and $x_t(k)$ and $y_t(k)$ are target position components. For conceptualization, the target is assumed to be moving at constant velocity. The target state dynamic equation is given by

$$X_t(k+1) = \Phi(k+1,k)X_t(k) \tag{1}$$

where $\Phi(k+1,k)$ is a transient matrix and is given by

$$\Phi(k+1,k) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ ts & 0 & 1 & 0 \\ 0 & ts & 0 & 1 \end{bmatrix} \tag{2}$$

where ts is time interval between two successive measurements. Let $B(k)$ and $R(k)$ represent the actual bearing and range respectively. Let $B_m(k)$ and $R_m(k)$ be bearing and range measurements generated by active sonar. The bearing measurement is the angle from ownship position to the target position, referenced (clockwise-positive) to the y-axis and is given by [7-11]

$$B_m(k) = B(k) + \gamma(k) \tag{3}$$

where

$$B(k) = \tan^{-1}\left(\frac{x_t(k) - x_0(k)}{y_t(k) - y_0(k)}\right) \tag{4}$$

where (x_0, y_0) are the position components of the ownship. $\gamma(k)$ is zero mean white Gaussian sequence with variance $\sigma_B^2(k)$.

The four dimensional state eqn. (1) and the nonlinear measurement of eqns. (3) and (4) define the bearings-only motion analysis problem. The nonlinear bearing measurement of eqns. (3) and (4) are manipulated to provide a pseudo bearing measurement that is 'linearly' related to the target state [10]. Eqn. (4) is rewritten as follows to obtain pseudo bearing measurement, $z'(k)$

$$\frac{\sin B(k)}{\cos B(k)} = \frac{x_t(k) - x_0(k)}{y_t(k) - y_0(k)} \tag{5}$$

Substituting eqn. (3) in the eqn. (5), straight forward formulation yields

$$x_t(k)\cos B_m(k) - y_t(k)\sin B_m(k) + r_s(k)\gamma(k) = x_0(k)\cos B_m(k) - y_0(k)\sin B_m(k) \tag{6}$$

where $r_s(k)$ is given by

$$r_s(k) = [x_t(k) - x_0(k)]\sin B_m(k) + [y_t(k) - y_0(k)]\cos B_m(k) \tag{7}$$

It is assumed that the magnitude of the noise $\gamma(k)$ is less than 2 deg. With this assumption $\cos\gamma(k)$ and $\sin\gamma(k)$ are equal to 1 and $\gamma(k)$ respectively. Again by simple algebraic formulation of the eqn. (7), we get measurement equation

$$z'(k) = H'(k)X_t(k) + \gamma'(k) \tag{8}$$

where the pseudo bearing measurement $z'(k)$ is given by

$$z'(k) = x_0(k)\cos B_m(k) - y_0(k)\sin B_m(k) \tag{9}$$

Measurement matrix for bearing measurement is given by

$$H'(k) = \begin{bmatrix} 0 & 0 & \cos B_m(k) & -\sin B_m(k) \end{bmatrix} \tag{10}$$

and $\gamma'(k) = \gamma(k)\, r_s(k)$. Let us define $X_t(n,k)$ as an estimate of target state vector, where n is number of samples. To find out initial estimate, $X_t(0,k)$, $z'(k)$ is modified as

$$z'(k) = H'(k)\phi(k,0)X_t(0,k) + \gamma'(k) \tag{11}$$

where

$$\phi(k,0) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ ts_1 + ts_2 + \ldots + ts_k & 0 & 1 & 0 \\ 0 & ts_1 + ts_2 + \ldots + ts_k & 0 & 1 \end{bmatrix} \tag{12}$$

Next we model the range measurement. $R_m(k)$, the measured range at time k, is given by

$$R_m(k) = R(k) + \varsigma(k) \tag{13}$$

where

$$R^2(k) = (x_t(k) - x_0(k))^2 + (y_t(k) - y_0(k))^2 \tag{14}$$

and $\varsigma(k)$ is a zero mean Gaussian sequence and is uncorrelated to the noise sequence in the bearing measurement. Eqn. (14) can be rewritten as follows to obtain pseudo range measurement, $z''(k)$

$$R(k) = (x_t(k) - x_0(k))\left(\frac{x_t(k) - x_0(k)}{R(k)}\right) + (y_t(k) - y_0(k))\left(\frac{y_t(k) - y_0(k)}{R(k)}\right) \\ = (x_t(k) - x_0(k))\sin B(k) + (y_t(k) - y_0(k))\cos B(k) \tag{15}$$

Substituting eqn. (3) and (13) in eqn. (15), we obtain

$$z''(k) = H''(k)X_t(k) + \varsigma'(k) \tag{16}$$

where the pseudo range measurement $z''(k)$ is given by

$$z''(k) = x_0(k)\sin B_m(k) + y_0(k)\cos B_m(k) + R_m(k) \tag{17}$$

The measurement matrix for range measurement is given by

$$H''(k) = [0 \quad 0 \quad \sin B_m(k) \quad \cos B_m(k)] \tag{18}$$

and

$$\varsigma'(k) = \varsigma(k) - \gamma(k)[\cos B_m(k)(x_t(k) - x_0(k)) - \sin B_m(k)(y_t(k) - y_0(k))] \tag{19}$$

To find out initial estimate, $X_t(0,k)$, $z''(k)$ is modified as

$$z''(k) = H''(k)\phi(k,0)X_t(0,k) + \varsigma'(k) \tag{20}$$

Consider the pseudo bearing and range measurements in matrix form as

$$Z(k) = [z'(1)z''(1)z'(2)z''(2)z'(3)z''(3)\ldots z'(k)z''(k)] \tag{21}$$

Let us use the familiar Least Square Estimator equation to find out the initial estimate of target state [11]

$$\hat{X}_t(0,k) = \left[A^T(k,0)A(k,0)\right]^{-1} A^T(k,0)Z(k) \tag{22}$$

Calculation of $A^T(k,0)A(k,0)$ is given below. $A(k,0)$ can be written as

$$A(k,0) = [H'(1)\phi(1,0) \quad H''(1)\phi(1,0) \quad H'(2)\phi(2,0) \quad H''(2)\phi(2,0) \quad \ldots] \tag{23}$$

Using eqn. (12) we can write that

$$\phi(1,0) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ ts_1 & 0 & 1 & 0 \\ 0 & ts_1 & 0 & 1 \end{bmatrix}$$

$$\phi(2,0) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ ts_1 + ts_2 & 0 & 1 & 0 \\ 0 & ts_1 + ts_2 & 0 & 1 \end{bmatrix}$$

and so on. Using eqn (10), we can write that

$$H'(1)\phi(1,0) = [0 \quad 0 \quad \cos B_m(1) \quad -\sin B_m(1)]\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ ts_1 & 0 & 1 & 0 \\ 0 & ts_1 & 0 & 1 \end{bmatrix} \\ = [ts_1 \cos B_m(1) \quad -ts_1 \sin B_m(1) \quad \cos B_m(1) \quad -\sin B_m(1)] \tag{24}$$

Similarly

$$H'(2)\phi(2,0) = [(ts_1 + ts_2)\cos B_m(2) \quad -(ts_1 + ts_2)\sin B_m(2) \quad \cos B_m(2) \quad -\sin B_m(2)] \tag{25}$$

It leads to

$$H'(k)\phi(k,0) = \left[(ts_1 + ts_2 + \ldots + ts_k)\cos B_m(k) \quad -(ts_1 + ts_2 + \ldots + ts_k)\sin B_m(k) \quad \cos B_m(k) \quad -\sin B_m(k) \right]$$

and

$$H''(k)\phi(k,0) = \left[(ts_1 + ts_2 + \ldots + ts_k)\sin B_m(k) \quad (ts_1 + ts_2 + \ldots + ts_k)\cos B_m(k) \quad \sin B_m(k) \quad \cos B_m(k) \right] \tag{26}$$

Then using eqn. (23), $A(k,0)$ can be written as

$$A(k,0) = \begin{bmatrix} ts_1 \cos B_m(1) & -ts_1 \sin B_m(1) & \cos B_m(1) & -\sin B_m(1) \\ ts_1 \sin B_m(1) & ts_1 \cos B_m(1) & \sin B_m(1) & \cos B_m(1) \\ (ts_1 + ts_2)\cos B_m(2) & -(ts_1 + ts_2)\sin B_m(2) & \cos B_m(2) & -\sin B_m(2) \\ (ts_1 + ts_2)\sin B_m(2) & (ts_1 + ts_2)\cos B_m(2) & \sin B_m(2) & \cos B_m(2) \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix} \tag{27}$$

$A^T(k,0)A(k,0)$ can be written as

$$A^T(k,0)A(k,0) = \begin{bmatrix} \ldots & \left(\sum_{i=1}^{k} ts_i\right)\cos B_m(i) & \left(\sum_{i=1}^{k} ts_i\right)\sin B_m(i) & \ldots \\ \ldots & \left(\sum_{i=1}^{k} -ts_i\right)\sin B_m(i) & \left(\sum_{i=1}^{k} ts_i\right)\cos B_m(i) & \ldots \\ \ldots & \cos B_m(i) & \sin B_m(i) & \ldots \\ \ldots & -\sin B_m(i) & \cos B_m(i) & \ldots \end{bmatrix} \times$$

$$\begin{bmatrix} \vdots & \vdots & \vdots & \vdots \\ \left(\sum_{i=1}^{k} ts_i\right)\cos B_m(i) & \left(\sum_{i=1}^{k} -ts_i\right)\sin B_m(i) & \cos B_m(i) & -\sin B_m(i) \\ \left(\sum_{i=1}^{k} ts_i\right)\sin B_m(i) & \left(\sum_{i=1}^{k} ts_i\right)\cos B_m(i) & \sin B_m(i) & \cos B_m(i) \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix} = \begin{bmatrix} p & 0 & q & 0 \\ 0 & p & 0 & q \\ q & 0 & k & 0 \\ 0 & q & 0 & k \end{bmatrix} \tag{28}$$

where

$$p = (ts_1)^2 + (ts_1 + ts_2)^2 + (ts_1 + ts_2 + ts_3)^2 + \ldots = \sum_{i=1}^{k} (ts_i)^2 \tag{29}$$

and

$$q = (ts_1) + (ts_1 + ts_2) + (ts_1 + ts_2 + ts_3) + \ldots = \sum_{i=1}^{k} (ts_i) \tag{30}$$

Let

$$PSI = A^T(k,0)A(k,0) \tag{31}$$

then

$$PSI = \begin{bmatrix} p & 0 & q & 0 \\ 0 & p & 0 & q \\ q & 0 & k & 0 \\ 0 & q & 0 & k \end{bmatrix} \tag{32}$$

Same procedure can be used to find out $A^T(k,0)Z(k)$. It is obtained by using eqn. (27) and eqn. (21)

$$A^T(k,0)Z(k)=\begin{bmatrix} \sum_{i=1}^{k} ts_i \cos B_m(i)z'(i)+\sum_{i=1}^{k} ts_i \sin B_m(i)z''(i) \\ \sum_{i=1}^{k} -ts_i \sin B_m(i)z'(i)+\sum_{i=1}^{k} ts_i \cos B_m(i)z''(i) \\ \sum_{i=1}^{k} \cos B_m(i)z'(i)+\sum_{i=1}^{k} \sin B_m(i)z''(i) \\ \sum_{i=1}^{k} -\sin B_m(i)z'(i)+\sum_{i=1}^{k} \cos B_m(i)z''(i) \end{bmatrix} \tag{33}$$

Let

$$G = A^T(k,0)Z(k) \tag{34}$$

Let us consider eqn. (22) again,

$$\hat{X}_t(0,k)=\left[A^T(k,0)A(k,0)\right]^{-1}A^T(k,0)Z(k)$$

Using eqn. (31) and eqn. (34), eqn. (22) can be written as

$$\hat{X}_t(0,k)=[PSI]^{-1}[G] \tag{35}$$

III. SEQUENTIAL IMPLEMENTATION OF PSI AND G MATRICES

So far the mathematical modelling describes batch processing. In this section, the equations are converted into sequential mode equations as follows. The effect of all the range and bearing measurements in each element of G matrix is maintained in the form of Recursive Sums. Whenever new range and bearing measurements are available, only calculations pertaining to the newly arrived measurements are to be carried out and added to the Recursive Sums. Let T represent the total time elapsed from obtaining first measurement from sonar up to the availability of k^{th} measurement and is given by

$$T = ts_1 + ts_2 + ts_3 + \ldots + ts_k \tag{36}$$

Let $B_m(k)$ and $R_m(k)$ be k^{th} bearing and range measurements respectively. At the starting of the trial, the ownship is assumed to be at $(0,0)$, the origin of x-y coordinate system. Let $x_0(k)$ and $y_0(k)$ be x and y components of ownship's position at k^{th} measurement. The elements of G matrix shown in eqn. (34) are converted into Recursive Sums as follows. Let the Recursive Sums be $SUMS[1]$ to $SUMS[8]$. After obtaining first measurement, these are given by

$$\begin{aligned}
SUMS[1]_1 &= \cos B_m(1)z'(1) \\
SUMS[2]_1 &= -\sin B_m(1)z'(1) \\
SUMS[3]_1 &= T\cos B_m(1)z'(1) \\
SUMS[4]_1 &= -T\sin B_m(1)z'(1) \\
SUMS[5]_1 &= \cos B_m(1)z''(1) \\
SUMS[6]_1 &= \sin B_m(1)z''(1) \\
SUMS[7]_1 &= T\cos B_m(1)z''(1) \\
SUMS[8]_1 &= T\sin B_m(1)z''(1)
\end{aligned} \tag{37}$$

After obtaining k^{th} measurement, these are

$$\begin{aligned}
SUMS[1]_k &= \cos B_m(k) z'(k) + SUMS[1]_{k-1} \\
SUMS[2]_k &= -\sin B_m(k) z'(k) + SUMS[2]_{k-1} \\
SUMS[3]_k &= T \cos B_m(k) z'(k) + SUMS[3]_{k-1} \\
SUMS[4]_k &= -T \sin B_m(k) z'(k) + SUMS[4]_{k-1} \\
SUMS[5]_k &= \cos B_m(k) z''(k) + SUMS[5]_{k-1} \\
SUMS[6]_k &= \sin B_m(k) z''(k) + SUMS[6]_{k-1} \\
SUMS[7]_k &= T \cos B_m(k) z''(k) + SUMS[7]_{k-1} \\
SUMS[8]_k &= T \sin B_m(k) z''(k) + SUMS[8]_{k-1}
\end{aligned} \tag{38}$$

The G matrix at kth measurement can be written as

$$G = \begin{bmatrix} SUMS[3]_k + SUMS[8]_k \\ SUMS[4]_k + SUMS[7]_k \\ SUMS[1]_k + SUMS[6]_k \\ SUMS[2]_k + SUMS[5]_k \end{bmatrix} \tag{39}$$

Once $X_t(0,k)$ is calculated, the state vector corresponding to the current measurement can be found using transient matrix. The range, course, bearing and speed of the target are calculated using the current state vector.

IV. TRACKING OF A MANOEUVRING TARGET

So far, it is assumed that target is not manoeuvring. In this section, this assumption is relaxed. Let us assume that the target moves at constant velocity with occasional manoeuvre as shown in Table 1. This problem can be easily solved by using fifteen to twenty measurements with sliding window technique. By sliding out the oldest sonar measurements, its corresponding own position, the array of SUMS are updated accordingly. This helps in maintaining the computations related to the latest 'N' sets of measurements where 'N' is the predefined window size of measurements. This is implemented by maintaining the record of all the necessary parameters in arrays data structure. The length of the measurements is decided by the accuracy of the results obtained in Monte-Carlo simulation.

V. SIMULATION AND RESULTS

The algorithm is realized using Matlab on a PC platform. The positions of target and ownship are updated every second. In general, the errors allowed in the estimated target motion parameters in an underwater environment for weapon control are 8% in range, 3° in course and 1 m/s in speed. These accuracies cannot be obtained with deterministic method using two sets of range and bearing measurements spread over time interval even 80 seconds (details are given in Appendix–A). Hence stochastic estimator like PLE is necessary to estimate the target motion parameters. The measurements with additive measurement noise, available to the PLE are according to range scales. The scenarios considered are shown in Table 1. In scenarios 1 and 2, target moves at constant velocity for a period of 240 seconds and then it manoeuvres in course at the rate of 3°/s. Let the noise in the bearing and range measurements are white Gaussian and their values are shown in Table 1. PLE is used to estimate target motion parameters. The simulated and estimated target paths for scenario 1 & 2 are shown in Fig. 1 & 2 respectively. For the purpose of comparison, the measurements are also applied to the Extended Kalman Filter. The results of scenarios in Monte-Carlo simulation with 100 runs are shown in Fig. 3 & 4.

In case of PLE, for scenario 1, it was observed that the required accuracies are obtained in estimated target course and speed after 5th and 3rd measurements respectively. Once the target manoeuvres, the estimated solution is disturbed. The target manoeuvre is completed around 270 seconds. Upon the target manoeuvre, the error in estimated course is increased and reduced to within limits from 35th measurement onwards. There is very small disturbance in estimated target speed during target manoeuvre. The error in estimated range is acceptable from the beginning of the trial. In case of scenario 2, the required accuracies in estimated course and speed are obtained from 3rd measurement onwards. Upon the target manoeuvre, the error in estimated course is increased and reduced to within limits from 27th measurement onwards.

In case of EKF, for scenario 1, it was observed that the required accuracies are obtained in estimated target course and speed after 6th and 3rd measurements respectively. Once the target manoeuvres, the estimated solution is disturbed. The target manoeuvre is completed around 270 seconds. Upon the target manoeuvre, the error in estimated course is increased and reduced to within limits from 37th measurement onwards. There is very small disturbance in estimated target speed during target manoeuvre. The error in estimated range is acceptable from the beginning of the trial. In case of scenario 2, the required accuracies in estimated course and speed are obtained from 7th measurement onwards. Upon the target manoeuvre, the error in estimated course is increased and reduced to within limits from 33rd measurement onwards. The required accuracies are obtained faster in PLE than in EKF.

Parameters	Scenario 1	Scenario 2
Initial Range (m)	5000	4500
Initial Bearing (deg)	55	240
Target Speed (m/s)	2.5	3
Target Course (deg)	250 to 180 at 240 seconds	150 to 90 at 240 seconds
Ownship Speed (m/s)	3	3
Ownship Course (deg)	90	180
Error in Range (rms)	10	10
Error in Bearing (rms)	0.5	0.5

Table 1: Scenarios chosen for evaluation of algorithm

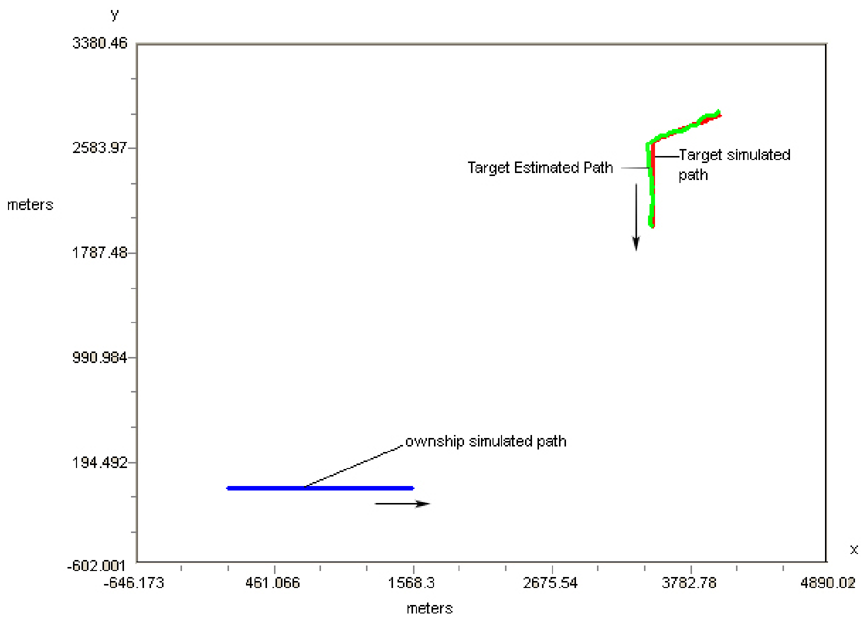

Fig. 1 Simulated and Estimated Target Paths of Scenario 1

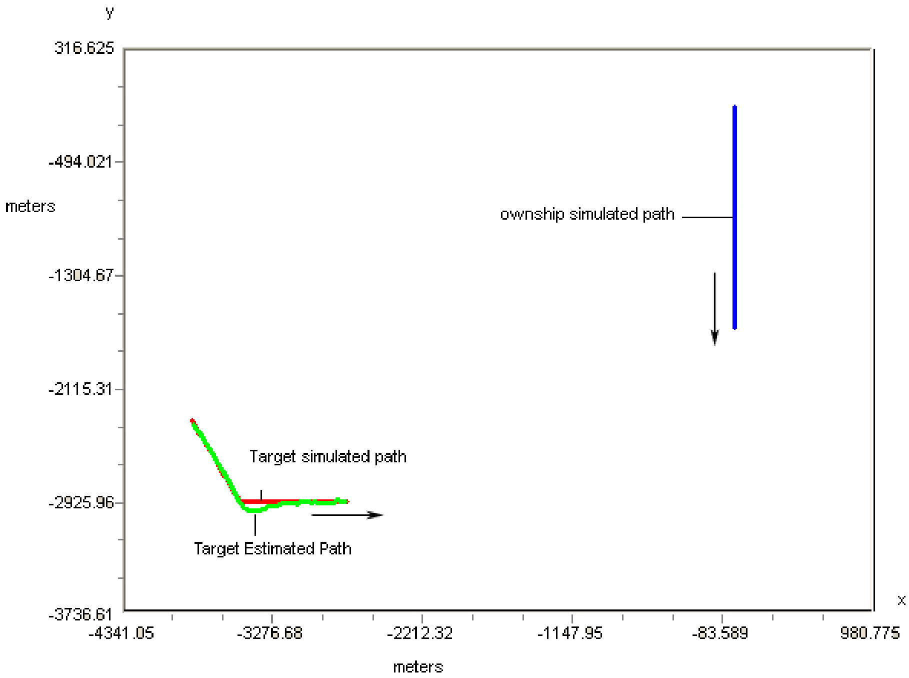

Fig. 2 Simulated and Estimated Target Paths of Scenario 2

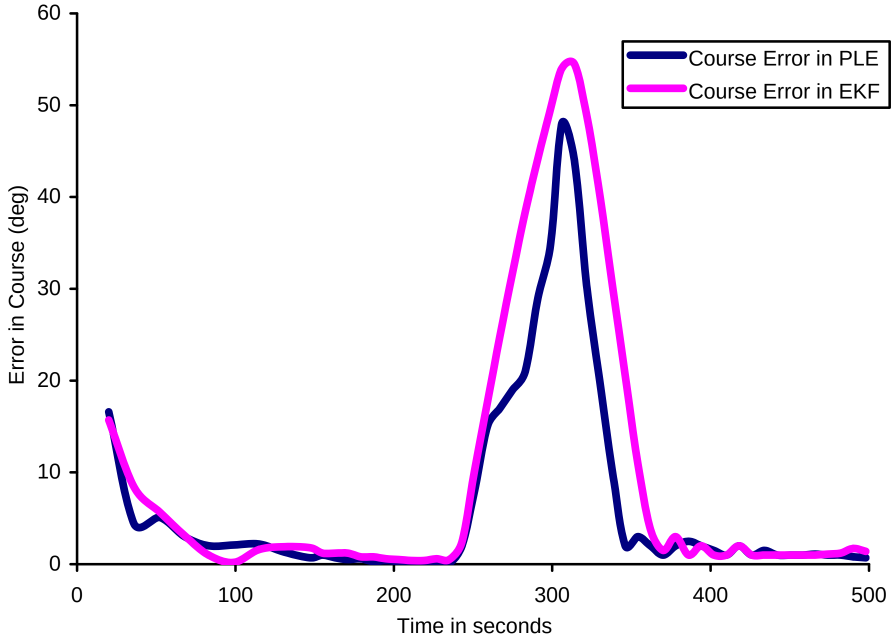

Fig. 3(a) Error in course estimate for scenario 1

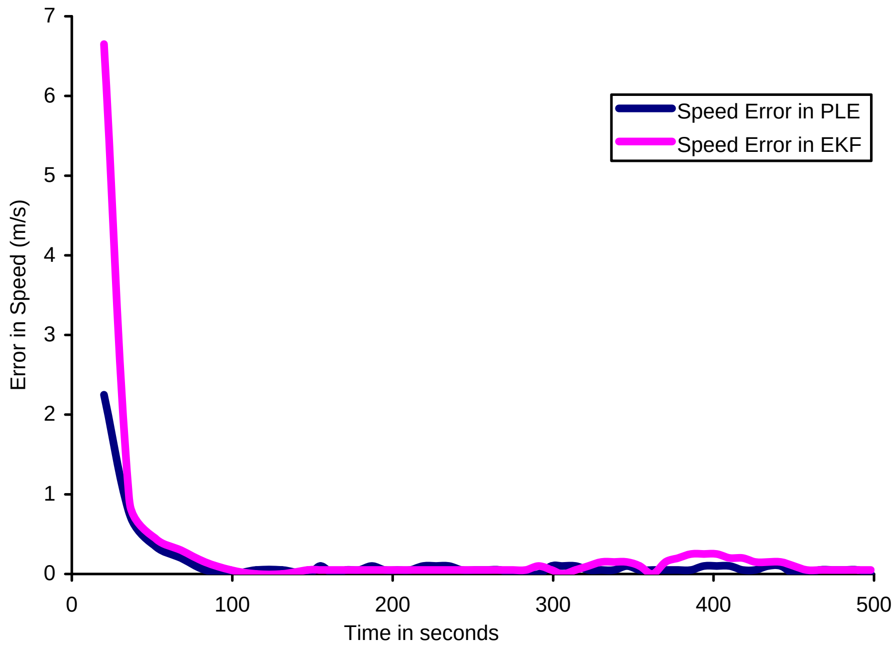

Fig. 3(b) Error in speed estimate for scenario 1

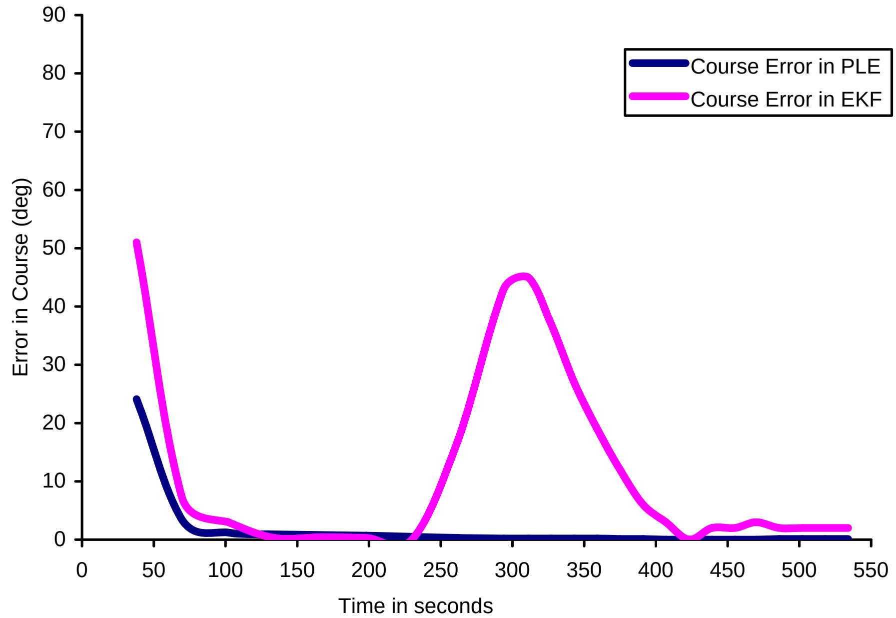

Fig. 4(a) Error in course estimate for scenario 2

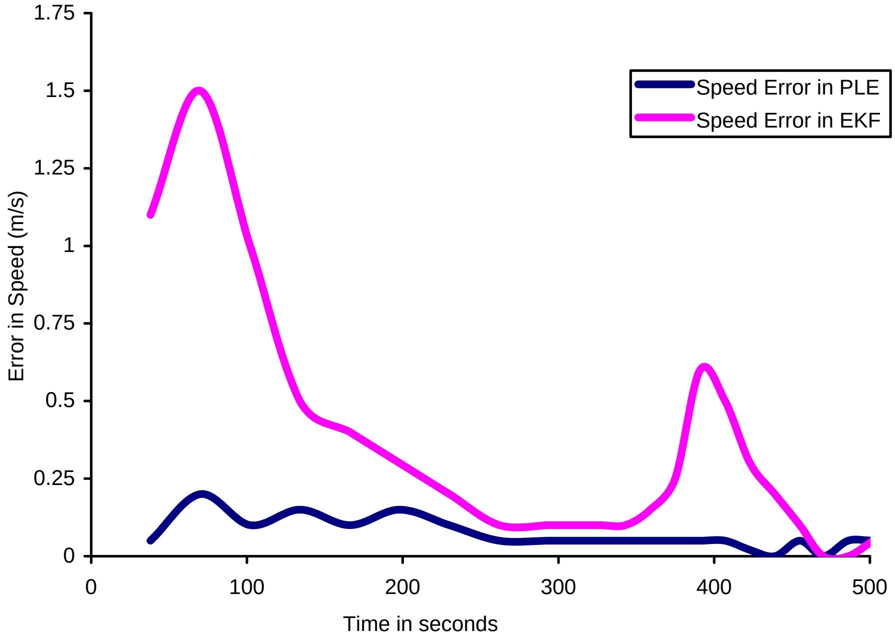

Fig. 4(b) Error in speed estimate for scenario 2

VI. LIMITATIONS OF THE ALGORITHM

The only limitation of the algorithm is that it cannot give the validity of the solution i.e., how far the solution is accurate, as the statistical characteristics of the noise in the input measurements are not considered in PLE. It is assumed that all measurements are correct. Due to the reverberations, the active sonar measurements may contain a number of outliers which can completely distort the solution when using the proposed method. This problem can be resolved if the measurements are available along with its variance to PLE. Then the measurements are weighted against its variance in Recursive Sums. If reverberation is more, variance of the error in the measurement is more, thereby; the incremental value of the SUMS with that measurement will be less. The Recursive Sums shown in eqn. (36) and (37) are modified as follows. After obtaining first measurement, the Recursive Sums, SUMS [1] to SUMS [8] are given by

$$\begin{aligned}
SUMS[1]_1 &= \cos B_m(1) z'(1) / \sigma_B^2(1) \\
SUMS[2]_1 &= -\sin B_m(1) z'(1) / \sigma_B^2(1) \\
SUMS[3]_1 &= T\cos B_m(1) z'(1) / \sigma_B^2(1) \\
SUMS[4]_1 &= -T\sin B_m(1) z'(1) / \sigma_B^2(1) \\
SUMS[5]_1 &= \cos B_m(1) z''(1) / \sigma_B^2(1) \\
SUMS[6]_1 &= \sin B_m(1) z''(1) / \sigma_B^2(1) \\
SUMS[7]_1 &= T\cos B_m(1) z''(1) / \sigma_B^2(1) \\
SUMS[8]_1 &= T\sin B_m(1) z''(1) / \sigma_B^2(1)
\end{aligned} \tag{40}$$

After obtaining k[th] measurement,

$$
\begin{aligned}
SUMS[1]_k &= \cos B_m(k) z'(k)/\sigma_B^2(k) + SUMS[1]_{k-1} \\
SUMS[2]_k &= -\sin B_m(k) z'(k)/\sigma_B^2(k) + SUMS[2]_{k-1} \\
SUMS[3]_k &= T\cos B_m(k) z'(k)/\sigma_B^2(k) + SUMS[3]_{k-1} \\
SUMS[4]_k &= -T\sin B_m(k) z'(k)/\sigma_B^2(k) + SUMS[4]_{k-1} \\
SUMS[5]_k &= \cos B_m(k) z''(k)/\sigma_B^2(k) + SUMS[5]_{k-1} \\
SUMS[6]_k &= \sin B_m(k) z''(k)/\sigma_B^2(k) + SUMS[6]_{k-1} \\
SUMS[7]_k &= T\cos B_m(k) z''(k)/\sigma_B^2(k) + SUMS[7]_{k-1} \\
SUMS[8]_k &= T\sin B_m(k) z''(k)/\sigma_B^2(k) + SUMS[8]_{k-1}
\end{aligned}
\tag{41}
$$

where

$$
\begin{aligned}
z'(k) &= x_0(k)\cos B_m(k)/\sigma_B^2(k) - y_0(k)\sin B_m(k)/\sigma_B^2(k) \\
z''(k) &= x_0(k)\cos B_m(k)/\sigma_B^2(k) + y_0(k)\sin B_m(k)/\sigma_B^2(k) + R_m(k)/\sigma_R^2(k)
\end{aligned}
\tag{42}
$$

and $\sigma_B^2(k)$ is variance of the error in k^{th} bearing measurement

$\sigma_R^2(k)$ is variance of the error in k^{th} range measurement

Many times, the variances of the errors in bearing & range measurements are not available. Hence the following method is adopted, though it is not 100% error free. The maximum relative velocity between the target and ownship is 13 m/s (with submarine speed of 3 m/s and ownship speed of 10 m/s). If target is at 4000 meters, the measurement interval between two measurements is not more than 6 seconds (8000 m / 1500 m/s). So, the range separation between the target and ownship within two measurements is 6*13 = 78 meters. Let at certain point of time, the range measurement be 3.2 km, the next range measurement should be within the magnitude of 3.2 ± 78 m. If it is not, then it is to be considered as invalid measurement. As range and bearing come together if range measurement is invalid, then corresponding bearing measurement was also invalid. Thus, there is no requirement of separate bearing gate for false bearing measurements. In this way, outlier problem up to certain extent is reduced.

The algorithm is easily extended to multi target tracking also as follows. The sonar measurements for each target with a particular identification number are passed on to the processing system. Concurrently multiple instances of PLE module are run on data of separate target tracks. Standard software techniques are used to realize the same.

VII. SUMMARY & CONCLUSION

In this paper, PLE algorithm developed by S. C Nardone, A.G. Lindgren and K. F. Gong [10] and S. K. Rao [11] is extended with sequential processing to reduce mathematical complexity and the memory requirements in tracking of a moving target. Here Recursive Sums are introduced and updated whenever a new bearing and range measurement is available. The algorithm works in a closed loop. The estimated target state vector at any instant is used to calculate the target motion parameters at that instant and updates the Recursive Sums for improved solution at next instant. The algorithm is evaluated against hundreds of scenarios in Monte-Carlo simulation and observed that convergence is obtained within 10 samples when target is at constant velocity. The algorithm is extended for tracking manoeuvring targets also. The results are presented for two scenarios using measurements from active sonar and the performance of this algorithm is compared with that of EKF. It was noticed that PLE generates required accurate solution faster. The estimated target motion parameters are useful to find out weapon preset parameters to release weapon onto the target. It is well known that PLE generates bias in the estimated range, particularly at long range scenarios. The effect of the bias is negligible, as weapons are highly sophisticated with homing capabilities. Therefore it is concluded that this algorithm finds a place for underwater active target tracking applications.

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APPENDIX-A

Using two sets of range and bearing measurements and the time difference between them, the target course, θ is given by

$$\theta = \tan^{-1}\sqrt{(P/Q)} \tag{A.1}$$

where

$$P = V_0\, ts \sin\psi + R(2)\sin B(1) - R(1)\sin B(2) \tag{A.2}$$

and

$$Q = V_0\, ts \cos\psi + R(2)\cos B(1) - R(1)\cos B(2) \tag{A.3}$$

where V_0 and ψ are ownship speed and target course respectively. Target speed is given by

$$V_t = \sqrt{\left(P^2 + Q^2\right)} \tag{A.4}$$

Let us consider scenario 1 in Table 1 without target manoeuvre. The positions of target and ownship are updated every second. The measurements with additive measurement are assumed to be available to the following deterministic method. Using two sets of range and bearing measurements spread over on time interval of 80 seconds, the target course and speed are calculated. The results obtained in Monte Carlo simulation are shown in Fig A.

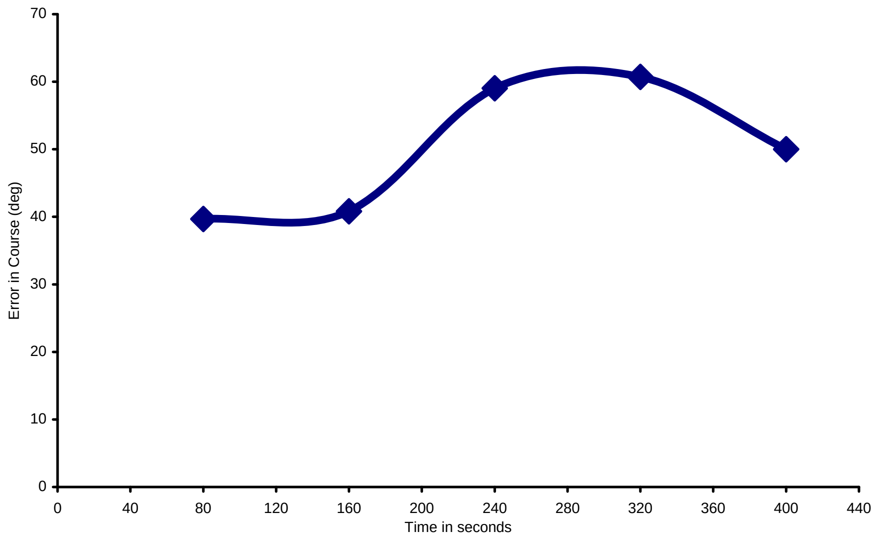

Fig. A.1(a) Error in course

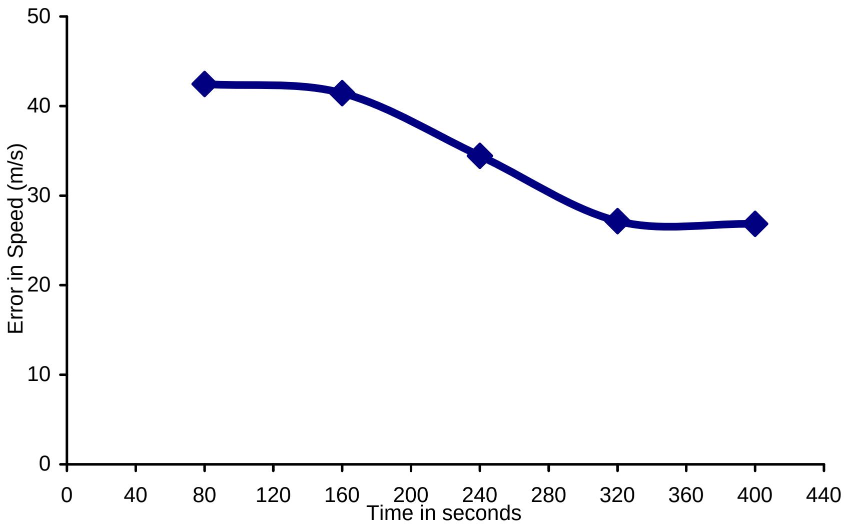

Fig. A. 1(b) Error in speed

From the results, it is clear that the required accuracies cannot be obtained with this deterministic process when the measurements are corrupted with the noise as shown in Table 1 and hence stochastic filtering is necessary to estimate target motion parameters.