# Probabilistic approach for the selection of the shallow foundation's safety factor

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*Abstract*— **In this paper, we present a new approach for estimating the risk of foundation's failure of which the design was made using the conventional safety factor. We also present the results of numerical simulations applied to a continuous spread footing foundation based on a grainy dry soil where the geotechnical characteristics are a random spatial variables. Indeed, we show that for the soil which has a high natural variability the probability of failure associated with the standard safety factor becomes important then, in this case, a good choice of the safety factor is necessary.**

**Keyword-** Shallow foundations, Safety factor, Natural soil's variability, Risk of failure, Probabilistic method.

## I. INTRODUCTION

Since the middle of the last century, geotechnical engineering developments places the control of geotechnical risks as a priority. Some authors illustrate this desire [1, 2, 3, 4 and 5]. In parallel, the complexity of projects, through the goals "cost / time / performance", is increasing and the lands chosen to receive them have, in the majority of cases, difficult geotechnical conditions. These unfavorable conditions could result from a high variability of soil properties and could make their recognition and analysis very complex which may be the source of further damage as shown in Fig.1.1. Our research work fall under the geotechnical risk control and it focus on the question of the natural soil variability effects on the stability of shallow foundations. Our works are based on the application of probability and statistics theories to assess and quantify those effects.

Fig.1.1. Illustration of the foundations's failure

The first reflections on the applicability of probabilities and statistics in the geotechnical engineering date from the year 1960. At that time the focus was on the relationship between parameters analysis, what is commonly called the study of correlations with the objective to facilitate geotechnical studies for the structures designing. Various statistical analyzes were undertaken at that time, some results can be found in the articles of Amar, Bagulin and Jézéquel [6, 7] and in the works of Magnan and Baghery [8, 9, 10, 11 and 12] and in the research report of Vidalie [13].

## II. METHODOLOGY

The calculation of the bearing capacity of shallow foundations from cohesion and internal friction coefficient is among the most well – known problem of the soil mechanics and all manuals make extensive references. The work of Bowles, Frank, Hansen, Magnan, Meyrhof, Genevois and Versic [14, 15, 16, 17, 18, 19 20 and 21] are good examples. For estimating the bearing capacity, we generally use the following formula:

$$Q_c = \frac{\gamma B}{2}\left(1 - 0{,}4\frac{B}{L}\right) N_\gamma(\emptyset) + \sigma_{v_0} N_q(\emptyset) + \left(1 + 0{,}2\frac{D}{B}\right)\left(1 + 0{,}2\frac{B}{L}\right) c\, N_c(\emptyset)$$

As mentioned, the study focuses on the continuous spread footing foundations based on a dry powdery soil. In this case we have:

$$Q_c = \left[\frac{\gamma B}{2} N_\gamma(\emptyset)\right]$$

The few published studies on this subject (Magnan and Baghery [8, 9, 10, 11 and 12]) had shown that it is essential to take into account the scales of soil variability if we want to make a realistic estimating. Indeed, the probabilities obtained are much too high to be realistic (about 30%). In this work we propose a new approach for the calculating of the foundations failure probability. Basing on the exploitation of the analytical properties of the bearing capacity coefficients (Fig.2.1), our method, as we will illustrate in the next sections, reduce significantly the discordance between the estimated failure probabilities and the realistic failure probabilities without tacking in account the scales of soil properties variability.

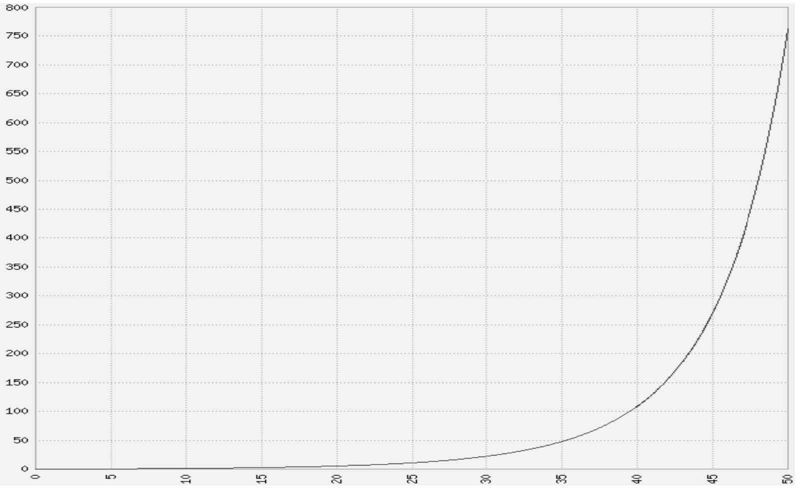

Fig.2.1 Curves of the coefficient of bearing capacity $N_\gamma(\emptyset)$

Fig.2.1 shows that $N_\gamma$ varies continuously (in the analytical sense) with $\emptyset$ and following a strictly increasing monotonicity which means that $N_\gamma$ is a bijection. Then the foundation failure condition could be translated this way:

$$F_p = Probability\ [\ Q_c^* < \frac{Q_c}{Fs}\ ].$$

Where,

$Q_c^*$ : Real bearing capacity.

$Q_c$ : Estimated bearing capacity

Fs : Safety factor which is in general equal to 3

Then,

$$\mathrm{F_p} = \mathrm{Probability}\left[\left(\mathrm{Q_c}^* = \frac{\gamma^* \mathrm{B}}{2} \mathrm{N_\gamma}(\emptyset^*)\right) < \left(\frac{\mathrm{Q_c}}{\mathrm{Fs}} = \frac{\gamma_\mathrm{m} \mathrm{B}}{6} \mathrm{N_\gamma}(\emptyset_\mathrm{m})\right)\right]$$

So,

$$\mathrm{F_p} = \mathrm{P}\left[\mathrm{Q_c}^* < \frac{\mathrm{Qc}}{3}\right] = \mathrm{P}\left[\gamma^* \mathrm{N_\gamma}(\emptyset^*) < \frac{\gamma_\mathrm{m} \mathrm{N_\gamma}(\emptyset_\mathrm{m})}{3}\right]$$

If we note "e" as a nonzero real number, we obtain:

$$\mathrm{F_p} = \lim_{e \to 0}\left(\mathrm{P}\left[\gamma^* < \frac{\gamma_\mathrm{m}}{\mathrm{e}}\right] \mathrm{P}\left[\mathrm{N_\gamma}(\emptyset^*) < \frac{\mathrm{e}}{3} \mathrm{N_\gamma}(\emptyset_\mathrm{m})\right] + \mathrm{S(e)}\right)$$

Where,

$$\mathrm{S(e)} = \sum_{k=2}^{\infty}\left(\mathrm{P}\left[\gamma^* < \frac{\gamma_\mathrm{m}}{\mathrm{k.e}}\right]\left(\mathrm{P}\left[\mathrm{N_\gamma}(\emptyset^*) < \mathrm{k}\frac{\mathrm{e}}{3}\mathrm{N_\gamma}(\emptyset_\mathrm{m})\right] - \mathrm{P}\left[\mathrm{N_\gamma}(\emptyset^*) < (\mathrm{k}-1)\frac{\mathrm{e}}{3}\mathrm{N_\gamma}(\emptyset_\mathrm{m})\right]\right)\right)$$

Let's define "$Ø_k$" such as:

$$N_\gamma(Ø_k) = k\,\frac{e}{3}\,N_\gamma(Ø_m)$$

Then we obtain the final formula for the failure probability

$$F_p = \lim_{e\to 0}\left(P\left[\gamma^* < \frac{\gamma_m}{e}\right]P[Ø^* < Ø_1] + S(e)\right)$$

Where,

$$S(e) = \sum_{k=2}^{\infty}\left(P\left[\gamma^* < \frac{\gamma_m}{k.e}\right](P[Ø^* < Ø_k] - P[Ø^* < Ø_{k-1}])\right)$$

## III. RESULTS AND DISCUSSIONS

In this section, we visualize the numerical simulations results for the failure probabilities calculations.

### *A. Results*

Fig.3.1 shows the non linear increasing of the failure probability related to $\sigma_1$and also her insensitivity according to the variability $\sigma_2$ of the soil density. In the other hand, as we said before, the values of failure probabilities obtained are more realistic than those obtained using the older methods. This difference between results can be explained by the fact that in our approach we didn't assume hypothesis concerning the distribution of the values of $N_\gamma(Ø)$.

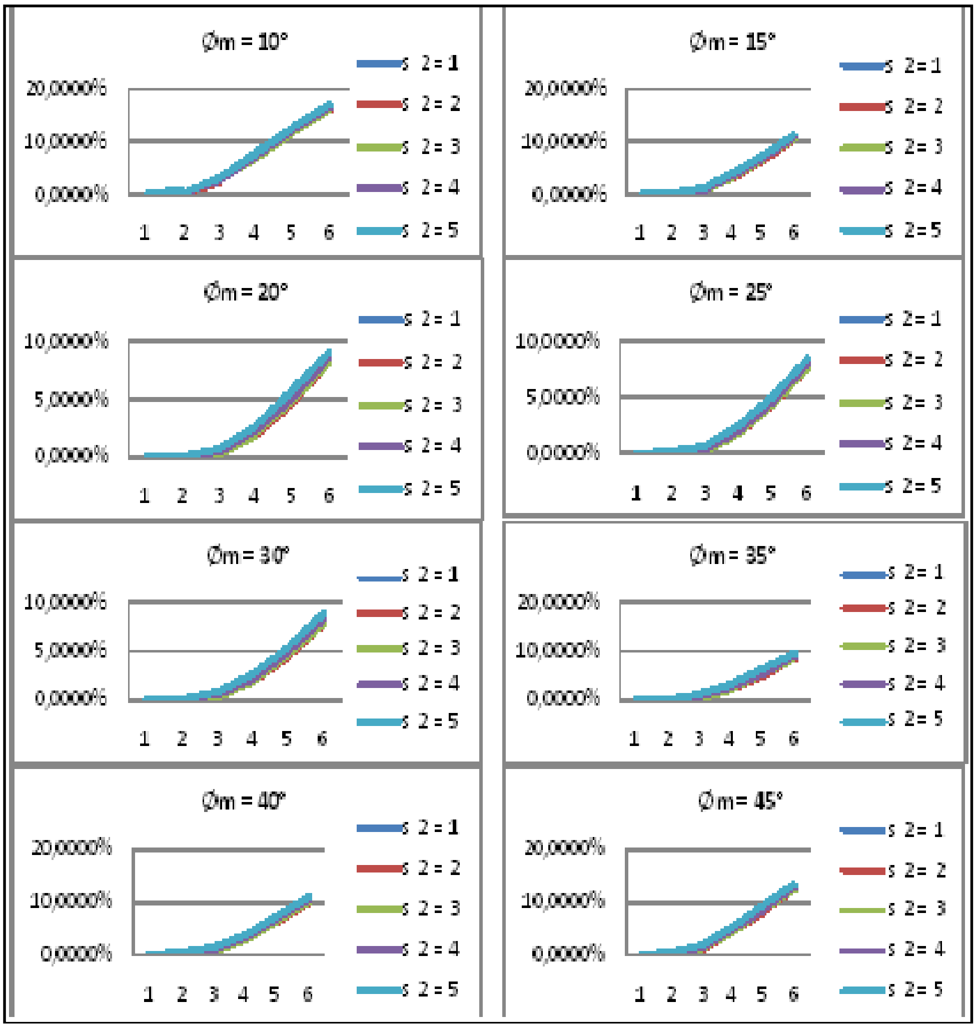

Fig.3.1 Curves representing the failure probability as a function of $\sigma_1$ and $\sigma_2$ for different values of $Ø_m$,

In the other hand, Fig.3.2 shows that the failure probability follows a convex curves as a function of the average value $Ø_m$. We also note that the failure probability admits, independently of $\sigma_1$ and $\sigma_2$, the same minimum value in $Ø_m$ = 25°.

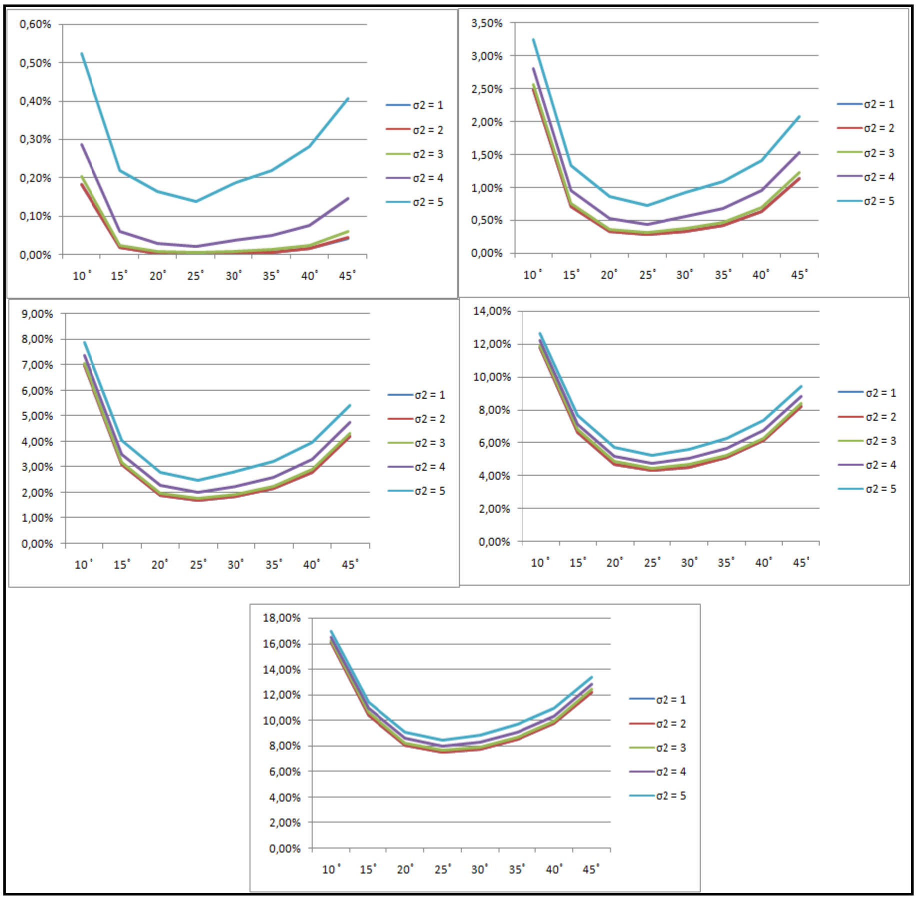

Fig.3.2. Curves representing the failure probability as a function of $\varnothing_m$ and $\sigma_2$ for different values of $\sigma_1$

Fig.3.3 gives, for different values of $\sigma_1$, the curves representing the failure probability as a function of $\varnothing_m$.

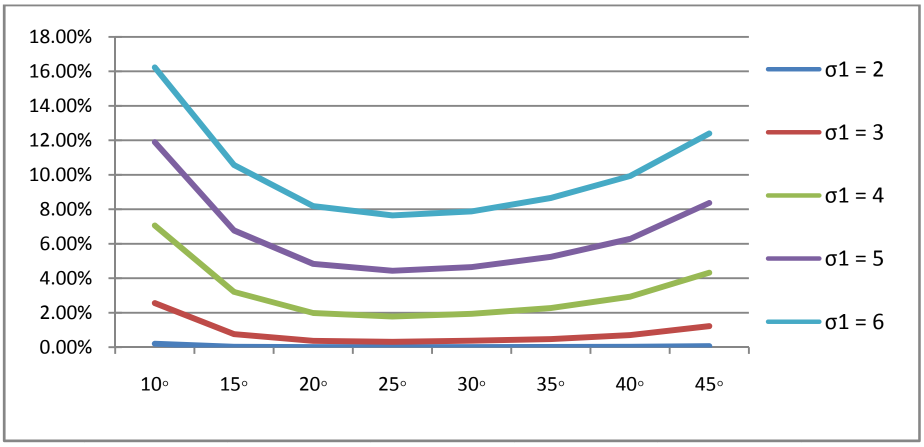

Fig.3.3. Curves representing the failure probability as a function of $\varnothing_m$ for different values of $\sigma_1$

### *B. Discussions*

Results show that the failure probability depends on the level of soil variability in particular that of the internal friction coefficient Ø. We have :

- For soils with low variability, the choice of safety factor Fs is justified by the fact that the failure is improbable.
- For soils with medium variability, the failure probability takes values which, depending on $\varnothing_m$, varies from 0 to 5%. This corresponds to a relatively low risk of failure.
- For soils with high variability, the risk of failure becomes important with values of approximately 10 – 20% and therefore the safety factor Fs = 3 is not adapted to this situation.

TABLE I. ABSTRACT FOR THE RESULTS

| Soil with | Failure Probability | Comments |
|---|---|---|
| Low Variability | 0 % | Improbable Failure |
| Medium Variability | About 5 % | Moderate Probability |
| High Variability | About 10 to 20 % | High Probability |

The fact that the failure probability has a minimum in $\text{Ø}_m$= 25° can be seen as a consequence of the variations mode of the bearing capacity coefficient $N_\gamma(\text{Ø})$. In fact, as illustrated by Fig.2.1, for $\text{Ø} \geq 25°$, the monotoncity of $N_\gamma(\text{Ø})$ becomes more and more pronounced (positive acceleration). That means, for a small decreasing $\Delta\text{Ø}$ corresponds a big drop $\Delta N_\gamma$, which logically increases the value of the failure probability. On the other hand, for the values $\text{Ø} \leq 25°$, the failure probability grows, despite small variations of $N_\gamma$ in this area of values, which can be justified by the ratio between $\text{Ø}_m$ and $\sigma_1$, which, becoming very important in this area of values, increases the values of the failure probability. So for $\text{Ø} = 25°$, all favorable conditions come together to give low values to the failure probability.

## IV. CONCLUSION

The method presented provided realistic estimates of the risk of foundations failure and shows the interest of taken into account the levels of the natural soil variability in the selection of safety factors especially in the case of soils with high natural variability. Therefore, this method can be integrated in the foundation design process in order to control the geotechnical risks and uncertainties.

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## REFERENCES

[1] Tbatou. T, Rougui. M., El youbi. M, (2014), The deterministic assessment of the effect of the soil-structure interaction on a vulnerable impact of a reinforced concrete building | L'évaluation déterministe de l'effet de l'intéraction sol-structure sur l'impact vulnérabiliste d'un bâtiment en béton armé, Journal of Materials and Environmental Science 5 (6), pp. 1951

[2] Baba.K, Bahi.L, Ouadif.L, Akhssas.A, (2012), Slope Stability Evaluations by Limit Equilibrium and Finite Element Methods Applied to a Railway in the Moroccan Rif, Open Journal of Civil Engineering, 2 (01), pp 27-32

[3] Nehab. N, Baba. K, Ouadif. L, Cherradi. C, Bahi. L, (2014), Soft soil strengthening by stone columns: case of the embankment under the bridge "Moulay Youssef"(Rabat/Salé) http://dx.doi.org/10.1051/matecconf/20141102012

[4] Ouadif. L, Bahi. L, Baba. K, Akhssas. A, (2012), Study of Stability of a Highway Fill Application of the Analytical and the Finite Elements Methods, International Journal of Engineering Research and Applications (IJERA) Vol. 2, Issue 1, Jan-Feb 2012, pp. 904-910

[5] Akhssas. A, Bahi. L, Lahmili. A, Ouaif. L, (2011), Study of the stability of rock slope, Canadian Journal on Scientific and Industrial Research 2 (2), pp. 35-39.

[6] Amar. S, Baguelin. F, Jezequel J. – F. (1973), Utilisation des corrélations. Documents de la Table ronde sur les corrélations des paramètres en mécanique des sols. École centrale des Arts et Manufactures, Paris, pp. 1 – 39 à 1 – 44.

[7] Amar. S, Jezequel J. – F. (1972), Essais en place et en laboratoire sur sols cohérents. Comparaisons des résultats. Bull, liaison Labo. P. et Ch., Paris, 58, 12 pages.

[8] Magnan J.-P., Baghery S. ( 1982b), Statistiques et probabilités en mécanique des sols. Etat des connaissances. Laboratoire Central des Ponts et Chaussées, Rapport de recherche LPC, 109, 187 pages.

[9] Magnan J - P. ( 1986), Approche probabiliste du calcul des fondations. Comptes rendus, Séminaire CLAROM « Approche probabiliste et applications aux ouvrages en mer», Pau, oct., 13 pages.

[10] Magnan J - P. (1982), Les méthodes statistiques et probabilistes en mécanique des sols. Presses de l'École Nationale des Ponts et Chaussées, Paris, 203 pages.

[11] Magnan J - P. (1988), Place des analyses probabilistes dans la pratique des projets de géotechnique. Comptes rendus, Symposium sur la conception des ouvrages de génie civil basée sur la fiabilité (Symposium on reliability – based design in civil engineering), École Polytechnique Fédérale, Lausanne, pp. 303 – 310.

[12] Magnan J - P. (1989), Variabilité des propriétés des sols et dimensionnement des ouvrages. Comptes rendus, 12ieme Congrès international de Mécanique des sols et des travaux de fondations. Rio-de-Janeiro, vol. 2, pp. 853 – 854.

[13] Vidalie J – F. (1977), Relation entre les propriétés physico – chimiques et les caractéristiques mécaniques des sols compressibles. Laboratoire central des Ponts et Chaussées, Paris, Rapport de recherche LPC, 65.

[14] Bowels J – E. (1996) . Foundation Analysis and Design. 5ieme Ed, Mc Graw – Hill book Company, New York.

[15] Frank. R. (1999). Calcule des fondations superficielles et profondes. Presse de l'Ecole Nationale des Ponts et Chaussées, Paris.

[16] Hansen J – B (1970). A Revised and Extended Formula for Bearing Capacity. Danish Geotechnical Institute. Bulletin No. 28, p21.

[17] Meyerhof G – G.(1951). The Ultimate Bearing Capacity of Fondations. Geotechnique, 2(4) : 301 – 332.

[18] Meyerhof G – G. (1965). Shallow Foundation ASCE J. Soil Mech. Found. Div., 91 (SM2) : 21 – 31.

[19] Versic A - S. (1973). Analysis of Ultimate Loads of Shallow Foundations ASCEJ.

[20] Magnan J – P. – Résistance au cisaillement. C 216 (1991). Traité Construction, volume C 2I.

[21] Genevois B. (1984), Rôle de la variabilité spatiale des paramètres de sol sur la capacité portante des fondations superficielles. Thèse de Docteur – ingénieur, École centrale des Arts et Manufactures, Chatenay-Malabry, 144 pages.

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