Robust MRI Brain Image Segmentation Method: A Hybrid Approach using Level Set and Fuzzy C-Means Clustering

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Abstract— **Advances in medical imaging technologies have given rise for effective diagnostic procedures. The acquisition promptness and resolution enhancements of imaging modalities have given physicians more information, less invasively about their patients. Active contours are used to segment, match and track images of an atomic structure by manipulating constraints derived from the image data together with prior knowledge about the location, size, and shape of these structures. The level set method is referred as a part of active contour family. The major disadvantages of level set method are initialization of controlling parameters and time complexity. The proposed method adopts Robust Spatial Kernel Fuzzy C-Means (RSKFCM) and Lattice Boltzmann Method (LBM) to overcome these drawbacks. RSKFCM is based on standard Fuzzy C-Means algorithm which uses Gaussian RBF kernel function as distance metric and incorporates spatial information. The LBM uses the energy function to determine and reduce the actual processing time which addresses the time complexity. The proposed system combines both RSKFCM and LBM to form a hybrid approach, and the system is tested on a large set of MRI brain images and the experimental results are found to be improved with respect to time complexity.**

Keywords- Fuzzy C-Means, Spatial Fuzzy C-Means, Kernel Fuzzy C-Means, Spatial Kernel Fuzzy C-Means, Robust Spatial Kernel Fuzzy C-Means, Level Set Method, Lattice Boltzmann Method

I. INTRODUCTION

Medical imaging has undergone advancement in the previous decade [1]. The process of medical imaging involves representation of the body's interior organs, which helps in medical examination and other medical purposes. There are many techniques for medical imaging at present like X-ray, CT's and MRI [2]. The data produced through these methods are applied in different areas of medical analysis like molecular imaging, nanotechnology etc.

Segmentation is the core part in medical image processing. Image segmentation is the process in which an image is subdivided into its constituent parts. Segmentation process can be performed based on observing sets of pixels whereby pixels representing a region can be identified according to some homogeneity criteria like color, texture or intensity so as to locate and identify boundaries in an image. Segmentation of medical images is complex due to several reasons such as characteristics of imaging modality, geometry of anatomy, partial volume effect, presence of artefacts and noise. Applications of segmentation in medical image processing are to locate tumors and other pathologies. Through segmentation, other processes like boundary analysis, intensity detection and abnormality identification are carried out. These processes allow a physician's task of diagnosing the stage of the tumor to be much easier.

In the Literature survey that we have done, we could find that many researchers proposed segmentation methods. Traditional segmentation methods can be classified into three categories: threshold based [3] region based and edge based. However poor contrast, lack of spatial information, and noise in the images caused the segmentation to be a hectic process. Clustering techniques on segmentation process is being used from early decades; Canny [4] used an extended FCM algorithm for edge detection to segment the images. Pham and Prince [5] proposed an algorithm using Fuzzy C means with presence of intensity homogeneities on MRI images. Chuang [1] altered the FCM method, and proposed an algorithm called MS-FCM for MRI brain image segmentation, which overcomes the problem of noise sensitivity in the traditional FCM by allowing spatial information to be utilized. Zhou et al., [6] proposed fuzzy c mean based on mean shift which relates similar to the level set algorithm. Zanaty et al., [7] proposed alternative for Euclidean distance in standard FCM by introducing new kernelized distance metric (KFCM) which consider spatial constrains as well. The combination of Spatial FCM (SFCM) and Level Set method for image segmentation results in better outcomes. Li et al., [8] proposed the very same method for medical image segmentation. As the traditional FCM uses Euclidean distance, it lacks in producing accurate results on segmentation. Abisha and Shiji [9] proposed kernel induced distance metric for the better segmentation in which it uses both spatial and kernel information. It reduces the problem raised due to the noise and outliers. Traditional FCM alone lacks in producing better results for medical image segmentation since it doesn't uses spatial information as said in above papers. Combining SFCM and KFCM together will provide optimistic results since it uses both the spatial and kernel information. Arunakumar and Harish [10] implemented a combination of spatial information with distance as kernel metric calling this method as SKFCM, which produced a robust results and this clustering algorithm was named as RSKFCM.

The Level Set method helps in finding the shapes and boundaries efficiently in an image. Suri [11] used Level Set Method for MRI scanned brain image segmentation which gave faster and better results. Paragios [12] introduced knowledge-based constraints by dealing with the local deformations in Level Set. Mitchell [13] introduced flexible and extensible Level Set Methods. Li et al., [14] proposed re-initialization method which is applied to periodically replace the degraded level set function with a signed distance function. The level set method introduced in the earlier decade is depended on the position of the initial contour, and the evolving curve can be trapped into local minima. The method proposed by Chan and Vese [15] is also not suitable for parallel programming because the average intensities inside and outside the contour is computed at each iteration. These results into more CPU time consumption by increasing inter process communications. Inspired by the better performance of the RSKFCM method, Sudharshan Duth [16] proposed a method to initialize the level set parameter based on Robust Spatial Kernel Fuzzy C-Means (RSKFCM).

Balla et al., [17] proposed a new method which aims at the evolving curve to be stopped according to the membership degree of the current pixel to be inside or outside of the active contour. The proposed method is developed with the help of the modified fuzzy C-means (FCM) and Lattice Boltzmann Method). LBM can handle problem of time consumption as the curvature implicitly computed and the algorithm is simple for parallel programming. LBM acts as alternative solver for Level Set Equation.

In this paper we make use of RSKFCM and Lattice Boltzmann Method for medical image segmentation. RSKFCM is based on standard FCM algorithm, which considers spatial information and uses Gaussian RBF kernel function as distance metric. Classical methods take more CPU time for solving LSE. An alternative approach to solve Level Set Equation (LSE) is Lattice Boltzmann Method (LBM). LBM is a numerical framework for modelling Boltzmann particle dynamics on a 2-D or 3-D lattice which overcomes the problem of classical methods and produces accurate results. It was first designed to solve macroscopic fluid dynamics challenges. The method is accurate both in time and in space up to second order. LBM has more advantages like parallelizability and simplicity as it is local and explicit in nature.

The rest of the paper is organized as follows: in section 2, we give a brief background about RSKFCM and its algorithm, level set method and Lattice Boltzmann method. Section 3, introduces our algorithm and gives the mathematical analysis of the proposed algorithm. Section 4 presents the experimental results. Finally, we provide concluding remarks in Section 5.

II. BACKGROUND

A. *Robust Spatial Kernel FCM (RSKFCM)*

Clustering and segmentation are very close terms in image processing. Clustering techniques separate pixels having the same characteristics. The separation is based on parameters like spatial information and distance metrics. The result of clustering is mapped to spatial domain as separated regions in segmentation process. In clustering the data is divided into clusters based on some similarity measures like distance, connectivity, intensity. Fuzzy c-means is well known clustering method where data points are clustered based on membership function assigned to each of them. The traditional FCM is not suitable for revealing non Euclidean structure, as the algorithm does not utilize the spatial information. By using spatial information into the membership function to improve the segmentation results Spatial FCM (SFCM) was introduced which is more robust to noises and outliers but only removes the noise partially. A kernel parameter, using all data points in the collection is determined in Kernel FCM (KFCM). Original Euclidean distance in the FCM is replaced by a kernel-induced distance in it. KFCM with spatial constraints (SKFCM), it provides better and moreover faster segmentation result than FCM and SFCM alone. RSKFCM is based on standard FCM algorithm, which incorporates spatial information and uses Gaussian RBF kernel function. It produces good results as it uses multiple advantages of different methods [10].

1) *RSKFCM Algorithm*

The working of RSKFCM algorithm is based on the following steps:

1) Distributes the pixel of the input image into data set X and initialize the value of centers Ɛ, m
2) Compute all membership values u_{ij} of each pixel against centers such as

$$u_{ij} = \frac{(1-K(x_j, v_i))^{-1/(m-1)}}{\sum_{k=1}^{c}(1-K(x_j, v_k))^{-1/(m-1)}}$$

3) Compute the new membership value w_{ij}

$$w_{ij} = \frac{u_{ij}{}^{p} s_{ij}{}^{q}}{\sum_{k=1}^{c} u_{kj}{}^{p} s_{kj}{}^{q}}$$

$$s_{ij} = \sum_{k \in NK(x_j)} u_{ik}$$

Where $NK(x_j)$ represents a square window cantered on pixel in the x_j spatial domain

4) Calculate the objective function J as follows:

$$J = 2\sum_{i=1}^{c}\sum_{j=1}^{N} w_{ij}^{m}(1-K(x_j, v_i))$$

5) Calculate new centre values v_i

$$v_i = \frac{\sum_{j=1}^{n} w_{ij}^{m} x_j}{\sum_{j=1}^{n} w_{ij}^{m}}$$

6) Evaluate the threshold of termination condition $\{J(i) - J(i=1)\} <$ ℇ, where ℇ is a termination criterion. Stop if it is satisfied otherwise go to step 2.

B. Level Set Method (LSM)

The LSM is a numerical technique for tracking interfaces and shapes. It has the advantage of handling automatically topological changes of the tracked shape as it uses an implicit representation of active contours. In 2-D image segmentation, the LSM represents a closed curve as the zero level set of, called the level set function. The evolution of the curve starts from an arbitrary starting contour and evolves itself driven by the LSE which can be seen as a convection–diffusion equation

$$\frac{\partial \theta}{\partial t} + \vec{V} \cdot \nabla \phi = b\Delta\phi \tag{1}$$

Where $\nabla\phi$ and $\Delta\phi$ are the gradient and the Laplacian of ϕ respectively. The term $b\Delta\phi$ is called artificial viscosity (Sethian suggested replacing it with $bk\,|\,\Delta\phi\,|$ which is better for handling the evolution of lower dimensional interfaces [17]), and *k* is the curvature of the distance function. The LSE can therefore be written as

$$\frac{\partial \theta}{\partial t} + \vec{V} \cdot \nabla \phi = bk\,|\,\Delta\phi\,| \tag{2}$$

C. Lattice Boltzmann Method (LBM)

The LBM is a numerical framework for modelling Boltzmann particle dynamics on a 2-D or 3-D lattice, which was first designed to solve macroscopic fluid dynamics challenges. The method is accurate both in time and in space up to second order. The Advantages of LBM are parallelizability and simplicity due to local and explicit nature. LBM can handle problem of time consumption as the curvature is implicitly computed and the algorithm is simple for parallel programming.D2Q9 method, in which the 2D image is partitioned into eight links with its neighbours and one link for the cell itself. The LBM evolution equation can be written as follows using the Bhatnagar, Gross and Krook model [18]

$$f_i(\vec{r}+\vec{e}_i,t+1)=f_i(\vec{r},t)+\frac{1}{T}[f_i^{eq}(\vec{r},t)-f_i(\vec{r},t)] \tag{3}$$

Where τ represents the relaxation time determining the kinematic viscosity ϑ of the fluid by

$$\vartheta=\frac{1}{3}\left(\tau-\frac{1}{2}\right) \tag{4}$$

and f_i^{eq} is the equilibrium particle distribution defined as

$$f_i^{eq}\left(\rho,\vec{u}\right)=\rho\left(A_i+B_i(\vec{e}_i\cdot\vec{u})+C_i(\vec{e}_i\cdot\vec{u})^2+D_i(\vec{u})^2\right) \tag{5}$$

where A_i to D_i are constant coefficients depending on the geometry of the lattice links, ρ and $\vec{u}$ are the macroscopic fluid density and velocity, respectively, computed from the particle distribution as

$$\rho=\sum_i f_i \qquad \vec{u}=\frac{1}{\rho}\sum_i f_i\vec{e}_i \tag{6}$$

For modeling typical diffusion computations, the equilibrium function can be simplified as follows [20]

$$f_i^{eq}\left(\rho,\vec{u}\right)=\rho A_i \tag{7}$$

In the case of D2Q9 model, A_i=4/9 for the zero link, A_i=1/9 for the axial links, and A_i=1/36 for the diagonal links. Now, the relaxation time τ is determined by the diffusion coefficient γ defined as

$$\gamma=\frac{2}{9}(2\tau-1) \tag{8}$$

As shown in [19], LBM can be used to solve the parabolic diffusion which can be recovered by the Chapman-Enskog expansion

$$\frac{\partial\rho}{\partial t}=\gamma\nabla\cdot\nabla\rho \tag{9}$$

In this case, the external force can be included as follows

$$f_i\longleftarrow f_i+\frac{2_\tau-1}{2_\tau}B_i(\vec{F}.\vec{e}_i) \tag{10}$$

Moreover, thus, (9) becomes

$$\frac{\partial\rho}{\partial t}=\gamma\nabla\cdot\nabla\rho+F \tag{11}$$

Replacing ρ by the signed distance function ϕ, the LSE can be recovered

III. PROPOSED METHOD

Level set method is a part of the active contour family. Level set methods have shown tremendous results for medical image segmentation. The main drawbacks of Level set methods are the results depends on the initialization of controlling parameters and time complexity. In the proposed method we overcome these two drawbacks. To initialize the controlling parameter we used Robust Spatial Kernel Fuzzy C-Means (RSKFCM). RSKFCM is based on standard FCM algorithm, which incorporates spatial information and uses Gaussian RBF kernel function. To overcome the second drawback we make use of Lattice Boltzmann Method (LBM). As done in [18], the bias field is incorporated into the RSKFCM framework by modelling the observed image as follows:

$$Y_i=X_iG_i\forall i\in\{1,2,...,N\} \tag{12}$$

where Y_i, X_i, and G_i are the observed intensity, true intensity, and *gain* field at the i^{th} pixel, respectively. N is the total number of pixels in the magnetic resonance image. The artefact can be modelled as an additive *bias* field by applying a logarithmic transformation to both sides of equation (12) [18],

$$y_i = x_i + \beta_i \forall i \in \{1, 2, ..., N\} \tag{13}$$

where y_i and x_i are the observed and true log-transformed intensities at the i^{th} voxel, respectively, and β_i is the bias field at the i^{th} voxel. By incorporating the bias field model into an RSKFCM framework, we will be able to iteratively estimate both the true intensity and the bias field from the observed intensity. By substituting equation 13 into RSKFCM objective function equation the clustering criterion to minimize in the presence of bias field becomes a constrained optimization problem

$$J(W,V,B,Y) = \sum_{k=1}^{c}\sum_{i=1}^{N} w_{ki}^{p} \| -\beta_i - v_k \|^2$$

$$\sum_{k=1}^{c} w_{ki} = 1 \; \forall i \qquad 0 \le w_{ki} \le 1 \; \forall k, i \tag{14}$$

where $Y = \{y_i\}_{i=1}^{N}$ is the observed image and $B = \{\beta_i\}_{i=1}^{N}$ is the bias field image.

In a continuous form, the aforementioned criterion can be written as

$$J(W,V,B,Y) = \sum_{k=1}^{c} \int_{\Omega_k} W_k^p(x,y) \| Y(x,y) - v_k \|^2 \, dxdy$$

$$\sum_{k=1}^{c} W(x,y) \; = 1 \; \forall x, y \qquad 0 \le W_k(x,y) \le 1 \; \forall k, x, y \tag{15}$$

Consider the two-phase level set although the method can be easily extended to more than two phases. The image domain Ω is segmented into two disjoint regions Ω_1 and Ω_2, i.e., $c = 2$. In this case, we can introduce a level set function as follows:

$$J(W,V,B,Y,\phi) = \int_{\Omega} W_1{}^p(x,y) \| Y(x,y) - B(x,y) - v_1 \|^2 \; H(\phi) dxdy$$

$$+ \int_{\Omega} W_2{}^p(x,y) \| Y(x,y) - B(x,y) - v_2 \|^2 \; (1 - H(\phi)) dxdy$$

$$W_1(x,y) + W_2(x,y) = 1 \; \forall x, y \quad 0 \le W_k(x,y) \le 1 \; \forall k, x, y \tag{16}$$

where ϕ is a signed distant function. The aforementioned term $J(U, V,B, Y, \phi)$ is used as the data link in our energy functional which is defined as follows:

$$E(W,V,B,Y,\phi) = J(W,V,B,Y) + v \mid C \mid \tag{17}$$

Where $v|C|$ is a regularization term with v >0 being a fixed parameter and C being a given curve which is represented implicitly as the zero level of φ and $|C|$ is the length of C and can be expressed by the following equation[21]

$$\mid C \mid = \int_{\Omega} \mid \nabla H(\phi) \mid dxdy \tag{18}$$

As done in [21], to obtain LSE, we minimize $E(W,V,B,Y,\phi)$ with respect to ϕ. For fixed U, V and B, we use the gradient descent method

$$\frac{\partial \phi}{\partial t} = -\frac{\partial E}{\partial \phi} \tag{19}$$

We thus obtain three necessary conditions,

$$W_k^*(x,y) = \frac{1}{\sum_{l=1}^{c} \left(\frac{\| Y(x,y) - B(x,y) - v_k \|}{\| Y(x,y) - B(x,y) - v_l \|} \right)^{\frac{2}{(p-1)}}} \tag{20}$$

$$v_k^* = \frac{\int_\Omega W_k^p(x,y)(Y(x,y)-B(x,y))dxdy}{\int_\Omega W_k^p(x,y)dxdy} \tag{21}$$

$$B^*(x,y) = Y(x,y) - \frac{\sum_{k=1}^{c} W_k^p(x,y)v_k}{\sum_{k=1}^{c} W_k^p(x,y)} \tag{22}$$

By using the gradient projection method of Rosen [22], we can replace δ (ϕ) by |∇ ϕ | in the proposed LSE, and as ϕ is a distance function, we have |∇ ϕ | = 1 equation 23 and will stay at each step since an adaptive approach is not used and the distant field is valid in the whole domain. Thus, the proposed LSE becomes

$$\begin{aligned}\frac{\partial\phi}{\partial t} &= W_1^p(x,y)\,\|\,Y(x,y)-B(x,y)-v_1\,\|^2 \\ &-W_2^p(x,y)\,\|\,Y(x,y)-B(x,y)-v_2\,\|^2 + v\,div(\nabla\phi) \\ & W_1(x,y)+W_2(x,y)=1\forall x,y \quad 0\le W_k(x,y)\le 1\forall k,x,y\end{aligned} \tag{23}$$

Replacing ρ by the signed distance function ϕ, (11) becomes

$$\frac{\partial\phi}{\partial t} = \gamma\, div(\nabla\phi) + F \tag{24}$$

By setting the external force

$$F = \lambda(W_1^p(x,y)\,\|\,Y(x,y)-B(x,y)-v_1\,\|^2 - W_2^p(x,y)\,\|\,Y(x,y)-B(x,y)-v_2\,\|^2) \tag{25}$$

Where λ is a positive parameter; we can see that equation 23 is only a variation formula of equation 24 and, thus, can be solved by the LBM with the above defined FEF. The choice of parameter *p is* a great importance for the segmentation result.

1) Algorithm

The computational steps are as follows

1) Initialize the distance function ϕ and class centroid values *v*1 and *v*2. Initialize *B* with zeros.
2) Compute Level set functions $W_1^p(x,y)$ and $W_2^p(x,y)$ using (20)
3) Compute *v*1 and *v*2 with (21)
4) Compute B with the equation (22)
5) Compute the external force using (25)
6) Include the external force based on (25)
7) Resolve the convection-diffusion equation with LBM with (3)
8) Accumulate $f_i(\vec{r},t)$ values at each grid point by equation 6, which generates an updated distance value at each point.
9) Find the contour
10) If the segmentation is not done, increase the value of λ and go back to step 5

IV. Experimental Results

This section evaluates the performance of the proposed RSKFCM. The MRI image of the brain chosen for the experiment is available in three bands: T1-weighted, proton density (pd)-weighted and T2-weighted. The normal brain images are obtained from Brainweb database [24]. In this paper, we use the transversal slice map, the slice thickness is 1 mm and the size is 217 x 181 pixels. We used the same weighting exponent m=2.0, and a 3x3 square of the brain image pixels. We implemented and simulated all the algorithms with MatlabR R2013a.

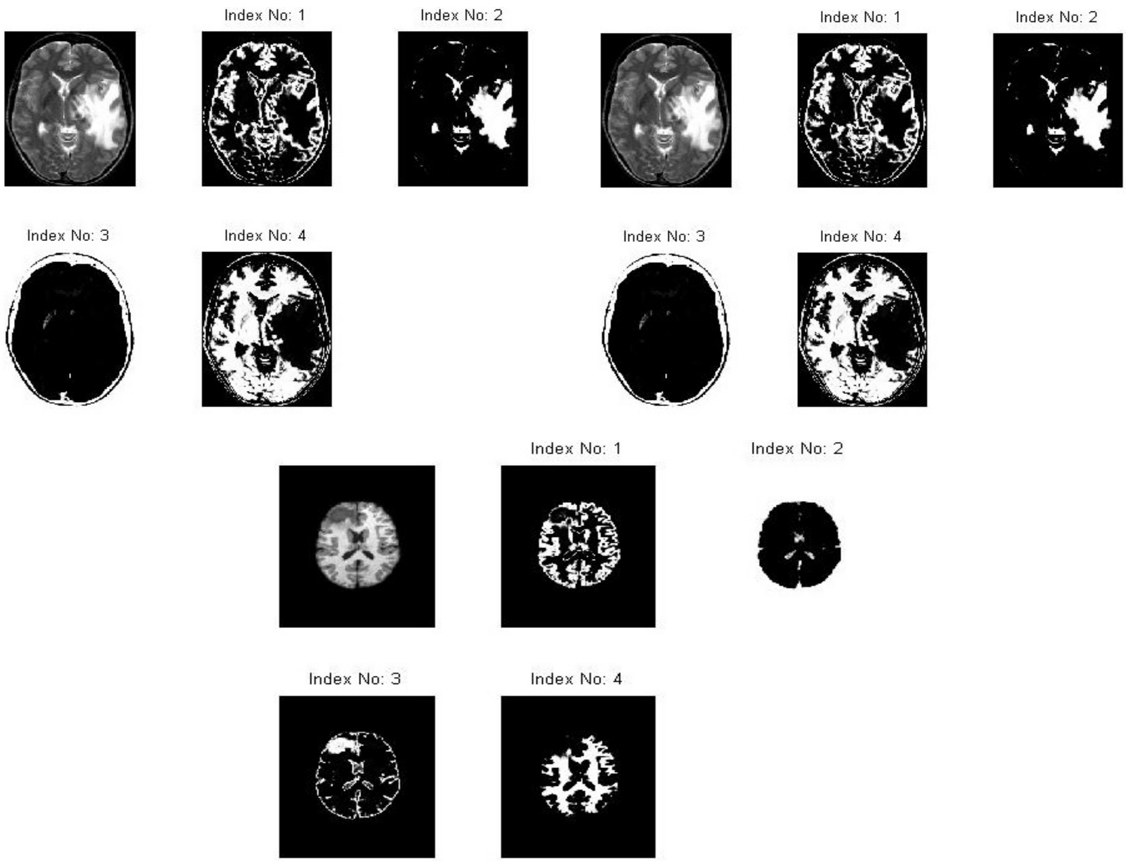

Fig.1. Initial segmentation using RSKFCM

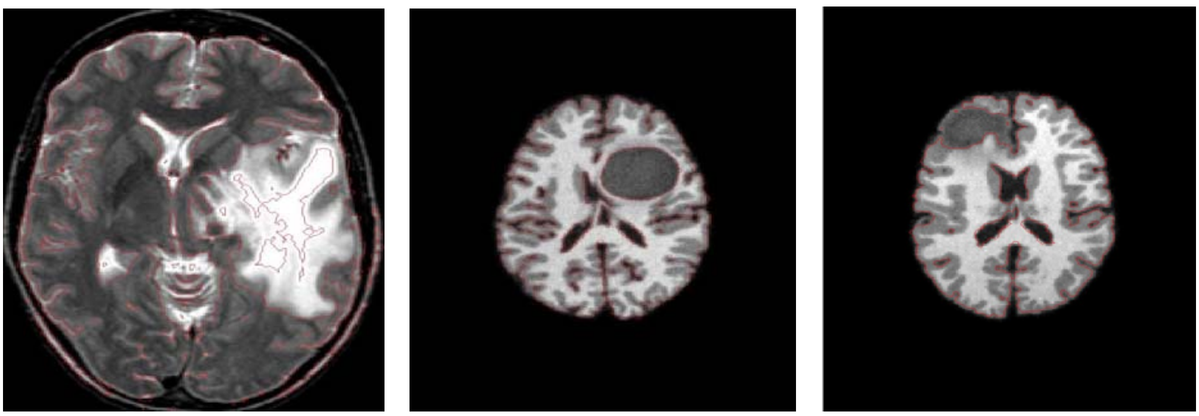

Fig.2. Final segmentation using proposed method

Figure 1 shows the initial segmentation results using RSKFCM method. Figure 2 shows the final segmentation result of the proposed method. In summary, the proposed method gives a good segmentation result. Since, RSKFCM incorporate spatial information and uses kernel metric, it shows less susceptible to different types of noise. Hence it is suitable to initiate level set evolution for image segmentation. The below Table1 shows the achieved CPU time in seconds for the proposed method with other existing methods which proves reduction in time complexity.

Table1- (a) Segmentation results of the CV method (b) Segmentation results of Balla et al., (c) Segmentation results of proposed method.

Methods		Image 1	Image 2	Image 3
		CPU time (sec)		
a	Chan - Vese	164.36	93.28	102.7410
b	Balla et al.,	0.79	0.78	0.77
c	Proposed Method	0.60	0.69	0.72
Image dimensions 483 X 321				

V. CONCLUSION

In this paper, we proposed a hybrid method for image segmentation. The proposed method is based on level set method. The proposed method utilizes RSKFCM algorithm to initialize the controlling parameters of level set and LBM method to reduce the time complexity. Performance evaluation has been carried out on MRI brain images. The results were confirmed promising efficient.

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