SHORTEST PATH ARC LENGTH NETWORK USING TRIANGULAR INTUITIONISTIC FUZZY NUMBER

A.D.CHANDRASEKARAN[#1], K.GANESAN[*2], S.BALAMURALITHARAN[#3]
[1,2,3] Faculty of Science and Humanities, Department of Mathematics,
SRM University, Kattankulathur – 603 203, TAMIL NADU, INDIA.
[1]Email: adchandru1977@gmail.com
[2]Email: gansan_k@yahoo.com
[3]Email: balamurali.maths@gmail.com

***Abstract*—In this paper, an intuitionistic fuzzy shortest path is presented to find the optimal path in a network which a fuzzy number, instead of a positive integer is assigned to each arc length. The algorithm is based on the idea that firstly from all the shortest paths from source to destination, an arc with shortest length is computed and then the Euclidean distance is computed for all the paths with the arc of minimum distance. Finally an illustrative numerical example is given to express the proposed work.**

Keywords: Intuitionistic Fuzzy Set (IFS), Intuitionistic Fuzzy Shortest Path, Intuitionistic Fuzzy Number (IFN)

I. INTRODUCTION

The shortest path problem is the basic network problem, the real number is assigned to each edges. However, it seems that in the literature there is little investigation on aggregation operators for aggregating a collection of papers [1-23]. The Fuzzy shortest path problem was first analyzed by Dubois and Prade[2], using fuzzy number instead of a real number is assigned to each edges. He used Floyd's algorithm and Ford's algorithm to treat the fuzzy shortest path problem. Although in their method the shortest path length can be obtained, maybe the corresponding path in the network doesn't exist. In this paper we propose an intuitionistic fuzzy number instead of fuzzy number. An algorithm is based on the idea that from all the shortest paths from source node to destination node, an edge with shortest length is computed and the Euclidean distance is computed for all the paths with the edge of minimum distance is the shortest path for membership and non-membership values.

Okada and Soper [15] developed an algorithm based on the multiple labeling approach, by which a number of non-dominated paths can be generated. Besides, the multiple labeling approaches are an exhaustive approach, and it needs to compare all the possible paths from the source node to the other nodes. Klein [13] proposed a dynamical programming recursionbased fuzzy algorithm. Tzung-Nan Chuang and Jung-Yuan Kung (2006*)* [7] found the fuzzy shortest path length in a network by means of a fuzzy linear programming approach. Shu MH, et al. [5], proposed fuzzy shortest path length procedure that can find fuzzy shortest path length among all possible paths in a network. It is based on the idea that a crisp number is the minimum if and only if any other number is larger than or equal to it [18]. The algorithm is based on the idea that shortest length is computed and then the Euclidean distance is derived for all the paths with the arc of optimal distance [3].

This paper is organized as follows: In section 2 provides preliminary concepts required for analysis. In section 3, an intuitionistic fuzzy shortest path length procedure is given. In section 4 and 5 intuitionistic fuzzy shortest path problem is explained using an illustrative example and conclusion in section 6.

II. PRELIMINARY AND DEFINITIONS

The concept of an intuitionistic fuzzy set was introduced by Atanassov [1] to deal with vagueness, which can be defined as follows.

2.1 Intuitionistic Fuzzy Set

Let X is a universe of discourse. An Intuitionistic Fuzzy Set (*IFS*) T in X is given by $T=\{x,\mu_T(x),\gamma_T(x)/x\in X\}$ where $\mu_T(x):X\to[0,1]$ and $\gamma_T(x):X\to[0,1]$ determine the degree of membership and non-membership of the element $x\in X, 0\le\mu_T(x)+\gamma_T(x)\le 1$.

2.2 Intuitionistic Fuzzy Graph

Let X is a universe, containing fixed graph vertices and let $V\subset X$ be a fixed set. Construct the *IFS* $V=\{x,\mu_v(x),\gamma_v(x)/x\in X\}$ where the functions $\mu_V(x):X\to[0,1]$ and $\gamma_V(x):X\to[0,1]$ determine the degree of membership and non-membership to set V of the element (vertex) $x\in X$, such that $0\le\mu_T(x)+\gamma_T(x)\le 1$.

2.3 Intuitionistic Fuzzy Number

Let $T=\{x,\mu_T(x),\gamma_T(x)/x\in X\}$ be an IFS, then we call $(\mu_T(x),\gamma_T(x))$ an Intuitionistic Fuzzy Number (IFN). We denote it by $(\langle x,y,z\rangle,\langle l,m,n\rangle)$ where $\langle x,y,z\rangle$ and $\langle l,m,n\rangle\in F(I), I=[0,1], 0\le z+n\le 1$.

2.4 Intuitionistic Fuzzy Number

A triangular intuitionistic fuzzy number 'A' is denoted by $A=\{(\mu_A,\gamma_A)/x\in\mathbb{R}\}$, where μ_A and γ_A are triangular fuzzy numbers with $\gamma_A\le\mu_A^c$. Equivalently, A Triangular Intuitionistic Fuzzy Number (TIFN) 'A' is given by $A=(\langle x,y,z\rangle,\langle l,m,n\rangle)$ with $\langle l,m,n\rangle\le\langle x,y,z\rangle^c$ that is either $l\ge y, m\ge z$ (or) $m\le x, n\le y$ are membership and non-membership fuzzy numbers of A.

The additions of two TIFN are as follows.

For two triangular intuitionistic fuzzy numbers

$T_1=(\langle x_1,y_1,z_1\rangle:\mu_{T_1},\langle l_1,m_1,n_1\rangle:\gamma_{T_1})$ and $T_2=(\langle x_2,y_2,z_2\rangle:\mu_{T_2},\langle l_2,m_2,n_2\rangle:\gamma_{T_2})$

with $\mu_{T_1}\ne\mu_{T_2}$ and $\gamma_{T_1}\ne\gamma_{T_2}$, define $T_1+T_2=\begin{pmatrix}\langle x_1+x_2,y_1+y_2,z_1+z_2\rangle:Min(\mu_{T_1},\mu_{T_2}),\\ \langle l_1+l_2,m_1+m_2,n_1+n_2\rangle:Max(\gamma_{T_1},\gamma_{T_2})\end{pmatrix}$.

III. PROCEDURE FOR AN INTUITIONISTIC FUZZY SHORTEST PATH LENGTH

In this paper the arc length in a network is considered to be an intuitionistic fuzzy number, namely triangular intuitionistic fuzzy number. The shortest path length procedure is based on the Chuang and Kung method.

Step:1 Compute all the possible path lengths L_i from $i=1,2,3,\dots n.$ Where $L_i=(\langle a_i',b_i',c_i'\rangle,\langle l_i',m_i',n_i'\rangle)$

Step:2 Initialize $L_{\min}=(\langle a,b,c\rangle,\langle l,m,n\rangle)=L_1=(\langle a_1',b_1',c_1'\rangle,\langle l_1',m_1',n_1'\rangle)$

Step:3 Initialize $i=2$.

Step:4(i) Calculate $\langle a,b,c\rangle$ for membership values by

$$a=\min(a,a_i');\quad b=\begin{cases} b, & if\ b\le a_i' \\ \dfrac{bb_i'-aa_i'}{(b+b_i')-(a+a_i')} & if\ b>a_i' \end{cases}\ \text{and}\ c=\min(c,b_i').$$

Step:4(ii) Calculate $\langle l,m,n\rangle$ for non-membership values by

$$l=\min(l,l_i');\quad m=\begin{cases} m, & if\ m\le l_i' \\ \dfrac{mm_i'-ll_i'}{(m+m_i')-(l+l_i')} & if\ m>l_i' \end{cases}\ \text{and}\ n=\min(n,m_i').$$

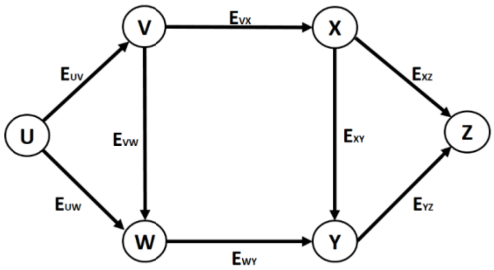
Step:5 Set $L_{\min}=(\langle a,b,c\rangle,\langle l,m,n\rangle)$ as computed in Step 4(i) & 4(ii)

Step:6 Replace $i=i+1$

Step:7 if $i<n+1$, then go to Step 4(i) & 4(ii)

IV. NUMERICAL EXAMPLE

In order to illustrate the above procedure consider a small network shown in figure, where each arc length is represented as a triangular intuitionistic fuzzy number.



Consider a network with the triangular intuitionistic fuzzy arc lengths as shown below. The arc lengths are assumed to be

$E_{UV} = \left(\langle 17,30,42\rangle, \langle 37,50,56\rangle\right)$; $E_{UW} = \left(\langle 25,35,49\rangle, \langle 37,51,56\rangle\right)$; $E_{VW} = \left(\langle 33,49,56\rangle, \langle 52,61,69\rangle\right)$;

$E_{VX} = \left(\langle 33,41,53\rangle, \langle 42,55,64\rangle\right)$; $E_{WY} = \left(\langle 14,21,29\rangle, \langle 24,31,39\rangle\right)$; $E_{XY} = \left(\langle 14,38,52\rangle, \langle 44,52,54\rangle\right)$;

$E_{XZ} = \left(\langle 20,28,36\rangle, \langle 30,40,49\rangle\right)$; $E_{YZ} = \left(\langle 23,35,52\rangle, \langle 43,55,65\rangle\right)$.

Step:1 In the above network there are four possible paths $(n = 4)$ from source node U to destination node Z. The possible path lengths L_i for $i = 1,2,3,4$ are as follows: $W_1 : U - V - X - Y - Z$; $\mathrm{L}_1 = \left(\langle 87,144,199\rangle, \langle 166,212,239\rangle\right)$

$W_2 : U - V - X - Z$; $\mathrm{L}_2 = \left(\langle 70,99,131\rangle, \langle 109,145,169\rangle\right)$

$W_3 : U - V - W - Y - Z$; $\mathrm{L}_3 = \left(\langle 87,135,179\rangle, \langle 156,197,229\rangle\right)$

$W_4 : U - W - Y - Z$; $\mathrm{L}_4 = \left(\langle 62,91,130\rangle, \langle 104,137,160\rangle\right)$.

Iteration – I:

$$L_{\min} = \left(\langle a,b,c\rangle, \langle l,m,n\rangle\right) = L_1$$

Step:2 Initialize

$$= \left(\langle a_1^{'}, b_1^{'}, c_1^{'}\rangle, \langle l_1^{'}, m_1^{'}, n_1^{'}\rangle\right)$$
$$= \left(\langle 87,144,199\rangle, \langle 166,212,239\rangle\right)$$

Step:3 Initialize $i = i + 1 = 2$

Step:4(i) Calculate $\langle a,b,c\rangle$ for membership values by $a = \min\left(a, a_2^{'}\right) = \min(87,70) = 70.$

if $b > a_2^{'}$; $b = \dfrac{bb_2^{'} - aa_2^{'}}{(b + b_2^{'}) - (a + a_2^{'})} = \dfrac{(144)(99) - (87)(70)}{(144 + 99) - (87 + 70)} = 94.95$ and

$c = \min\left(c, b_2^{'}\right) = \min(199,99) = 99.$

Step:4(ii) Calculate $\langle l,m,n\rangle$ for non-membership values by $l = \min\left(l, l_2^{'}\right) = \min(166,109) = 109$;

if $m > l_2^{'}$ then $m = \dfrac{mm_2^{'} - ll_2^{'}}{(m + m_2^{'}) - (l + l_2^{'})}$ and $n = \min\left(n, m_2^{'}\right) = \min(239,145) = 145.$

$$= \frac{(212)(145) - (166)(109)}{(212 + 145) - (166 + 109)} = 154.22$$

Step:5 Set $L_{\min} = \left(\langle a,b,c\rangle, \langle l,m,n\rangle\right) = \left(\langle 70,94.95,99\rangle, \langle 109,154.22,145\rangle\right)$ as computed in Step 4(i) & 4(ii)

Step:6 Replace $i = i + 1 = 3$

Step:7 if $i < n + 1 = 5,$ then go to Step 4(i) & 4(ii)

Iteration – II:

Step:4(i) Calculate $\langle a,b,c\rangle$ for membership values by $a=\min\left(a,a_3^{'}\right)=\min\left(70,87\right)=70$

if $b>a_3^{'}$; then, $b=\dfrac{bb_3^{'}-aa_3^{'}}{(b+b_3^{'})-(a+a_3^{'})}=\dfrac{(94.95)(135)-(70)(87)}{(94.95+135)-(70+87)}=92.23$ and

$c=\min\left(c,b_3^{'}\right)=\min\left(99,135\right)=99.$

Step:4(ii) Calculate $\langle l,m,n\rangle$ for non-membership values by $l=\min\left(l,l_3^{'}\right)=\min\left(109,156\right)=109$;

if $m<l_3^{'}$ then $m=154.22$ and $n=\min\left(n,m_3^{'}\right)=\min\left(145,197\right)=145.$

Step:5 Set $\begin{aligned}L_{\min}&=\left(\langle a,b,c\rangle,\langle l,m,n\rangle\right)\\&=\left(\langle 70,92.23,99\rangle,\langle 109,154,145\rangle\right)\end{aligned}$ as computed in Step 4(i) & 4(ii)

Step:6 Replace $i=i+1=4$

Step:7 if $i<n+1=5$, then go to Step 4(i) & 4(ii)

Iteration – III:

Step:4(i) Calculate $\langle a,b,c\rangle$ for membership values by $a=\min\left(a,a_4^{'}\right)=\min\left(70,62\right)=62$

if $b>a_4^{'}$; $\begin{aligned}b&=\dfrac{bb_4^{'}-aa_4^{'}}{(b+b_4^{'})-(a+a_4^{'})}\\&=\dfrac{(92.23)(91)-(70)(62)}{(92.23+91)-(70+62)}=79.11\end{aligned}$ and $c=\min\left(c,b_4^{'}\right)=\min\left(99,91\right)=91.$

Step:4(ii) Calculate $\langle l,m,n\rangle$ for non-membership values by $l=\min\left(l,l_4^{'}\right)=\min\left(109,104\right)=104$;

if $m>l_4^{'}$ then $\begin{aligned}m&=\dfrac{mm_4^{'}-ll_4^{'}}{(m+m_4^{'})-(l+l_4^{'})}\\&=\dfrac{(154.22)(137)-(109)(104)}{(154.22+137)-(109+104)}=125.19\end{aligned}$ and $n=\min\left(n,m_4^{'}\right)=\min\left(145,137\right)=137.$

Step:5 Set $L_{\min}=\left(\langle a,b,c\rangle,\langle l,m,n\rangle\right)$ as computed in Step 4(i) & 4(ii)

Step:6 Replace $i=i+1=5$

Step:7 if $i\geq n+1=5$, STOP the procedure.

Finally we obtain an Intuitionistic Shortest Path Length $\begin{aligned}L_{\min}&=\left(\langle a,b,c\rangle,\langle l,m,n\rangle\right)\\&=\left(\langle 62,79.11,91\rangle,\langle 104,125.19,137\rangle\right).\end{aligned}$

V. SHORTEST PATH ALGORITHM

Our aim is to determine an intuitionistic fuzzy shortest path length $L_{\min}=\left(\langle a,b,c\rangle,\langle l,m,n\rangle\right)$ and the shortest needed to traverse from source to destination. By combining an intuitionistic fuzzy shortest length method with similarity measure, an algorithm is as follows. An algorithm for intuitionistic fuzzy shortest path:

Step:1 Find out all the possible paths from source node U to destination node Z and calculate the corresponding path lengths L_i for $i=1,2,\ldots,n.$

Step:2 Determine $L_{\min}=\left(\langle a,b,c\rangle,\langle l,m,n\rangle\right)$ by using an intuitionistic fuzzy shortest path length procedure.

Step:3 Compute the Euclidean distance d_i for $i=1,2,\ldots,n.$ between all the possible path and $L_{\min}=\left(\langle a,b,c\rangle,\langle l,m,n\rangle\right).$

Step:4 Conclude the shortest path with path having minimum Euclidean distance.

Applying this algorithm to the numerical example network. The first two steps have been already calculated in section – 4.Then step 3is to find out the similarity degree $S\left(L_{\min},L_i\right)$

between $L_{\min}$ and L_i for $i = 1,2,\ldots,n.$ by means of similarity measure $S\left(A,B\right)=\sum_{K=1}^{M}\frac{\left[1-\left|A(x_k)-B(x_k)\right|\right]}{m}.$ In order to get the accuracy in similarity degree, we should let a generic element U denoted by μ_i for $i = 1,2,\ldots,n.$

Step:1 From Section – 4, already we have four possible path lengths are as follows:

$$W_1 : U-V-X-Y-Z;\ \mathrm{L}_1=\left(\langle 87,144,199\rangle,\langle 166,212,239\rangle\right)$$

$$W_2 : U-V-X-Z;\ \mathrm{L}_2=\left(\langle 70,99,131\rangle,\langle 109,145,169\rangle\right)$$

$$W_3 : U-V-W-Y-Z;\ \mathrm{L}_3=\left(\langle 87,135,179\rangle,\langle 156,197,229\rangle\right)$$

$$W_4 : U-W-Y-Z;\ \mathrm{L}_4=\left(\langle 62,91,130\rangle,\langle 104,137,160\rangle\right).$$

Step:2 From Section – 4, also we have computed minimum path length is

$$\begin{aligned}L_{\min} &=\left(\langle a,b,c\rangle,\langle l,m,n\rangle\right)\\ &=\left(\langle 62,79.11,91\rangle,\langle 104,125.19,137\rangle\right).\end{aligned}$$

Step:3 Determine the Euclidean distance between all the four path lengths L_i for $i = 1,2,3,4$ and

$$\begin{aligned}L_{\min} &=\left(\langle a,b,c\rangle,\langle l,m,n\rangle\right)\\ &=\left(\langle 62,79.11,91\rangle,\langle 104,125.19,137\rangle\right).\end{aligned}$$

$$d\left(L_1,L_{\min}\right)=\left(\begin{array}{l}\left\langle\sqrt{\left(87-62\right)^2+\left(144-79.11\right)^2+\left(199-91\right)^2}\right\rangle,\\ \left\langle\sqrt{\left(166-104\right)^2+\left(212-125.19\right)^2+\left(239-137\right)^2}\right\rangle\end{array}\right)$$

$$=\left(\langle 128.45\rangle,\langle 147.59\rangle\right).$$

$$d\left(L_2,L_{\min}\right)=\left(\begin{array}{l}\left\langle\sqrt{\left(70-62\right)^2+\left(99-79.11\right)^2+\left(131-91\right)^2}\right\rangle,\\ \left\langle\sqrt{\left(109-104\right)^2+\left(145-125.19\right)^2+\left(169-137\right)^2}\right\rangle\end{array}\right)$$

$$=\left(\langle 45.38\rangle,\langle 37.97\rangle\right)$$

$$d\left(L_3,L_{\min}\right)=\left(\begin{array}{l}\left\langle\sqrt{\left(87-62\right)^2+\left(135-79.11\right)^2+\left(179-91\right)^2}\right\rangle,\\ \left\langle\sqrt{\left(156-104\right)^2+\left(197-125.19\right)^2+\left(229-137\right)^2}\right\rangle\end{array}\right)$$

$$=\left(\langle 107.20\rangle,\langle 127.77\rangle\right).$$

$$d\left(L_4,L_{\min}\right)=\left(\begin{array}{l}\left\langle\sqrt{\left(62-62\right)^2+\left(91-79.11\right)^2+\left(130-91\right)^2}\right\rangle,\\ \left\langle\sqrt{\left(104-104\right)^2+\left(137-125.19\right)^2+\left(160-137\right)^2}\right\rangle\end{array}\right)$$

$$=\left(\langle 40.77\rangle,\langle 25.85\rangle\right).$$

Step:4 Conclude the shortest path with the path having minimum Euclidean distance for member and non-member by examining the Euclidean distance d between L_i for $i = 1,2,3,4$ and

$$L_{\min}=\left(\langle a,b,c\rangle,\langle l,m,n\rangle\right).$$

VI. CONCLUSION

In this paper we investigated an algorithm for solving shortest path problem on a network with fuzzy arc lengths. The algorithm can be useful to decision makers. We have developed an example with the help of a minimum Euclidean distance. From the above computations we can observe that Path $W_4 : U - W - Y - Z$ has the shortest Euclidean distance for membership and non-membership. Hence the shortest path from source node U to destination node Z is $W_4 : U - W - Y - Z$.

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AUTHOR PROFILE

Author1:A.D. Chandrasekaran is currently Assistant Professor, Department of Mathematics, Faculty of Science and Humanities, SRM University. Currently he is pursuing Ph.D. at SRM University under the guidance of Dr.K.Ganesan. He received his M. Phil Mathematics from Madurai Kamaraj University. His research area includes Fuzzy optimization, Intuitionistic Fuzzy Graphs and Fuzzy Non-linear equations. He has presented his research articles in two international conferences. His academic experience is about 18 years in teaching. He is currently working in the area of Triangular Intuitionistic Fuzzy Numbers.

Author2:Dr. K. Ganesan is currently Professor, Department of Mathematics, Faculty of Science and Humanities, SRM University. He received his Ph.D. in Mathematics from the University of Madras, Chennai. His academic experience is about 26 years in teaching. His research interest includes Fuzzy optimization and Decision making, Interval Mathematics, Intuitionistic Fuzzy Numbers, Intuitionistic Fuzzy Graphs, Operation research, Differential equations etc. He published more than 60 research papers in international journals and conferences. He is a regular reviewer for various international journals published by Elsevier Science, Springer, World Scientific Publications Company Private Ltd etc. He has organized many national conferences and workshops. He is recently successfully completed a BRNS funded research project for Department of Atomic energy, Government of India. He is one of the authors of more 10Mathematical books. He is guiding more than 10 students in this area.

Author3: Dr. S. Balamuralitharan was born on 30TH May 1979, in Thirunelveli, Tamil Nadu. He received Ph.D degree in Mathematics from B.S. Abdur Rahman University in the year 2013. He is working as an Assistant Professor in SRM University, kattankulathur-603 203 in the Department of Mathematics from June 2013 to till date. His area of interests includes Biology and nonlinear Differential Equation. He has published 30 papers in the International Journals, presented 10 papers in the International Conferences (including Malaysia & China) and presented 11 papers in the National Conferences. The e-mail ID is: balamurali.maths@gmail.com and the contact number is: 9841922575. He is guiding one student in his area.