# AT-SITE FLOOD ANALYSIS USING EXPONENTIAL AND GENERALIZED LOGISTIC MODELS IN PARTIAL DURATION SERIES (PDS)

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***Abstract* - Flood Frequency Analysis (FFA) uses both flood peak series, i.e. Annual Maximum Series (AMS) and Partial Duration Series (PDS). AMS analyses number of peaks equal to number of observation years using a single best fit probability distribution, while PDS analyses peaks over a threshold value adopting Poisson distribution (PD) and negative binomial (NB) distribution counting occurrences of peaks (μ) over threshold and exponential distribution (ED), generalized logistic distribution (GLD) or Pareto distribution (GP) for magnitude. Both ED and GLD method are used to analyze at-site extreme flood information in the PDS model at Champua gauging site under Baitarani Basin, Odisha (India). Performances of AMS/PD-ED, AMS/NB-ED, AMS/PD-GLD, AMS/NB-GLD are compared in terms of uncertainty of T-year event estimator. Results show AMS/PD-ED model yields lower variance of T-year estimate than AMS/NB-ED model, while AMS/NB-GLD gives less variance in comparison to AMS/PD-GLD. Results indicate that in case of ED, flood quintiles are acceptable for μ≤8 and T≤500 within 95% confidence limit, while they are comparable for μ≤3 and T≤100 in case of GLD distribution. To summarize, NB distribution is preferred for number of flood peaks if coupled with GLD for flood exceedances and Poisson distribution in case of ED for flood exceedances values.**

**Keywords -**Annual Maximum Series, Partial Duration Series, Exponential Distribution, Generalized Logistic Distribution

## I. INTRODUCTION

Design flood estimation corresponding to a specified risk of flood inundation or the stability of water resource structures is an important part of the engineering practice and flood frequency analysis (FFA) is an efficient tool to quantify the magnitude of the extreme hydrologic events. The relation between discharge and the return period is unique for every gauging station. There are two approaches commonly used for probabilistic analysis of extreme flood magnitudes, i.e., the Annual Maximum Series (AMS) and Partial Duration Series (PDS). The AMS model considers the annual maximum flood of each year in a series that has as many elements as there are years in the data record. In contrast, the PDS model considers all flood peaks above a certain threshold level. The AMS model results in loss of information as some peaks that are not annual maximum (AM) are still relatively high and are not considered in AMS analysis and very low discharge values can be part of AMS. The AMS sample is usually rather small as compared to PDS sample where more than one flood per year may be included. PDS models are used for flood frequency analysis (FFA) where the flood record is short. If the threshold is increased to certain limit, recurrence interval of large events computed using AMS and PDS models tend to converge. Since the PDS sample is defined by all peaks that lie above a certain truncation level, assuring the independence of data series and choosing an appropriate threshold value is of prime importance.

The methods followed for formulating these models are quite different e.g. an AMS model uses a cumulative distribution function (cdf) to model the flood extremes, whereas the PDS model uses two probabilistic models: (a) one for the probability of occurrence of peaks above a threshold, and (b) the cdf, ~ for modeling the flood exceedance. Generally more than one distribution may fit the data well and selecting the best model can be difficult (Salas et al.2012). Different countries have formulated guidelines regarding use of various distributions. Further the behavior of the method also depends on the estimation technique for distribution parameters which are estimated through different methods such as methods of moments (MOM), methods of probability weighted moments(PWM)/L-moments and maximum likelihood estimation (MLE). The present study is focused on at-site flood analysis using the PDS model for quantile estimation using the L-moment method with two probability distribution function, i.e. ED and GLD. The numbers of exceedances are modeled as a discrete distribution whereas the exceedances magnitudes are modeled as continuous distribution and their total probability leads to flood magnitude of desired return period ($Q_T$). Considering the number of flood peaks above the threshold ($Q_0$) as random, Shane and Lynn (1964) assumed this number to follow a PD. Since the process following a PD assumes that events are independent and occur uniformly throughout the interval of observations, it necessitates that there is no clustering of events (Stark and Woods, 1986). Thus, the dependence of flood peaks in PDS may well have an effect on the ability of PD to describe the number of peaks above threshold. Besides PD (Todorovic & Zelenhasic 1970[29], NERC 1975[21], Cunnane 1979[5], Lang *et al.* 1997[16], ÖnÖz and Bayazit 2001 [23], Bhunya *et al.* 2012[2]), Negative Binomial (NB) ( Cunnane 1979[5], Lang *et al.* 1997[16], ÖnÖz and Bayazit 2001[23] , Bhunya *et al.* 2012[2]) and Binomial (Lang *et al.* 1997[16], ÖnÖz and Bayazit 2001[23]) can also be chosen. Studies by NERC (1975) [21,22] and Cunnane (1979) [5] on 26 streams in Great Britain indicated that the number of peaks occurring each year is not a Poisson variate since its variance was significantly greater than its mean, which violates the basic assumption of PD. As NB distribution has this property, it was used by Lang. (1999) [15] and ÖnÖz and Bayazit (2001) [23] in PDS modeling.

Many commonly used distributions are approximately exponential with a stretched upper tail, for which ED was first used by Kirby (1969) [13] to fit the flood exceedances in a PDS model. Although alternate frequency distributions have been proposed in the past, e.g. the Gamma distribution (Zelenhasic 1970[30]), Weibull distribution (Ekanayake and Cruise 1993[7]), Generalized logistic model (Bhunya *et al*. 2012[2]) in PDS models, and the most popular is the generalized Pareto (GP) distribution which has the ED as a special case (Fitzgerald 1989[8]; Madsen *et al.* 1997[18]; Martins and Stedinger 2000[19], 2001[20]; Bhunya *et al.* 2013[3]). All these studies use GP distribution as the flood exceedance model with PD arrival rate for deriving the PDS model. Recently ÖnÖz and Bayazit (2001) [23]compared the advantages of NB and Poisson distributions in PDS model when ED is used as the flood exceedances model. Bhunya *et al.* (2012[2], 2013[3]) found that the Poisson distribution is better than the NB distribution in cases where the mean and variance of the annual number of exceedances are small. However, their studies stressed the importance of threshold selection as main criteria for the performance of the PDS models rather than the choice of any particular distribution to model the arrival rates.

With the above background, the present study compares the merits of PD and the NB distribution as models for the occurrences of peaks exceeding a threshold in PDS context, considering ED and GLD to model the flood exceedances. Estimation of the ED and GLD parameters by the method of L-moments (Greenwood *et al.* 1979[9]; Hosking 1986[10]; Hosking & Wallis 1997[11], Shabri & Jemain 2013[27]) is formulated for different thresholds. The performances of the AMS/PD-GLD and AMS/NB-GLD and also AMS/PD-ED and AMS/NB-ED models are compared using the variances of T-year estimates on field data. These are also compared with the corresponding AMS analysis considering the maximum annual peak values matching to the period of data available.

## II. PROBABILITY DISTRIBUTIONS USED IN ANALYSIS

### A. *Exponential Distribution*

The exponential distribution is a special case of the Gamma family of distributions (Rao & Hamed 2000[24]) and the probability distribution function is given by

$$F(Q) = 1 - e^{\frac{-(Q-\xi)}{\beta}}, \quad \xi \le Q \le \infty \tag{1}$$

$\xi$ is the lower bound and β is the scale parameter.
Eqn.1 in the inverse form is expressed as

$$Q_T = \xi - \beta \ln(1 - F(Q)) = \xi - \beta \ln\left(\frac{1}{T}\right) \tag{2}$$

The parameters $\xi$ and β in terms of L-moments are given as

$$\xi = \lambda_1 - \beta \text{ and } \beta = 2\lambda_2 \tag{3}$$

$$Var(Q_T) = \frac{\beta^2}{N}\left[1 + 2K_T + \frac{4}{3}K_T{}^2\right], \text{ where } K_T = \ln(T) - 1 \tag{3i}$$

$K_T$ is the growth factor for exponential distribution.

B. *Generalized Logistic Distribution*

The probability distribution function of a three parameter generalized logistic distribution is defined by Hosking and Wallis (1997) as

$$F(Q) = \begin{matrix} -\frac{1}{k}\ln\left(1 - \frac{k(Q-\xi)}{\alpha}\right) & , k \ne 0 \\ \frac{(Q-\xi)}{\alpha} & , k = 0 \end{matrix} \tag{4}$$

$\xi, \alpha, k$ are the location, scale and shape parameters, respectively.
The T year event $Q_T$ is defined as

$$Q_T = \xi + \frac{\alpha}{k}\left(1 - (T-1)^{-k}\right) = \xi\left[1 + \frac{\alpha}{\xi k}\left(1 - (T-1)^{-k}\right)\right] = \xi K_T \tag{5}$$

$K_T$ is the growth factor for GLD. The location parameter $\xi$ is equated to the distribution median to the sample median m ($\xi$ = m ).

$$var(Q_T) = \frac{\alpha^2}{N}\left(3 + 2.3361\, K_T{}^2\right) \tag{6}$$

with m=$\lambda_1$ and α=$\lambda_2$ when expressed in L-moment estimators.

Ahmad *et al*. (1988) [1], Hosking and Wallis (1997) [11], Rao and Hamed (2000) [24] expressed the general form of the distribution to be used in flood exceedances model

$$F(Q) = \left\{1 + \exp\left[-\frac{(Q - Q_0) - m}{\alpha}\right]\right\}^{-1} \tag{7}$$

and the flood quantile is expressed as $Q_T = Q_0 + m + \alpha \ln(T-1)$ (8)

C. *Poisson Distribution*

In the PDS model, all peak events above a threshold level are considered and the number of these peaks in a given year is assumed to follow a Poisson distribution having the probability mass function:

$$P(X = r) = \frac{e^{-\mu N}(\mu N)^r}{r!} \qquad r = 0, 1, 2 \ldots \tag{9}$$

where P(X= r) is the probability of r events in N years, μ is the average number of threshold exceedances. Eqn.(9) is true as long as the flood peaks are independent (Langbein 1949[17]). For n number of observed exceedances in N years, μ is equal to n/N. For Poisson distribution, PWM estimations and the population estimate of μ is given by

$$\hat{\mu} = \frac{n}{N} \qquad (10)$$

The mean and variance of r are defined as follows:

$$E(r) = \mu = n/N = Var(r) = \sigma^2 \quad ; \quad n \geq 5 \qquad (11)$$

For μ > 5 the distribution of r is symmetrical and it asymptotically approaches a normal distribution (Johnson and Kotz 1973[12]). For no flood exceedances to occur in a given year, substituting r = 0 in Eq. (1), the following is obtained

$$P(r = 0) = \frac{e^{-\mu}(\mu)^0}{0!} = e^{-\mu} \qquad (12)$$

which is the probability distribution function (pdf) of an exponential distribution. The cumulative mass function of Eq. (9) for N years is given by

$$F(X) = 1 - \mu e^{-\mu N} \quad ; \quad N > 0 \qquad (13)$$

where X is a variate. Assume that a threshold level $Q_0$ is chosen, corresponding to a mean annual number of exceedances of Poisson distribution μ ; then at any higher threshold Q > $Q_0$, the number of exceedances $r'$ is Poisson distributed with the following parameter (Flood Studies Report 1975)

$$\mu' = \mu \; [1 - F(Q)] \qquad (14)$$

Since F (Q) ≤ 1, it follows that $\mu' \leq \mu$ i.e. the truncated mean of exceedances above the new threshold q decreases, which is quite obvious.

D. *Negative Binomial Distribution*

Binomial random variable is a count of number of successes in certain number of trials, whereas the negative binomial distributed variables are literally opposite to that; here the number of successes are predetermined and the number of trial are random. The NB distribution is the probability of having to wait X(= r+k-1) trials to obtain r-1 successes and the success in r+k trial. If X has a NB distribution with parameters p and r , the probability mass function is given by,

$$P(x = r) = \binom{r+k-1}{r-1} p^r (1-p)^k \quad ; r > 1 \text{ and } 0 \leq p \leq 1 \qquad (15)$$

In Eq.(15) p is the constant probability of a success in any independent trial . P(X=r) gives the probability that the variate X is equal to r. Because at least r trials are required to get a success, the range of Eq.(7) is from r to ∞. The probability that there is no flood exceedance in any given year is obtained by making r-1 = 0 in Eq. (15) as

$$P(r - 1 = 0) = \binom{k-1}{0}(1-p)^k = (1-p)^k \qquad (16)$$

Similarly, the probability of number of occurrences of exceedances less than r is given by

$$P[x \leq r] = \sum_{r=0}^{r} \binom{r+k-1}{r} p^r (1-p)^k \qquad (17)$$

The value of p and r are given by (Spigel 1987) as

$$E(Q - Q_0) = \mu = \frac{r(1-p)}{p} \text{ and } p = \frac{E(Q - Q_0)}{V(Q - Q_0)} \qquad (18)$$

## III. PDS BASED ANNUAL MAXIMUM FLOODS

The annual maximum flood in the PDS model is determined from the total probability of average number of flood exceedances and the exceedances value above the threshold. The probability distribution function F(Q) is given by (Shane and Lynn 1964)

$$F(Q) = \sum_{r=0}^{\infty} P_\mu(r)(G(y))^r \quad \text{where } y = Q - Q_o \qquad (19)$$

A. *AMS/PD-ED Model*

When number of exceedances are poissonian and their magnitudes are exponential (AMS/PD-ED) (Zelenhasic 1970[30]; ÖnÖz and Bayazit 2001[23])

$$F(Q)=\sum_{r=0}^{\infty} e^{\mu}\mu^{r}\left[1-\exp\left(-\frac{y}{\beta}\right)\right]^{r}=\exp\left\{-\mu\exp\left[-\left(\frac{Q-Q_0}{\beta}\right)\right]\right\} \tag{20}$$

$\beta$ is the single parameter of exponential distribution = E(y) = $\mu$ = Var(y)$^{0.5}$
Eq.(20) is the Gumbel or EV1 distribution widely used in FFA.

On simplification of Eq.(20) with $F(Q)=1-\frac{1}{T}$ , T being the return period, we get

$$Q_T = Q_0 + \beta\ln\mu + \beta Y_T \tag{21}$$

where $Y_T = -\ln\left(-\ln\left(1-\frac{1}{T}\right)\right)$ is the Gumbel reduced variate.

Cunnane (1973) [4] obtained the following expression for the asymptotic sampling variance of the estimate $Q_T$ computed from N-year long observations:

$$\mathrm{Var}(Q_T)=\frac{\beta^2}{\mu N}\left\{1+[\ln\mu+Y_T]^2\right\} \tag{22}$$

Rosberg (1985) introduced a small sample correction factor to the above formula which is of minor importance for $\mu N > 10$.

B. *AMS/NB-ED Model*

When number of exceedances are negative binomial and their magnitudes are exponential (AMS/NB-ED) (ÖnÖz and Bayazit 2001[23])

$$F(Q)=\sum_{r=1}^{\infty}\binom{r+k-1}{r-1}p^{r}(1-p)^{k}\left[1-\exp\left(-\frac{y}{\beta}\right)\right]^{r}=(1-p)^{k}\left\{1-p\left[1-\exp\left(-\frac{Q-Qo}{\beta}\right)\right]\right\}^{k} \tag{23}$$

which gives

$$Q_T = Q_0 + \beta\ln\left(\frac{p}{1-p}\right)-\beta\ln\left(\left(1-\frac{1}{T}\right)^{-1/r}-1\right) \tag{24}$$

And the variance is given as

$$\mathrm{Var}(Q_T)=\frac{\beta^2}{N}\left\{\begin{array}{l}\left(2-\frac{2}{r}-\frac{1}{pr}\right)\frac{1}{p^2}+\frac{1-p}{pr}\times\ln^2\left[\frac{(1-p)\left(F(Q)^{-1/r}-1\right)}{p}\right]\\ +\frac{2r-\frac{(1+p)^2}{p}}{p^2r^3}\frac{F(Q)^{-2/r}\ln^2F(Q)}{\left(F(Q)^{-1/r}-1\right)^2}-\frac{2}{p^2r^2}\left[\frac{(1+p)^2}{p}-2p\right]\frac{F(Q)^{-1/r}\ln F(Q)}{F(Q)^{-1/r}-1}\end{array}\right\} \tag{25}$$

C. *AMS/PD-GLD Model*

When number of exceedances are poissionian and their magnitudes are GLD distributed (AMS/PD-GLD) (Bhunya *et al.* 2012[2])

$$F(Q)=\sum_{r=0}^{\infty}\frac{e^{-\mu}(\mu)^r}{r!}\left(\left[1+\exp\left(-\frac{Q-Qo-m}{\alpha}\right)\right]^{-1}\right)^{r}=\exp\left(-\exp\left[-\frac{Q-Qo-[m+\alpha\ln(\mu)]}{\alpha}\right]\right) \tag{26}$$

Eq.(26) reduces to

$$Q_T = Q_0 + \alpha \ln(\mu) + m + \alpha Y_T \tag{27}$$

The variance is given by

$$\mathrm{Var}(Q_T) = \frac{\alpha^2}{N}\left[\frac{1}{\mu} + 0.7107(\ln \mu + Y_T)^2 + 3\right] \tag{28}$$

D. *AMS/NB-GLD Model*

When number of exceedances are negative binomial and their magnitudes are GLD distributed (AMS/NB-GLD) (Bhunya *et al.* 2012[2])

$$F(Q) = \sum_{k=0}^{\infty}\binom{r+k-1}{r-1}(p)^r(1-p)^k\left(\left[1+\exp\left(-\frac{Q-Qo-m}{\alpha}\right)\right]^{-1}\right)^r = \left(\left[1+\exp\left(-\frac{Q-Qo-m}{\alpha}\right)\right]^{-1}\right)^r \tag{29}$$

Eqn.(29) gives the flood of a given return period as follows

$$Q(T) = Q_0 + m - \alpha \ln\left\{\left(1-\frac{1}{T}\right)^{-1/r} - 1\right\} \tag{30}$$

$$\mathrm{Var}(Q_T) = \left\{-\alpha p \ln(F(Q))\frac{F(Q)^{-1/r}}{\left(F(Q)^{-1/r}-1\right)r^2}\right\}^2\left(2r - \frac{(2-p)^2}{1-p}\right)\left(\frac{r}{(1-p)^2 N}\right) + 3\frac{\alpha^2}{N} + \left[-\ln\left(F(Q)^{-1/r}-1\right)\right]^2\left(0.7107\frac{\alpha^2}{N}\right) \tag{31}$$

## IV. STUDY AREA

The gauging station at Champua on River Baitarani (Fig.1) is selected for the present study. River Baitarani up to this gauging site records the river hourly gauge values and the discharges at 8.00 AM everyday and the records are available for the period 1991 to 2013 (23 years). The hourly discharges are computed from the Gauge Discharge curve prepared using the recorded daily gauge and discharge values during the monsoon period. For the PDS model, the maximum daily maximum discharge values are used for analysis, whereas the yearly maximum values are considered for AMS analysis. Table.1 provides the discharge statistic of River Baitarani at Champua.

TABLE 1

Flow parameters of the GD site

| Period | Annual. Discharge ($m^3/s$) | | Mean(μ) ($m^3/s$) | SD(σ) ($m^3/s$) | Skewness (ϒ) |
|---|---|---|---|---|---|
| | Maximum | Minimum | | | |
| 1991-2013 | 2756.38 | 156.79 | 919.90 | 612.04 | 1.19 |

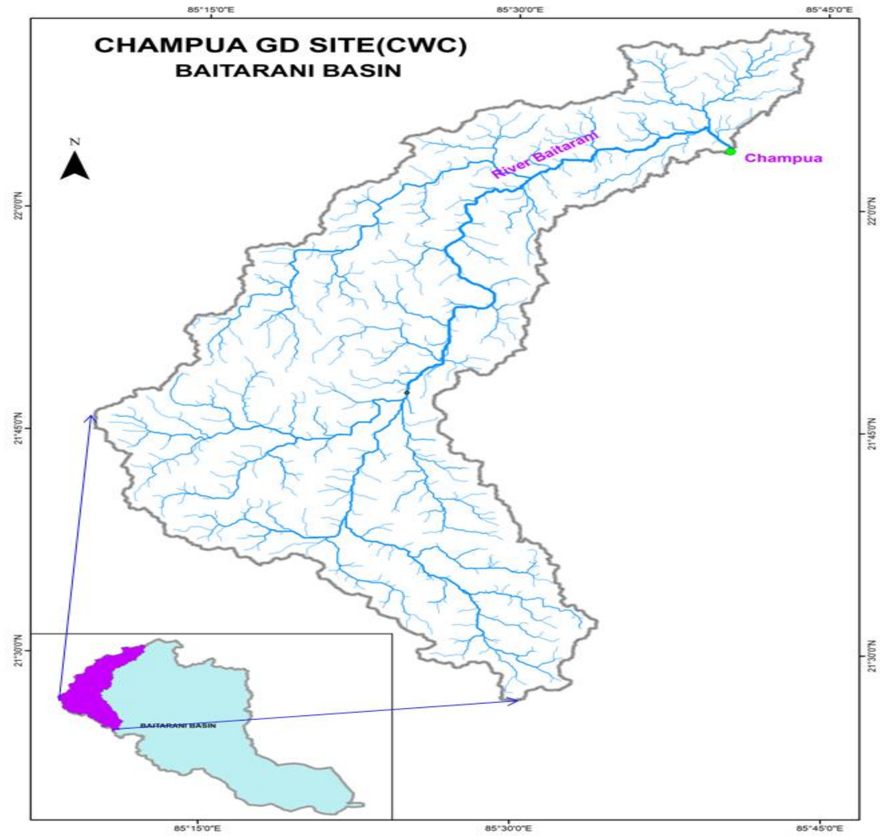


Fig 1 Index map of Baitarani Basin (GD site at Champua)

## V. RESULTS AND DISCUSSION

The performance of exponential (ED) and generalized logistic distribution (GLD) are compared on the basis of their variance ratio defined as

$$R_i = \frac{\mathrm{Var}(Q_T)_{*/ED}}{\mathrm{Var}(Q_T)_{*/GLD}} \qquad \text{i = 1 to 4 as defined in Table.6 (* is PD/NB)}$$

If the value of $R_i$ for a given period is greater than one, it indicates that GLD yields less variance than the ED when used for fitting the flood exceedances in PDS and the value equal to one is the ideal. The results with respect to the data used for AMS/PD-ED, AMS/NB-ED, AMS/PD-GLD and AMS/NB-GLD analysis are given in Tables. 2-5.

As indicated in Table.6, the variance ratio $R_1$ indicates that AMS/PD-ED yields lesser variance than AMS/NB-ED for all mean number of exceedances except for $\mu$=2, $\sigma^2$=4.727 for T=5 to 500. Earlier Cunnane (1979) [5] has found that there is no satisfactory improvement by employing binomial distribution instead of Poisson distribution to account for flood exceedances in PDS. In a similar study ÖnÖz and Bayazit (2001) [23] also concluded that flood estimates based on NB distribution is almost identical to those obtained using Poisson distribution. They used Var($Q_T$) as an index to compare the AMS/PD-ED and AMS/NB-ED model. Similarly in case of GLD, AMS/PD-GLD gives more variance compared to AMS/NB-GLD except for $\mu$=1.478, $\sigma^2$=2.351. The $R_3$ values further indicate that the variance of AMS/PD-ED model is smaller than AMS/PD-GLD model when the $\mu \geq 4$.The $R_4$ values indicate the AMS/NB-ED always generates excess variance than AMS/NB-GLD model for $\mu \leq 6$. But since it is seen that when $\mu \leq 3$, the POT and AMS model agrees with each other to a greater extent. In the present at-site analysis, the findings are also in agreement with that of Cunnane (1979) [5] and ÖnÖz and Bayazit (2001) [23]. When the value of number of exceedances ($\mu$) is in between 1.478 to 3, the standard deviation ($\sigma$) is nearly equal to the value of $\mu$ which is the basic property of a Poisson distribution. The variance ratio justifies that AMS/PD-ED model performs better than the AMS/PD-GLD model. In case GLD is to be used, AMS/NB-GLD is preferable to AMS/PD-GLD. In the entire analysis it is seen that PDS based AMS model results are more than the AMS model for $\mu$=1.478, $\sigma^2$=2.351. Figure 2-5 give a better visual match between the AMS/ ED versus AMS/PD-ED and AMS/GLD versus AMS/PD-GLD model up to T≈100 respectively which are quite comparable. Plots of AMS and PDS models are given in Figure 2-5. It reveals that in case of ED the flood quintiles are quite acceptable for μ≤8 and T≤500 within 95% confidence limit, while they are comparable for μ≤3 and T≤100 in case of GLD distribution.

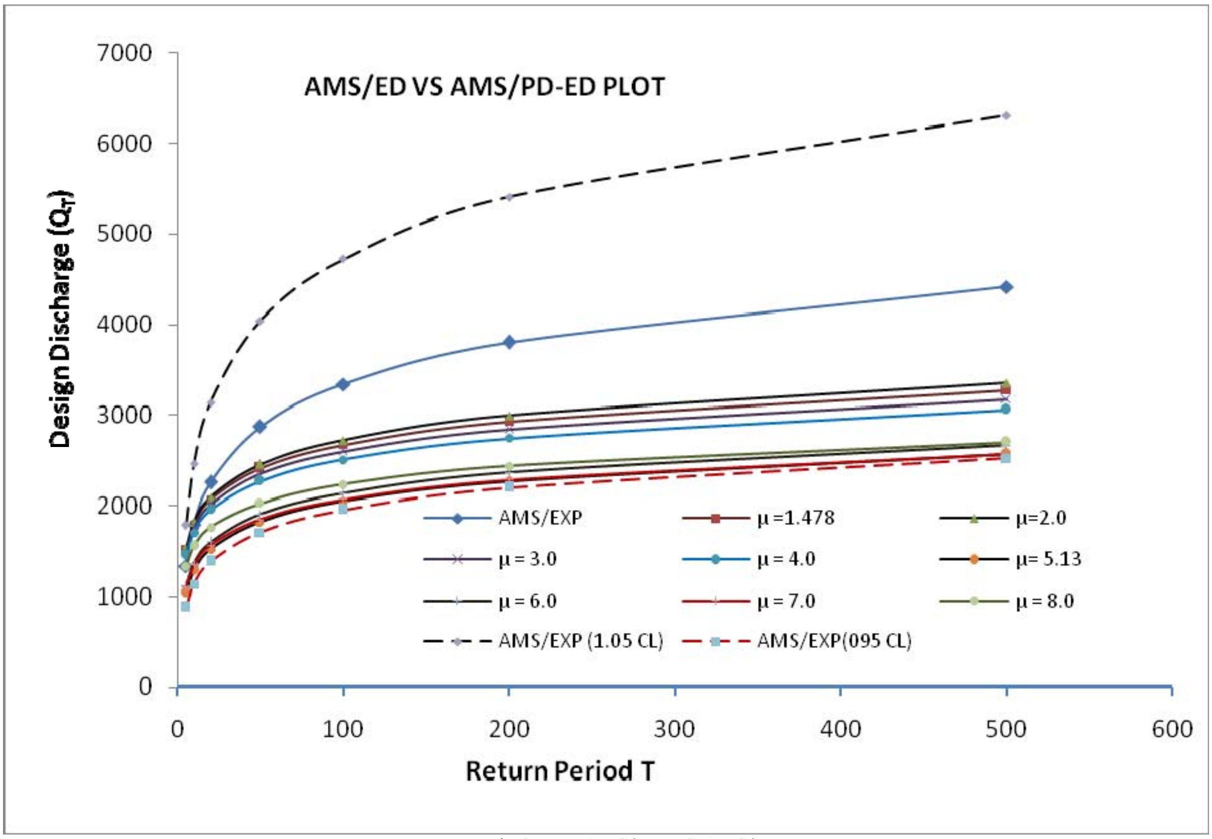


Fig 2 AMS/ED VS AMS/PD-ED

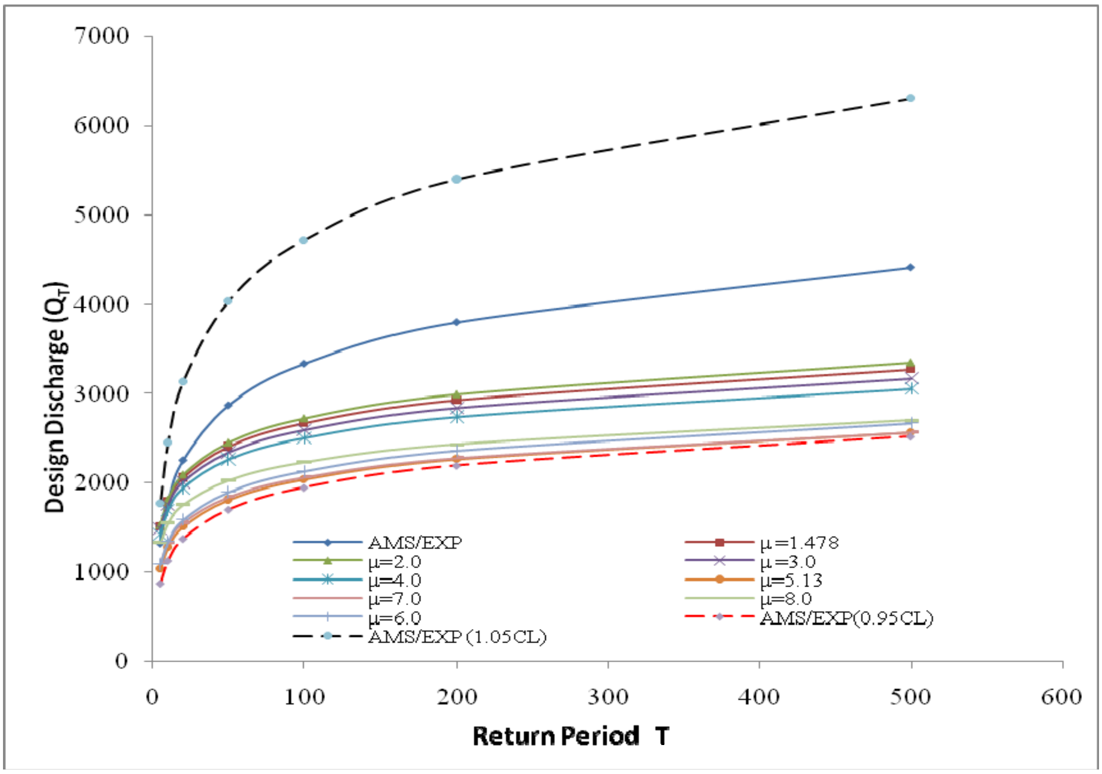


Fig 3 AMS/ED VS AMS/NB-ED

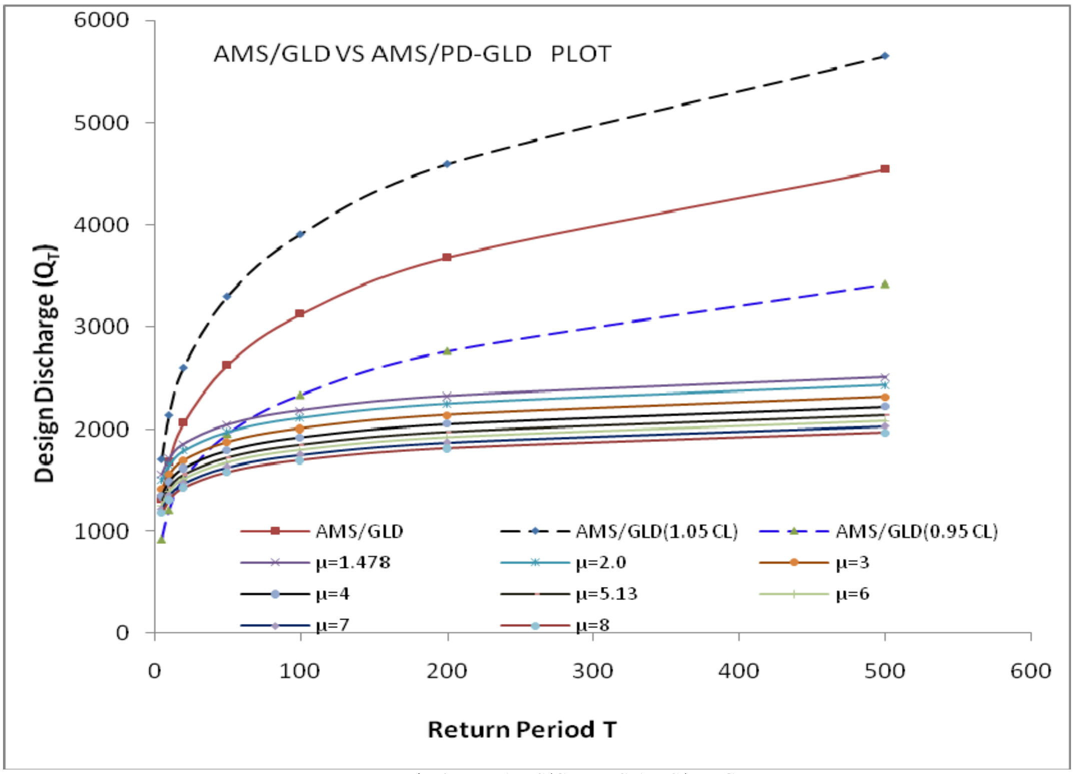


Fig 4 AMS/GLD VS AMS/PD-GLD

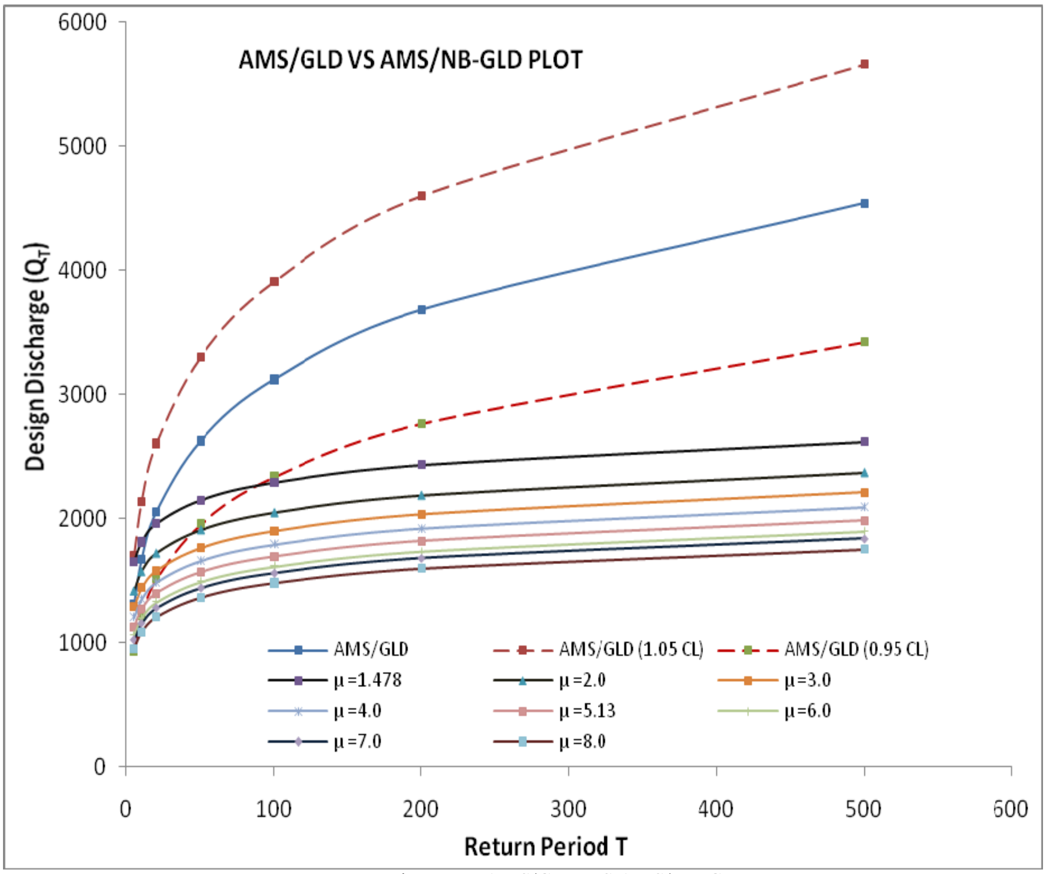


Fig 5 AMS/GLD VS AMS/NB-GLD

## VI. CONCLUSION

The analysis presented here focused on the choice between exponential distribution and the generalized logistic distribution both in the AMS and PDS model. In the AMS model the flood quantiles are computed using the annual maximum observed floods and in PDS model using the flood exceedances above a threshold coupled with Poisson and negative binomial distribution for mean number of flood exceedances. The variances of the flood quantile estimates are compared through $R_1$ to $R_4$ values in order to determine the superiority of the distributions employed in the study in the PDS format and also compared with the AMS model. The choice of the most efficient T-year event estimator model is not only dependent on whether it is AMS or PDS, but also the method of parameter estimation of the probability distributions used. Here in the present case only L-moment method of parameter estimation has been followed. It is seen that these ratios are not dependent on the years of data available but is a function of the mean and variance and the distribution parameters. The difference between the estimations under AMS and PDS model becomes prominent with increase in T years and with increase in value of $\mu$. A good match between the quantile values in PDS has been noticed so long the value of $\mu$ is less than 3. The values of the ratio of $Var(Q_T)$ as given in Table.3 infers that AMS/NB-GLD gives less variance of the $Q_T$ compared to AMS/NB-ED for the field data of Champua gauging site under Baitarani basin. This is in quite agreement to the finding of Bhunya *et al.* (2012) [2] who used data of a different region. Similarly in case of flood exceedances of exponential distribution, AMS/PD-ED model performs better than the AMS/NB-ED model for the present field data. The advantage of Poisson distribution over negative binomial distribution has been studied by ÖnÖz and Bayazit (2001) [23]. He has reported that the flood estimates and their corresponding variance based on negative binomial distribution when combined with ED for the flood exceedances are almost identical to those obtained using PD. The present finding is also in agreement with ÖnÖz and Bayazit (2001) [23], so also Kirby (1969) [13] and Cunnane (1979) [5]. To summarize the NB distribution should be preferred for number of flood peaks if to be coupled with GLD for flood exceedances and Poisson distribution in case of ED for flood exceedances values.

TABLE.2

Design Discharge ($Q_T$) and Var ($Q_T$) for AMS/PD-ED

| μ= | 1.478 | | 2.000 | | 3.000 | | 4.000 | | 5.130 | | 6.000 | | 7.000 | | 8.000 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| var= | 2.351 | | 4.727 | | 8.273 | | 12.091 | | 17.027 | | 22.727 | | 27.545 | | 37.545 | |
| Qo = | 805.097 | | 677.600 | | 553.700 | | 469.446 | | 400.648 | | 344.554 | | 313.350 | | 294.884 | |
| **T** | **$Q_T$** | **Var($Q_T$)** | **$Q_T$** | **Var($Q_T$)** | **$Q_T$** | **Var($Q_T$)** | **$Q_T$** | **Var($Q_T$)** | **$Q_T$** | **Var(QT)** | **QT** | **Var(QT)** | **QT** | **Var(QT)** | **QT** | **Var(QT)** |
| 5 | 1512.57 | 18842.90 | 1525.92 | 18897.25 | 1485.36 | 14442.68 | 1452.28 | 11760.09 | 1036.71 | 9708.87 | 1109.92 | 9355.58 | 1101.54 | 7767.75 | 1332.97 | 6313.78 |
| 10 | 1793.38 | 32850.84 | 1816.20 | 31435.43 | 1754.42 | 22757.50 | 1707.82 | 17929.77 | 1280.79 | 14432.51 | 1357.77 | 13706.00 | 1335.43 | 11228.01 | 1550.61 | 9026.93 |
| 20 | 2062.74 | 50647.00 | 2094.64 | 46904.83 | 2012.50 | 32704.92 | 1952.94 | 25181.80 | 1514.92 | 19912.45 | 1595.52 | 18715.61 | 1559.79 | 15185.74 | 1759.37 | 12113.24 |
| 50 | 2411.40 | 80020.99 | 2455.05 | 71933.84 | 2346.56 | 48447.65 | 2270.23 | 36508.38 | 1817.97 | 28385.40 | 1903.26 | 26416.48 | 1850.19 | 21237.11 | 2029.59 | 16811.58 |
| 100 | 2672.67 | 106720.45 | 2725.13 | 94391.35 | 2596.89 | 62364.78 | 2507.99 | 46430.48 | 2045.07 | 35755.06 | 2133.86 | 33086.80 | 2067.81 | 26458.43 | 2232.09 | 20852.57 |
| 200 | 2932.99 | 137316.67 | 2994.22 | 119921.00 | 2846.31 | 78037.60 | 2744.88 | 57538.55 | 2271.33 | 43967.23 | 2363.63 | 40499.28 | 2284.63 | 32245.77 | 2433.84 | 25322.06 |
| 500 | 3276.43 | 183782.28 | 3349.24 | 158419.26 | 3175.37 | 101473.64 | 3057.41 | 74059.99 | 2569.85 | 56129.35 | 2666.76 | 51449.21 | 2570.69 | 40774.55 | 2700.02 | 31895.63 |

TABLE. 3

Design Discharge ($Q_T$) and Var ($Q_T$) for AMS/NB-ED

| μ= | 1.478 | | 2.000 | | 3.000 | | 4.000 | | 5.130 | | 6.000 | | 7.000 | | 8.000 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| var= | 2.351 | | 4.727 | | 8.273 | | 12.091 | | 17.027 | | 22.727 | | 27.545 | | 37.545 | |
| Qo = | 805.097 | | 677.600 | | 553.700 | | 469.446 | | 400.648 | | 344.554 | | 313.350 | | 294.884 | |
| **T** | **$Q_T$** | **Var($Q_T$)** | **$Q_T$** | **Var($Q_T$)** | **$Q_T$** | **Var($Q_T$)** | **$Q_T$** | **Var($Q_T$)** | **$Q_T$** | **Var($Q_T$)** | **$Q_T$** | **Var(QT)** | **QT** | **Var(QT)** | **QT** | **Var(QT)** |
| 5 | 1495.76 | 20582.42 | 1657.20 | 19854.12 | 1461.67 | 17056.87 | 1432.89 | 13895.38 | 1020.17 | 11507.86 | 1092.65 | 11291.73 | 1086.85 | 9348.89 | 1317.90 | 7835.28 |
| 10 | 1785.48 | 34825.55 | 1961.04 | 29845.20 | 1743.29 | 25640.31 | 1698.71 | 20233.98 | 1273.02 | 16346.47 | 1349.65 | 15753.67 | 1328.52 | 12888.05 | 1543.53 | 10619.20 |
| 20 | 2058.90 | 52796.07 | 2245.65 | 41612.66 | 2007.10 | 35749.85 | 1948.52 | 27588.54 | 1511.14 | 21896.13 | 1591.57 | 20829.69 | 1556.43 | 16893.09 | 1755.93 | 13746.85 |
| 50 | 2409.89 | 82311.21 | 2609.60 | 60075.50 | 2344.43 | 51611.45 | 2268.49 | 38989.70 | 1816.49 | 30419.38 | 1901.70 | 28577.96 | 1848.87 | 22978.18 | 2028.24 | 18474.30 |
| 100 | 2671.92 | 109070.88 | 2880.83 | 76330.78 | 2595.83 | 65576.53 | 2507.12 | 48941.58 | 2044.33 | 37808.97 | 2133.09 | 35266.93 | 2067.15 | 28212.73 | 2231.42 | 22526.61 |
| 200 | 2932.61 | 139702.76 | 3150.49 | 94605.97 | 2845.78 | 81276.92 | 2744.45 | 60066.66 | 2270.97 | 46032.47 | 2363.24 | 42689.96 | 2284.30 | 34007.52 | 2433.51 | 27002.47 |
| 500 | 3276.28 | 186193.53 | 3505.85 | 121907.56 | 3175.16 | 104731.98 | 3057.24 | 76599.76 | 2569.70 | 58202.29 | 2666.61 | 53647.06 | 2570.55 | 42541.36 | 2699.89 | 33580.34 |

TABLE.4
Design Discharge ($Q_T$) and Var ($Q_T$) for AMS/PD-GLD

| μ= | 1.478 | | 2.000 | | 3.000 | | 4.000 | | 5.130 | | 6.000 | | 7.000 | | 8.000 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| var= | 2.351 | | 4.727 | | 8.273 | | 12.091 | | 17.027 | | 22.727 | | 27.545 | | 37.545 | |
| Qo = | 805.097 | | 677.600 | | 553.700 | | 469.446 | | 400.648 | | 344.554 | | 313.350 | | 294.884 | |
| **T** | **$Q_T$** | **Var($Q_T$)** | **$Q_T$** | **Var($Q_T$)** | **$Q_T$** | **Var($Q_T$)** | **$Q_T$** | **Var($Q_T$)** | **$Q_T$** | **Var($Q_T$)** | **$Q_T$** | **Var($Q_T$)** | **QT** | **Var(QT)** | **QT** | **Var(QT)** |
| 5 | 1560.61 | 10995.05 | 1498.22 | 11768.79 | 1412.04 | 13080.71 | 1347.30 | 13819.19 | 1288.91 | 14274.80 | 1251.97 | 14524.89 | 1216.82 | 14852.12 | 1181.74 | 14784.44 |
| 10 | 1711.96 | 15269.43 | 1646.65 | 16429.04 | 1556.38 | 18182.80 | 1487.01 | 19061.59 | 1423.68 | 19525.10 | 1383.55 | 19752.66 | 1345.70 | 20078.83 | 1306.86 | 19883.49 |
| 20 | 1857.14 | 20699.75 | 1789.04 | 22178.78 | 1694.83 | 24286.69 | 1621.02 | 25223.68 | 1552.95 | 25616.01 | 1509.76 | 25772.57 | 1469.32 | 26056.97 | 1426.89 | 25683.84 |
| 50 | 2045.05 | 29662.92 | 1973.34 | 31481.69 | 1874.04 | 33946.66 | 1794.49 | 34847.95 | 1720.28 | 35033.62 | 1673.12 | 35026.47 | 1629.34 | 35197.54 | 1582.25 | 34513.83 |
| 100 | 2185.87 | 37809.98 | 2111.45 | 39828.80 | 2008.34 | 42486.42 | 1924.47 | 43278.81 | 1845.67 | 43224.94 | 1795.54 | 43042.00 | 1749.25 | 43084.32 | 1698.67 | 42108.37 |
| 200 | 2326.18 | 47146.10 | 2249.05 | 49317.78 | 2142.14 | 52103.49 | 2053.99 | 52717.40 | 1970.60 | 52352.71 | 1917.51 | 51949.37 | 1868.73 | 51826.08 | 1814.67 | 50508.25 |
| 500 | 2511.28 | 61324.61 | 2430.59 | 63627.00 | 2318.67 | 66484.20 | 2224.85 | 66755.77 | 2135.43 | 65870.81 | 2078.43 | 65107.58 | 2026.35 | 64708.77 | 1967.70 | 62862.49 |

TABLE.5
Design Discharge ($Q_T$) and Var ($Q_T$) for AMS/NB-GLD

| μ= | 1.478 | | 2.000 | | 3.000 | | 4.000 | | 5.130 | | 6.000 | | 7.000 | | 8.000 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| var= | 2.351 | | 4.727 | | 8.273 | | 12.091 | | 17.027 | | 22.727 | | 27.545 | | 37.545 | |
| Qo = | 805.097 | | 677.600 | | 553.700 | | 469.446 | | 400.648 | | 344.554 | | 313.350 | | 294.884 | |
| **T** | **$Q_T$** | **Var($Q_T$)** | **$Q_T$** | **Var($Q_T$)** | **$Q_T$** | **Var($Q_T$)** | **$Q_T$** | **Var($Q_T$)** | **$Q_T$** | **Var($Q_T$)** | **$Q_T$** | **Var($Q_T$)** | **QT** | **Var(QT)** | **QT** | **Var(QT)** |
| 5 | 1657.74 | 12246.48 | 1421.63 | 8048.19 | 1290.86 | 8980.19 | 1205.55 | 9312.80 | 1128.72 | 9258.81 | 1063.05 | 8685.10 | 1023.81 | 8807.80 | 955.23 | 7876.61 |
| 10 | 1813.89 | 17622.85 | 1578.15 | 12336.36 | 1441.94 | 13238.75 | 1350.88 | 13520.75 | 1268.32 | 13334.45 | 1199.48 | 12532.47 | 1156.97 | 12635.41 | 1084.95 | 11364.66 |
| 20 | 1961.25 | 24076.05 | 1724.22 | 17666.03 | 1583.47 | 18498.49 | 1487.46 | 18663.59 | 1399.80 | 18275.12 | 1327.90 | 17205.54 | 1282.56 | 17250.12 | 1207.07 | 15599.00 |
| 50 | 2150.43 | 34325.24 | 1910.62 | 26359.19 | 1764.44 | 27022.02 | 1662.39 | 26933.90 | 1568.40 | 26174.36 | 1492.54 | 24687.33 | 1443.70 | 24599.90 | 1363.63 | 22375.96 |
| 100 | 2291.66 | 43417.97 | 2049.42 | 34218.77 | 1899.30 | 34688.48 | 1792.85 | 34332.84 | 1694.20 | 33213.23 | 1615.37 | 31360.64 | 1563.97 | 31132.00 | 1480.44 | 28419.15 |
| 200 | 2432.16 | 53687.30 | 2187.36 | 43206.82 | 2033.39 | 43424.33 | 1922.60 | 42734.38 | 1819.33 | 41185.26 | 1737.54 | 38923.36 | 1683.63 | 38517.52 | 1596.63 | 35266.73 |
| 500 | 2617.39 | 69089.17 | 2369.11 | 56842.09 | 2210.09 | 56632.65 | 2093.61 | 55396.70 | 1984.28 | 53171.98 | 1898.59 | 50301.11 | 1841.36 | 49605.25 | 1749.78 | 45567.20 |

TABLE.6
Variance ratios (R ) of Design Discharges

| μ= | 1.478 | 2.000 | 3.000 | 4.000 | 5.130 | 6.000 | 7.000 | 8.000 |
|---|---|---|---|---|---|---|---|---|
| var= | 2.351 | 4.727 | 8.273 | 12.091 | 17.027 | 22.727 | 27.545 | 37.545 |
| Qo = | 805.097 | 677.600 | 553.700 | 469.446 | 400.648 | 344.554 | 313.350 | 294.884 |
| $R_1 = \frac{Var(Q_T)_{AMS/PD-ED}}{Var(Q_T)_{AMS/NB-ED}}$ | | | | | | | | |
| 5 | 0.915 | **0.952** | 0.847 | 0.846 | 0.844 | 0.829 | 0.831 | 0.806 |
| 10 | **0.943** | **1.053** | 0.888 | 0.886 | 0.883 | 0.870 | 0.871 | 0.850 |
| 20 | **0.959** | 1.127 | **0.915** | **0.913** | **0.909** | 0.899 | 0.899 | 0.881 |
| 50 | **0.972** | 1.197 | **0.939** | **0.936** | **0.933** | **0.924** | **0.924** | **0.910** |
| 100 | **0.978** | 1.237 | **0.951** | **0.949** | **0.946** | **0.938** | **0.938** | **0.926** |
| 200 | **0.983** | 1.268 | **0.960** | **0.958** | **0.955** | **0.949** | **0.948** | **0.938** |
| 500 | **0.987** | 1.300 | **0.969** | **0.967** | **0.964** | **0.959** | **0.958** | **0.950** |
| $R_2 = \frac{Var(Q_T)_{AMS/PD-GLD}}{Var(Q_T)_{AMS/NB-GLD}}$ | | | | | | | | |
| 5 | 0.898 | 1.462 | 1.457 | 1.484 | 1.542 | 1.672 | 1.686 | 1.877 |
| 10 | 0.866 | 1.332 | 1.373 | 1.410 | 1.464 | 1.576 | 1.589 | 1.750 |
| 20 | 0.860 | 1.255 | 1.313 | 1.351 | 1.402 | 1.498 | 1.511 | 1.647 |
| 50 | 0.864 | 1.194 | 1.256 | 1.294 | 1.338 | 1.419 | 1.431 | 1.542 |
| 100 | 0.871 | 1.164 | 1.225 | 1.261 | 1.301 | 1.372 | 1.384 | 1.482 |
| 200 | 0.878 | 1.141 | 1.200 | 1.234 | 1.271 | 1.335 | 1.346 | 1.432 |
| 500 | 0.888 | 1.119 | 1.174 | 1.205 | 1.239 | 1.294 | 1.304 | 1.380 |
| $R_3 = \frac{Var(Q_T)_{AMS/PD-ED}}{Var(Q_T)_{AMS/PD-GLD}}$ | | | | | | | | |
| 5 | 1.714 | 1.606 | 1.104 | 0.851 | 0.680 | 0.644 | 0.523 | 0.427 |
| 10 | 2.151 | 1.913 | 1.252 | **0.941** | 0.739 | 0.694 | 0.559 | 0.454 |
| 20 | 2.447 | 2.115 | 1.347 | **0.998** | 0.777 | 0.726 | 0.583 | 0.472 |
| 50 | 2.698 | 2.285 | 1.427 | **1.048** | 0.810 | 0.754 | 0.603 | 0.487 |
| 100 | 2.823 | 2.370 | 1.468 | **1.073** | 0.827 | 0.769 | 0.614 | 0.495 |
| 200 | 2.913 | 2.432 | 1.498 | **1.091** | 0.840 | 0.780 | 0.622 | 0.501 |
| 500 | 2.997 | 2.490 | 1.526 | 1.109 | 0.852 | 0.790 | 0.630 | 0.507 |
| $R_4 = \frac{Var(Q_T)_{AMS/NB-ED}}{Var(Q_T)_{AMS/NB-GLD}}$ | | | | | | | | |
| 5 | 1.681 | 2.467 | 1.899 | 1.492 | 1.243 | 1.300 | 1.061 | 0.995 |
| 10 | 1.976 | 2.419 | 1.937 | 1.497 | 1.226 | 1.257 | 1.020 | 0.934 |
| 20 | 2.193 | 2.356 | 1.933 | 1.478 | 1.198 | 1.211 | 0.979 | 0.881 |
| 50 | 2.398 | 2.279 | 1.910 | 1.448 | 1.162 | 1.158 | 0.934 | 0.826 |
| 100 | 2.512 | 2.231 | 1.890 | 1.426 | 1.138 | 1.125 | 0.906 | 0.793 |
| 200 | 2.602 | 2.190 | 1.872 | 1.406 | 1.118 | 1.097 | 0.883 | 0.766 |
| 500 | 2.695 | 2.145 | 1.849 | 1.383 | 1.095 | 1.067 | 0.858 | 0.737 |

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