Data Fusion in Underwater Environment

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Abstract - Submarines and Ships of this time are furnished with multi sensors (structure mounted array, towed array and so forth.,) making them contemporary in following multi focuses in submerged environment. This paper elucidates Data Fusion calculations, taking into account multi-sensor target information of stages in the arrangement adrift. Two-dimensional following is grasped utilizing Modified Gain Bearings just Extended Kalman Filter in each accessible channel. In this methodology, every sensor utilizes one estimator to remove a state vector and its related covariance grid from its separate sensor estimations. Every channel yield is transmitted over an information connection to combination focus, where track-to-track relationship and state vector combination are performed following composite target state vector. Sonar information Pre-handling diminishs the clamor adequacy, gets difference of the uproarious information, embeds missed heading with evaluated direction and gives assessed orientation if there should arise an occurrence of missed or erroneous bearing estimations.

Keywords: Kalman Filter, Target Tracking, Data Fusion,sensor,simulation,estimation.

I. INTRODUCTION

A sensor device receives signals and does the processing which triggers the measurements that are functions of the signals. Sensors provide information for the controllability of the system. However, a single sensor is deficient in obtaining all the reliable information. Furthermore, as the complexity of a system increases, so does the number and variety of sensors required, providing complete description and effective control of the system. Multisensor systems institute the state of the art applications in areas such as robotics, defence, aerospace and so on. A multisensor system may different sensors which creates the need of data fusion.

Both quantitative and qualitative sensor fusion methods have been advanced in the literature. Quantitative methods are based on probabilistic and statistical methods of modelling and combining information, including statistical decision theory, Bayesian analysis and filtering techniques [1].

This paper is confined to the study of fusioning the state estimates of KF tracking the same target. The process noise is assumed to be dependent and the state estimates can be combined to improve accuracy by using conditional expectations of the target state, given the appropriate information. Mutambara [1] constructed the conditional expectations for the optimal fusion of estimates from several filters at any time, 'n'. These exact formulae are complicated and not suitable for practical use. Further analysis shows that the expressions can be simplified for the limiting case as n tends to infinity. In particular the limiting formula [2] for combining two filters is identical to one introduced earlier by Bar-Shalom [3].

Bar-Shalom's fusion technique is used for the underwater application as follows. Submarines/ ships (also called as observers) use different types of sensors like hull mounted array (HMA), towed array (TA), etc., to find out target motion parameters. Modern observers remain in passive mode of listening for most of the time. This work presumed that observer is using hull mounted array and towed array in passive mode to track target ships / submarines. Therefore, each sensor has its own set of targets in the track. The indecisiveness is how to decide whether two tracks from different systems represent the same target. If it is known two such tracks represent the same target and comes the problem of processing data fusion (also called as track fusion) [4].

We considered a situation of tracking a target moving at constant velocity in an ocean environment. For transparency, target is assumed to be at constant velocity but, this algorithm can be exploited for maneuvering targets also. The target is being tracked by HMA and TA operating from an observer. Two Kalman filters, as shown in Fig.1 process HMA and TA data in two different channels and two estimated target state vectors are available. First it is checked whether these estimates are related to single target and if it is so, then all these estimates are combined to get a better or more smoothed estimate. Data fusion is elaborately discussed in reference [3] and author tries to extend the theory for sea environment. Kalman filter requires the statistical characteristics like mean and variance of the noise in the bearing measurements. This filter also assumes the noise follows Gaussian distribution. In general, bearing measurements in underwater are highly corrupted with noise and Kalman filter fails with this noise. Hence pre-processing of the measurements is required to make the noise Gaussian, to find out the covariance of the noisy measurements and also to reduce the amplitude of the noise.

Added to the above, in underwater, sometimes, auto tracking of the target fails and the passive sonar tracks the target in manual mode and hence the measurements are not available continuously. It was also observed in underwater that the sonar measurement sometimes is spurious (the difference between the present and previous measurement being very high) and the same is treated as invalid. Pre-processing of the measurements estimate the bearings and replaces the bad or missed bearings. This on-line pre-processing continuously monitors the variance of the noise in the measurements and if any measurement is with more variance than predefined, this process replaces the same with the estimated bearing.

The well known classical estimator, that is the Modified Gain Bearings only Extended Kalman filter (MGBEKF) [5] - [8] is a right candidate for this application. Extensive simulation is carried out and the results of MGBEKF are compared with that of Unscented Kalman Filter (UKF) [9]-[11]. It is observed that the results are almost same in both the situations. As the computation is less with MGBEKF w. r. to UKF and convergence time is slightly faster in MGBEKF than that of UKF, in this paper MGBEKF is used to estimate the target motion parameters. Section 2 describes mathematical modelling of the algorithm including the fusion of state vectors. Section 3 is about pre-processing of the measurements. The simulation of the algorithm and results obtained are presented in section 4. The limitations of the algorithm are discussed in section 5. Finally the paper is concluded in section 6.

II. MATHEMATICAL MODELLING

It is proposed to do data fusion of hull-mounted array and towed array state vector outputs. These sensors generate bearings of the target. The well known classical estimator, that is MGBEKF, is explored for this application. Whenever the input data is available, the filter estimate is updated. The alternative derivation of the modified gain function [5], [6] of Song and Speyer's extended Kalman filter is slightly modified. The mathematical modelling of input measurements, Kalman filter and the outputs is as follows. Let the target state vector be X_s (k) where

$$X_s(k) = [\dot{x}(k) \quad \dot{y}(x) \quad R_x(k) \quad R_y(k)]^T \tag{1}$$

where $\dot{x}(k)$ and $\dot{y}(k)$ are target velocity components and, R_X (k) and R_Y (k) are range components respectively. The target state dynamic equation is given by

$$X_S(k+1) = \phi \, X_S(k) + b(k+1) + \Gamma \omega(k) \tag{2}$$

where ϕ and b are transition matrix and deterministic vector respectively.

The transition matrix is given by

$$\phi = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ t & 0 & 1 & 0 \\ 0 & t & 0 & 1 \end{bmatrix}$$

Where t is sample time

$$b(k+1) = [0 \quad 0 \quad -(x_0(k+1) - x_0(k)) \quad -(y_0(k+1) - y_0(k))]$$

and

$$\Gamma = \begin{bmatrix} t & 0 \\ 0 & t \\ \dfrac{t^2}{2} & 0 \\ 0 & \dfrac{t^2}{2} \end{bmatrix} \tag{3}$$

where x_0 and y_0 are observer position components. The plant noise, $\omega(k)$ is assumed to be zero mean white Gaussian with $E[\omega(k)\omega'(j)] = Q\delta_{kj}$. True North convention is followed for all angles to reduce mathematical complexity and for easy implementation. The bearing measurement, B_m is modelled as

$$B_m(k+1) = \tan^{-1}\left(\frac{R_x(k+1)}{R_y(k+1)}\right) + \varsigma(k) \tag{4}$$

where $\varsigma(k)$ is error in the measurement and this error is assumed to be zero mean Gaussian with variance σ^2. The measurement and plant noises are assumed to be uncorrelated to each other. Eqn. (4) is a non-linear

equation and is linearized by using the first term of the Taylor series for R_x and R_y. The measurement matrix is obtained as $H(k+1) = \begin{bmatrix} 0 & 0 & \hat{R}_y(k+1|k)/\hat{R}^2(k+1|k) & -\hat{R}_y(k+1|k)/\hat{R}^2(k+1|k) \end{bmatrix}$ (5)

Since true values are not known, the estimated values of R_x and R_y are used in the above equation. The covariance prediction is

$$P(k+1|k) = \phi(k+1|k)P(k|k)\phi^T(k+1|k) + \Gamma Q(k+1) + \Gamma^T$$ (6)

The Kalman gain is

$$G(k+1) = P(k+1|k)H^T(k+1) * \left[\sigma^2 + H(k+1)P(k+1|k)H^T(k+1)\right]^{-1}$$ (7)

The state and its covariance corrections are given by

$$X(k+1|k+1) = X(k+1|k) + G(k+1) * \left[B_m(k+1) - h(k+1, X(k+1|k))\right]$$ (8)

where $h(k+1), X(k+1|k)$ is the bearing using predicted estimate at time index $k+1$

$$P(k+1|k+1) = \begin{cases} \left[I - G(k+1)g\left(B_m(k+1), X(k+1|k)\right)\right] * \\ P(k+1|k) * \\ \left[I - G(k+1)g\left(B_m(k+1), X(k+1|k)\right)\right]^T + \\ \sigma^2 G(k+1)G^T(k+1) \end{cases}$$ (9)

where g (.) is modified gain function as defined in [6]. The value of g is

$$g = \begin{bmatrix} 0 & 0 & \cos B_m/\left(\hat{R}_x \sin B_m + \hat{R}_y \cos B_m\right) & -\sin B_m/\left(\hat{R}_x \sin B_m + \hat{R}_y \cos B_m\right) \end{bmatrix}$$ (10)

Since true bearing is not available in practice, it is replaced by the measured bearing to compute the function g (.).
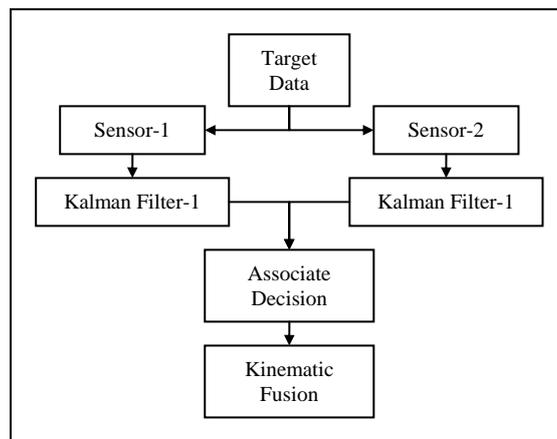

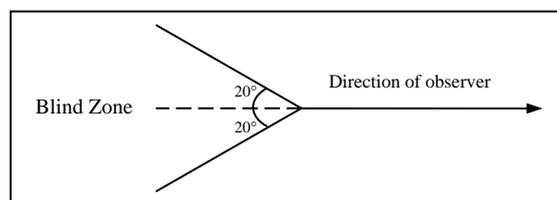Fig.1 Block diagram of state vector fusion
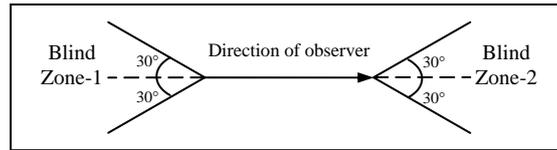

Fig. 2 (a). Blind zone for HMA
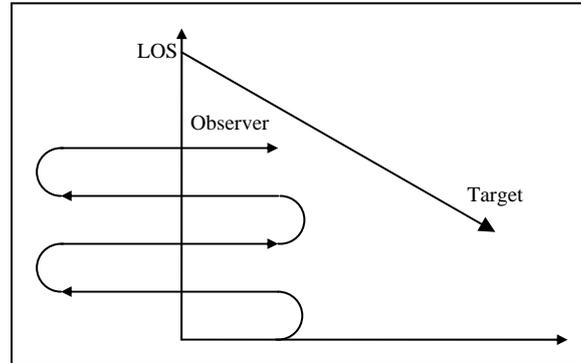
Fig. 2(b). Blind zone for TA

Fig. 3 TMA with single observation platform in S-manoeuvre

A. Fusion of State Vectors

Two separate identical MGBEKF's are used to process the HMA and TA data and at this stage it is assumed that the two estimates of target are available. Now it is to be checked, whether the HMA and TA are tracking different targets or same target. This process is called data fusion and is described as follows. Let $\hat{X}^i(k)$ be the state of a target estimated by a HMA and let $P^i(k)$ be covariance of $\hat{X}^i(k)$.

Let $\hat{X}^j(k)$ be the state of a target estimated by TA and let $P^j(k)$ be the covariance of $\hat{X}^j(k)$. Both are current estimates. It is useful to test the hypothesis that these estimates are for the same target or not. The difference in these estimates is given by [3].

$$\hat{\Delta}^{ij}(k) = \hat{X}^i(k) - \hat{X}^j(k) \tag{11}$$

Similarly the difference in the true states is given by

$$\Delta^{ij}(k) = X^i(k) - X^j(k) \tag{12}$$

Let $\quad \tilde{\Delta}^{ij}(k) = \Delta^{ij}(k) - \hat{\Delta}^{ij}(k)$

$$= \left(X^i(k) - \hat{X}^i(k)\right) - \left(X^j(k) - \hat{X}^j(k)\right)$$

$$= \left(\tilde{X}^i(k) - \tilde{X}^j(k)\right) \tag{13}$$

The following hypothesis is used.

If $H_0 : \Delta^{ij}(k) = 0$, then there is only one target and if $H_1 : \Delta^{ij}(k) \neq 0$ then there are two different targets. As the plant noise is not independent

$$T^{ij}(k) = E\left\{\left[\tilde{X}^i(k) - \tilde{X}^j(k)\right]\left[\tilde{X}^i(k) - \tilde{X}^j(k)\right]^T\right\}$$

$$= P^i(k) + P^j(k) - P^{ij}(k) - P^{ij'}(k) \tag{14}$$

where P^{ij} is the cross covariance. Assuming that the estimation errors are Gaussian, the test of H_0 vs H_1 is as follows: H_0 is accepted if $d = \hat{\Delta}^{ij}(k^T)\left[T^{ij}(k)\right]^{-1}\hat{\Delta}^{ij}(k) \leq \delta,$ else H_1 is accepted. The threshold is such that $P\{d > \delta | H_0\} = \alpha$, where $\alpha = 0.05$. The choice of this threshold is based on the fact that the above Gaussian assumption 'd' has a chi-square distribution with n_x degrees of freedom. If H_0 is accepted, then the two estimates $\hat{X}^i(k)$ and $\hat{X}^j(k)$ can be combined as follows.

$$\hat{X}^{ij} = \hat{X}^i(k) + \left[P^i(k) - P^{ij}(k)\right] * \left[P^i(k) + P^j(k) - P^{ij}(k) - P^{ij'}(k)\right]^{-1} * \left[\hat{X}^j(k) - \hat{X}^i(k)\right] \tag{15}$$

The covariance associated with the fused estimate is given by

$$M^{ij} = P^i(k) + \left[P^i(k) - P^{ij}(k)\right] * \left[P^i(k) + P^j(k) - P^{ij}(k) - P^{ij}{}'(k)\right]^{-1} * \left[P^i(k) - P^j(k)\right] \quad (16)$$

III. Pre-processing of the measurements

In underwater, noise in the bearing measurements is very high. The measurements available at every second are averaged over 20 seconds so that the variance of the noise is reduced by a factor of 20. The input to the MGBEKF is available at every 20th second and the target motion parameters are updated at 20 second interval. In under water, the speed of the vehicle is not much and so updating of the solution by 20 seconds will not hamper the weapon control process. If one or more measurements are not available during 20 seconds interval, the average will be done according to the no. of samples available.

If all the measurements are not available during 20 seconds interval, then 20 second average measurement is replaced with a bearing measurement estimated by using bearing rate, which is calculated with the measurements available till then the mean or bias in the measurements is assumed to be zero. If it is not zero, then the target motion parameters contain bias. At the pre-processing stage, there is no way to find out mean of the noise. If bias is present and is known by some means, all the measurements are subtracted by this mean. The measurement is bad or good, is determined by its variance of the noise. If variance is high than the assumed one, then it is treated as bad or not available measurement.

A. Variance of the Noise in the Measurement

The variance of error in each bearing measurement is calculated as follows. Consider a simple linear regression model for the bearings given by $b = a_0 + a_1 t + \varepsilon$ where a_0 and a_1 are regression coefficients, ε is distributed with zero mean and unknown variance σ^2, t is time variable and b is bearing. Here, a_0 represents the intercept on bearing axis and a_1, the bearing rate. Let there be n measurements in sample duration of 20 seconds. From simple regression analysis [12]

$$a_1 = \frac{\sum_{i=1}^{n}(b_i - \bar{b})(t_i - \bar{t})}{\sum_{i=1}^{n}(t_i - \bar{t})(t_i - \bar{t})} = \frac{\sum_{i=1}^{n}(db_i \cdot dt_i)}{\sum_{i=1}^{n}(dt_i \cdot dt_i)} \quad (17)$$

where dt and db are the changes in time and bearings respectively between two successive measurements, $\bar{t}$ and $\bar{b}$ represent the average of time and bearings respectively in the 20-second time sample. The variance of the noise is given by

$$\sigma^2 = \frac{\sum(b_i - \bar{b})^2 + a_1^2 \sum\left(\frac{b_i - (t_i - \bar{t})}{a_1}\right) - 2a_1 \sum\left((b_i - \bar{b})(t_i - \bar{t})\right)}{n} \quad (18)$$

The measurement is bad or good, is determined by its variance of the noise. If variance is high than the assumed one, then it is treated as bad or not available measurement.

B. Estimation of bearing

In general, the state vector is with $\dot{x}(k)$ & $\dot{y}(k)$ (target velocity components) and $\dot{x}(k)$ & y (k) (target position) components. By rotating the coordinate system such that $+y$ axis lies along the latest bearing entered, the state vector represents relative position and relative velocity at the time of entry of the latest bearing and it is given by $\left[\dot{B}(k) \quad \dfrac{\dot{R}(k)}{R(k)} \quad B(k) \quad 1\right]$,

where the first and second elements represent bearing rate and range rates respectively. Br represents the estimated error of the given input bearing and R represents the relative range of the target with reference to the observer.

Similar target state vector was utilized by Aidala and Hammel [13] in modified polar coordinate based Kalman filter. The estimated bearing at any time is given by the present bearing plus Br. If the bearing is missed at any time, then the present bearing is the previous bearing plus bearing rate multiplied by time between the samples (assuming the measurements are available with fixed interval). For obtaining this required state vector, covariance matrix of Pseudo Linear Estimator [14] is built up using Cartesian coordinate state vector and then converted to modified polar coordinate state vector in such a way that the $+y$ axis lies along the line of sight.

IV. Simulation and Results

Here HMA and TA are generating the bearing measurements of the target. As TA operates at low frequencies, it can obtain the contact of the target from longer ranges (sayn 10 km). HMA can give data of targets with maximum range 7 km. As the submarine targets can be observed from a maximum distance of 5 km, the range of the operation in simulation is limited to 5 km. The variance of the error in the bearing measurement is assumed to be uniform in case of HMA. In case of TA, it varies according to the look-angle. The blind zones in both cases are shown in Fig.2 and Fig.3. (Arrow in the figures indicate the direction of the observer.)

In these zones, the target cannot be tracked by sensors and hence   the bearings will be estimated as explained in the pre-processing. It is assumed that TA is towed behind the observer with a cable and length of the cable is known (say 400 meters). For the implementation of the algorithm, the initial estimate of target state vector is chosen as follows. As only bearing measurements are available, it is not possible to guess the velocity components of the target. So these components are each assumed as 5 m/sec, which are close to the realistic speeds of the vehicles in underwater. The range of the day is utilized in the calculation of initial position components of the target as follows

$$X(0|0) = \begin{bmatrix} 5 & 5 & 5000\sin B_m & 5000\cos B_m \end{bmatrix}^T \tag{19}$$

It is assumed that the initial estimate, $X(0|0)$ is uniformly distributed Then the elements of initial covariance can be written as

$$P(0|0) = Diagonal \left(4 * X_s(i)^2 / 12\right) \tag{20}$$

where i = 1,2,3 and 4.

Table-1

Parameter	Scenario 1	Scenario 2
Initial Range (m)	4000	4000
Initial Bearing (deg)	0	0
Target Speed (knts)	30	6
Target Course (deg)	135	135
Observer Speed (knts)	20	20
Observer Course (deg)	90	90
Error in the bearing (deg) (1σ)	1.0	2.0

The observer is assumed to be doing 'S' maneuver on the line of sight at a constant speed with a turning rate of 3 deg. /sec. The observer moves initially at 90 degrees course for a period of two minutes and then it changes its course to 270 degrees. At 7th , 12th and 17th minutes, the observer changes its course from 270 to 90, 90 to 270 and 270 to 90 degrees respectively, as shown in Fig. 3. It is assumed that the noise in the bearing measurements of HMA and TA is of additive zero mean Gaussian noise and the bearing measurements are generated continuously every half second. Pre-processing is carried out and the pre-processed bearing for every 20 seconds is passed on to tracking algorithms. As fourty samples are smoothed, the noise in the smoothed bearing follows almost Gaussian. The selection of tracking algorithm is carried out as follows. Three algorithms – MGBEKF, UKF and Particle Filter (PF) are considered for this nonlinear tracking problem. In real situation, multitarget tracking is to be carried out and as PF requires considerable time (and this much time is not allowed) for generation of target motion parameters, the remaining two algorithms are realized. Number of scenarios are tested by changing the course of the target in steps of 1 degree in such a way that the angle between the target course and line of sight is always less than 55 degrees, as only closing targets are of interest to the observer.

The initialization of target state vector and its covariance matrix, no. of Monte Carlo runs are chosen as same in the evaluation of MGBEKF and UKF for this application. For simplicity, the target is assumed to be moving at constant velocity. The results of these scenarios in Monte Carlo simulation are noted and it is found that the observability in the target motion parameters has taken place after the completion of the observer's first maneuver. In general, the error allowed in the estimated target motion parameters in underwater are eight percent in range, 0.2 degrees in bearing, three degrees in course and three meters/sec in velocity estimates. Around 80 % required solution is realized after observer's second maneuver and 90 to 95 % required solution is realized after the third. The simulation is carried out for a period of thirty minutes. For the purpose of

presentation, the results of UKF and MGBEKF against scenarios 1&2 as shown in Table 1. are presented in Fig. 4(a) to 4(c) and Fig. 5(a) to 5(c).
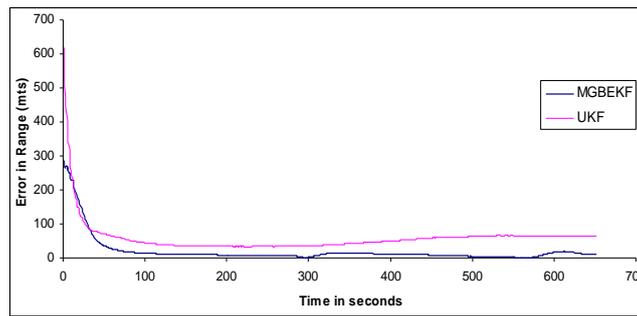

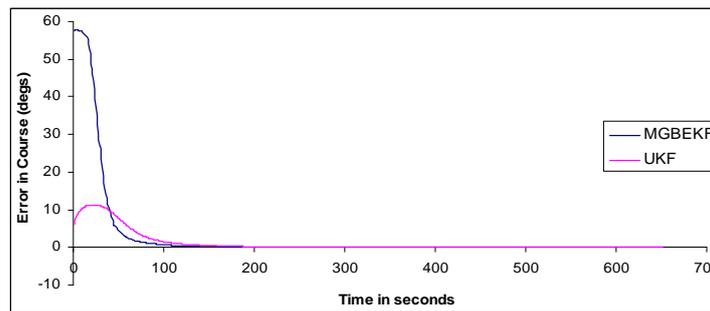Fig. 4(a). Error in range estimate

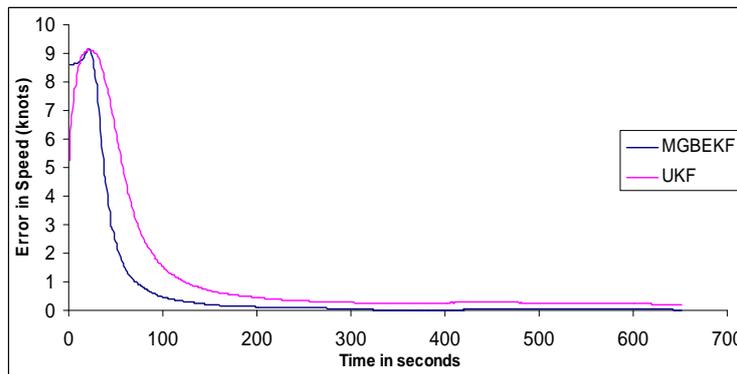Fig. 4(b). Error in course estimate

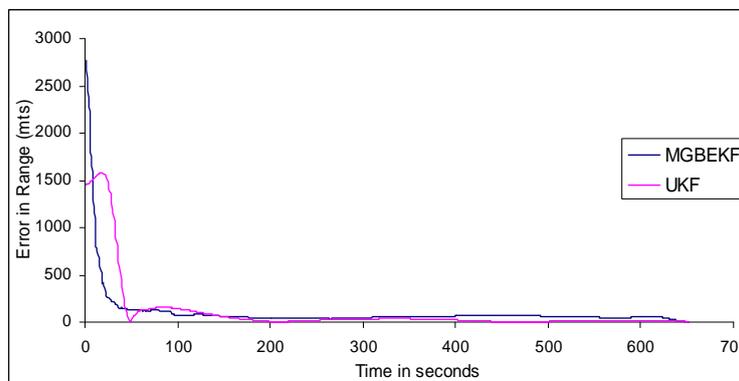Fig. 4(c). Error in speed estimate

Scenario – 2

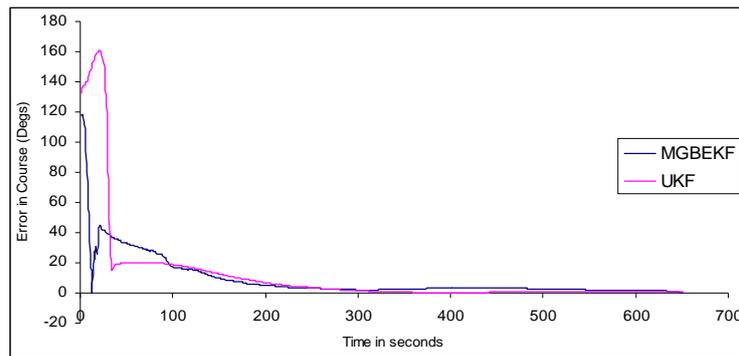Fig. 5 (a). Error in range estimate
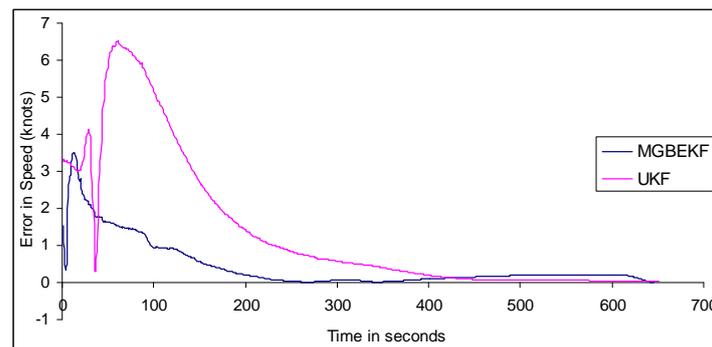
Fig. 5(b). Error in course estimate

Fig. 5(c). Error in speed estimate

The errors in the range, course and speed estimates are shown in these figures. From the results, it is observed that almost same results are obtained with both the estimators. In case of HMA the range, course and speed are getting converged with required accuracies at 147, 207 and 194 seconds for scenario 1 and at 194, 403 and 268 seconds for scenario 2 respectively. In case of TA the range, course and speed are getting converged with required accuracies at 163, 227 and 238 seconds for scenario 1 and at 197, 406 and 293 seconds for scenario 2 respectively. Though almost same results are obtained with UKF and MGBEKF, as MGBEKF is able to generate the solution a little bit fast, it is chosen as the algorithm for passive target tracking with data fusion. Now it is assumed that both sensors are utilized for the same scenario, that is scenario No.1 in Table 1. So, there are two Kalman filters working in parallel. The bearing measurements are corrupted with different magnitudes of noise, but maintaining the same Gaussian distribution and variance.

Data fusion process is carried out after obtaining convergence in the estimated solution. The parameter'd' (described in section 2) follows chi-square distribution and its theoretical value with four degrees of freedom is 0.96. The confidence level, α, is assumed as 0.05. To take care of false alarms, the decision is taken only when the value of 'd' is more than the theoretical threshold value consequently two times. In Monte-Carlo simulation it is observed that the value of d is never more than the threshold value 0.96. Hence it is decided that the two sensors are tracking the same target. So, the target state and its covariance are combined as per eqns. (15) & (16).

The estimated and simulated paths of the target are shown in Fig. 6. Now let us assume that HMA is tracking a ship target as described in the previous scenario and TA is tracking a submarine target, as described under scenario 2 in Table 1. It is observed that the value of d is obtained as 1.34, more than the threshold 0.96. Hence it is declared that the two sensors are tracking two targets. The observer path along with the estimated and simulated paths of the target are shown in Fig. 7. In figures 6 & 7, after around 300 seconds, estimated path of the target is almost coinciding with that of simulated path.
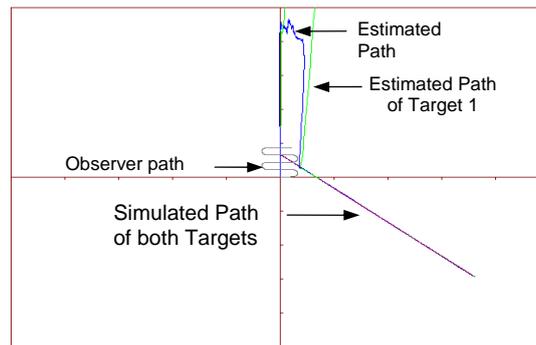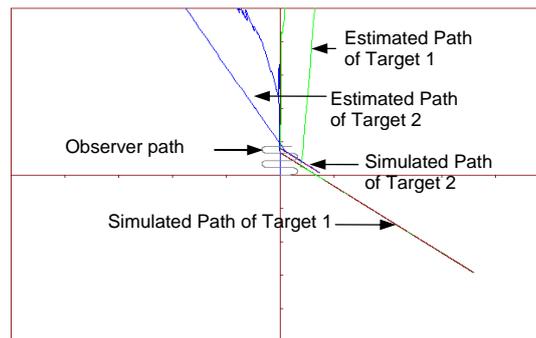
Fig.6. Data fusion of HMA and TA for scenario 1

Fig.7. Data fusion of HMA and TA for scenario 1 & 2

5. Limitations of the Algorithm

Angle on Target Bow (ATB) is the angle between the target course and line of sight. When ATB is more than 60 degrees, the distance between the target and observer increases as time increases and hence bearing rate decreases. The algorithm cannot provide good results when the measurement noise is more than 2° rms or when the target is going away from the observer (with ATB more than 60 degrees). In general, sonar can listen to a target when SNR is sufficiently high. When SNR becomes less, auto tracking of the target fails, the sonar tracks the target in manual mode and the measurements are not available continuously.

The bearings available in manual mode are highly inconsistent and are not useful for good tracking of the target. In underwater it is also possible that sonar measurement sometimes is spurious (the difference between the present and previous measurement being very high) and the same is treated as invalid. Pre-processing of the measurement process estimates the bearings and replaces the bad, missed or invalid measurements with the estimated bearings. In this algorithm, it is assumed that good track continuity is maintained over at least first four minutes of the simulation period (Minimum four minutes time is required to obtain the convergence in bearing rate). This means that propagation conditions are satisfactory during this period as well as track continuity is maintained during ownship maneuvers.

6. Conclusion

The authors excercised MGBEKF for underwater bearings only passive target tracking with data fusion, in particular state vector fusion, to find out whether there are two targets or one target. The author has attempted to pre - process the passive sonar bearing measurements to reduce and find out the statistical characteristics of the noise in the measurements, etc., so that the data can be effectively used for tracking an under water target by MGBEKF effectively. Each sensor has its own set of targets in the track. Method is given to decide whether two tracks from different systems represent the same target or not. If it is decided two such tracks represent the same target, then fused estimate of that target is presented.

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