A New Approach for Estimation of Penalty Parameter with Tolerance Stack-Up

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***Abstract*— Dimensional variations due to the geometric tolerance and joint clearance lead to tolerance stack-up. The multi-body dynamic analysis (MBD) of stack-up mechanism is contact driven. Making and breaking contact is a discontinuous event. It causes high impact force with discontinuous acceleration. There are two approaches for the estimation of normal force by virtue of contact. The impact function approach holds good when the actual penetration of colliding bodies is known. The restitution approach is used when exact impact values are not available. Both the models work on the penalty regularization method. The penalty regularization parameter is calculated from the material contact stiffness and the depth of penetration of the colliding bodies. A new technique is developed to estimate the penalty parameter for the clearance in the revolute joint. A methodology is described to estimate the penalty parameter, which is based on material stiffness and speed of the input link. A relation is established for the estimation of the penalty parameter. The estimated penalty parameter is the function of input speed; consequently it can be estimated for various input speeds with a single equation. Iterations of the kinematic and dynamic simulation are not required. Simulation time is also reduced by means of exact penalty regularization parameter. A case study of crank and rocker mechanism with the clearance revolute joint was simulated for the range of speed for the validation.**

Keywords: Restitution; Penalty parameter; Tolerance stack-up; Clearance joint

I. Introduction

Manufacturing limitations introduces the joint clearance in a kinematic revolute joint. The assembly of the components and the relative motion between then are possible by virtue of the joint clearance. In a mechanism with the tolerance stack-up; kinematic sensitiveness and tolerance sensitiveness both influences the performance of a mechanism. The kinematic sensitiveness refers to the variations in the velocity and the acceleration of mechanism with reference to position and input at a joint. In conventional kinematic analysis the dimensions of the individual members are the fixed and without the dimensional variations. Tolerance sensitiveness refers to the geometric variation of one assembly position relative to another. MBD analysis of stack-up mechanism is based on the contact force. The contact detection is important for the contact-impact condition. Once the contact is detected evaluation of contact forces can be done. This approach is known as penalty method or compliant method [4]. To simulate a kinematic mechanism, penalty regularization parameter is used as an input for the MBD analysis. The penalty regularization is a modelling technique in mechanics, in which a constraint is enforced mathematically by applying forces along the gradient of the constraint. In an impact analysis, when the contact between the bodies is detected, a contact force perpendicular to the plane of collision is applied. This force is typically applied as a spring-damper element [2, 3].

The Hertzian contact theory remains the foundation for almost all of the available force models, but by itself, it is not appropriate for most impacts in practice, due to the amount of energy dissipated during the impact. The Contact force models are based on the Hertzian law. A damping term used to accommodate the energy loss during the impact process for small or moderate impact velocities [2]. The penalty regularization has the advantage of simplicity for contact driven problems. Additional equations or variables are not required. It is particularly useful when treating intermittent contact. Additionally, a penalty formulation is easily interpreted [7]. Due to approximations and round-off errors, many numerical solutions do not satisfy the constraints exactly, a phenomenon known as “drift”. Penalty based formulations have also been used to control the drift phenomenon. The augmented Lagrangian formulation is probably the most robust and efficient method to solve the penalty based formulation [13].

Exact or practical value of penetration of the colliding bodies is essential for the impact force model. Impact force model and restitution or POSSION’s model requires the penalty regularization. An approach is given for the estimation of penalty parameter. A mathematical relation is modified in terms of magnitude of joint clearance and speed of the drive link. The estimation of the penalty parameter is useful for the contact analysis by restitution model, which also reduces the simulation time. The trends of results obtained from restitution model analysis are harmonious with the analysis results of ideal kinematic mechanism.

II. CONTACT PROPHESY

A large amount of normal force is generated, when two bodies come into the sudden contact. After impact the velocities of the contact bodies' changes its sign. The accelerations are almost discontinuous, and have a large spike. The bodies usually separate because of the contact forces or impulses. The energy loss during the collision is usually modelled as a damping force that is specified with a damping coefficient or a coefficient of restitution. The estimation of the normal contact force between colliding bodies is based on depth of penetration and penetration velocity between the two bodies. When a contact is active, the stiffness in the direction of the normal force is high; if a contact is inactive there is no stiffness in the direction of increasing contact.

There are two contact condition considered for contact prediction. The contacting bodies are assumed to be infinitely rigid, giving rise to the inequality condition $e_{max} \geq 0$. Subsequently, the bodies are assumed to be deformable[11]. At contact, interpenetration occurs, which is known as actual approach, denoted by d, and e_{max} is the clearance between pin and bearing. There are two conditions, when interpenetration occurs between colliding bodies, $d > 0$ and $e_{max} < 0$. Without inter-penetration $d = 0$ and $e_{max} > 0$. Combining both the conditions, $e_{max} + d \geq 0$, which implies $e_{max} = -d$ for the case of inter-penetration. Magnitude of d will depends on the contact model. Fig. 1 shows g and d with and without inter-penetration. Where R and r are the radius of bearing and pin respectively.

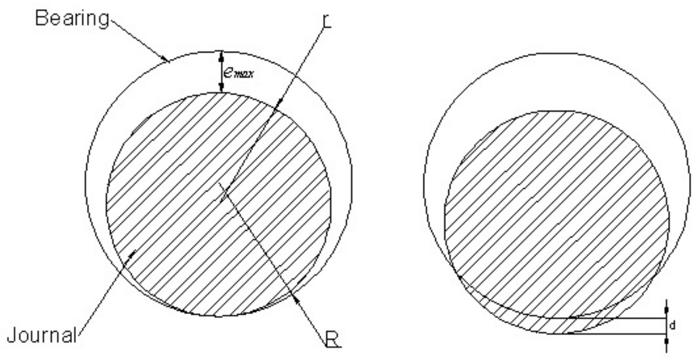

Fig. 1: Contact force model (i) Before Penetration (ii) After Penetration

The contact force has two important features. First, it has a continuous derivative, even at the contact surface. Penalty parameter P > 1; this feature improves stability during persistent contact. Second, is formally similar to the Hertz model, widely used in contact numerical models, which provide excellent results compared with experimental data in many applications [10].

An impulse is resulted from the collision of colliding bodies. It affects the momentum of the colliding bodies. Such contact exists for short period of time, which is impulsive or intermittent contact. For relatively long period of time, bodies which maintain continuous contact called persistent contact. There are two approaches to evaluate the normal force at the contact. These are impact force model and restitution model. Both force models result from a penalty regularization of the normal contact constraints. The force magnitude is a function of the constraint violation. Contact between rigid bodies theoretically requires that the two bodies not penetrate each other. This can be expressed as an inequality constraint. The contact force is the force at which mutual approach is not possible. These auxiliary constraints conditions is accomplished either through introduction of Lagrange multipliers or by penalty regularization [9].

The magnitude of the contact reaction force is equal to the product of material stiffness and penetration between contacting bodies. The disadvantage of the penalty regularization is setting of an appropriate penalty parameter. Furthermore, a large value for the material stiffness or penalty parameter can cause integration difficulties.

The auxiliary contact constraints are the impenetrability constraint, separating or normal force constraint, normal force be non zero at the contact and the persistency condition.

The impact force model can be obtained by replacing the first three auxiliary contact conditions with the Eq. (1) [7].

$$F_n = k(g^e) \tag{1}$$

Where, k is the material stiffness. The penalization becomes exact as k approaches infinity, but otherwise allows small violation of the impenetrability constraint.

The penalty regularization of the fourth contact constraint yields,

$$F_n = p\frac{dg}{dt} \tag{2}$$

In equation (2), p is the penalty parameter. The penalization is exact as p ≥ ∞, which carries the risk of ill conditioning. The equivalent contact stiffness K, is obtained by Goldsmith through the collision experiment of two spherical bodies is expressed as

$$K = \frac{4}{3\pi(\sigma_1 + \sigma_2)}\left[\frac{R_1 R_2}{R_1 - R_2}\right]^{\frac{1}{2}} \tag{3}$$

Where, R1 and R2 is the radius of two spheres respectively,

$$\sigma_1 = \frac{1 - \vartheta_1^2}{\pi E_1} \tag{4}$$

$$\sigma_2 = \frac{1 - \vartheta_2^2}{\pi E_2} \tag{5}$$

D is the damping coefficient and δ_n is the negative element normal deformation, can be expressed as

$$D = \frac{3K_c\,(1 - e^2)\,\delta_n}{4\bar{\delta}} \tag{6}$$

The penalty parameter is calculated with the stiffness of colliding bodies' material. Hartz's contact theory is used to estimate the contact stiffness. Hertz contact stiffness is calculated from Eq. (7). Dry clearance joint considered for the calculation of penalty value.

$$K_c = 2\ a\,E^* \tag{7}$$

Where, E^* is equivalent Young's Modulus of colliding materials, a is Hartz's contact radius. Mutual approach (Penetration) is calculated by the Eq. (8)

$$\delta = \frac{a^2}{R}\left[1 - \frac{2}{3}\left(\frac{a_0}{a}\right)^{3/2}\right] \tag{8}$$

The penalty regularization parameter is estimated form the developed Eq. (9)

$$P = K_c C_{ij}^e \tag{9}$$

Where, K_c is the Hartz contact stiffness in N/m, It is estimated based on the maximum tangential force acting at the revolute joint with respect to the drive velocity. C_{ij} is the maximum clearance at the revolute joint and 'e' is the force exponent. Force exponent is taken as 1.5.

III. Estimation of Penalty Parameter

Penalty parameter is calculated from the Eq. (9). The pin used at the revolute joint with clearance is colliding body. The impact force is generated due to the gravitational force and the tangential driving force due to the input velocity acting on the pin.

TABLE I. Tangential force acting on clearance joint pin.

Input Speed in RPM	Crank Coupler Pin force, N	Coupler Follower Pin force, N
100	0.01	0.01
200	0.04	0.04
300	0.09	0.10
400	0.17	0.18
500	0.26	0.27
600	0.38	0.39
700	0.52	0.54
800	0.67	0.70
900	0.85	0.89
1000	1.05	1.10
1100	1.27	1.33
1200	1.51	1.58
1300	1.78	1.85
1400	2.06	2.15
1500	2.37	2.47

The tangential force is estimated with the equations derived from the loop closure equations for one complete cycle of input link. Table I shows the estimated maximum tangential force for different input speed which is graphically represented in Fig. 2.

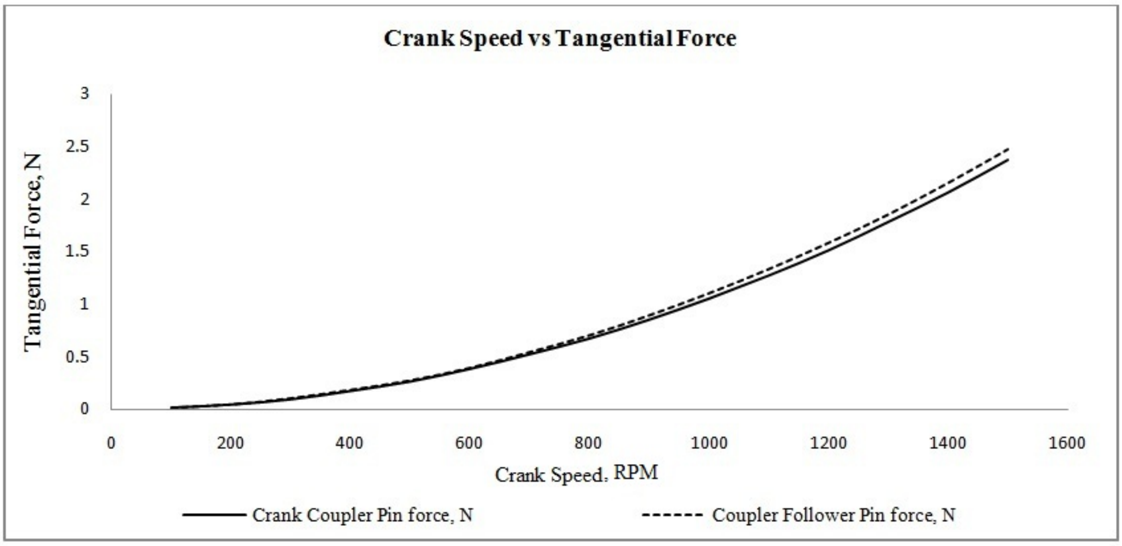

Fig. 2. Graphical plot of crank speed vs. tangential force

The contact stiffness is estimated by Eq. (7). Estimated penalty parameter for the range of speed with step size of 100 rpm is given in Table II, and graphically represented in Fig. 3. It shows that penalty parameter is varying linearly with drive link speed.

TABLE II. Penalty regularization parameter for clearance pin joint

Input Speed in RPM	Crank Coupler Pin	Coupler Follower Pin
100	97.33	99.40
200	194.67	189.85
300	292.00	300.17
400	389.33	402.72
500	486.66	493.23
600	584.00	592.79
700	681.33	697.54
800	778.66	794.18
900	875.99	895.50
1000	973.33	995.56
1100	1070.66	1094.71
1200	1167.99	1193.16
1300	1265.33	1291.09
1400	1362.66	1391.84
1500	1459.99	1491.83

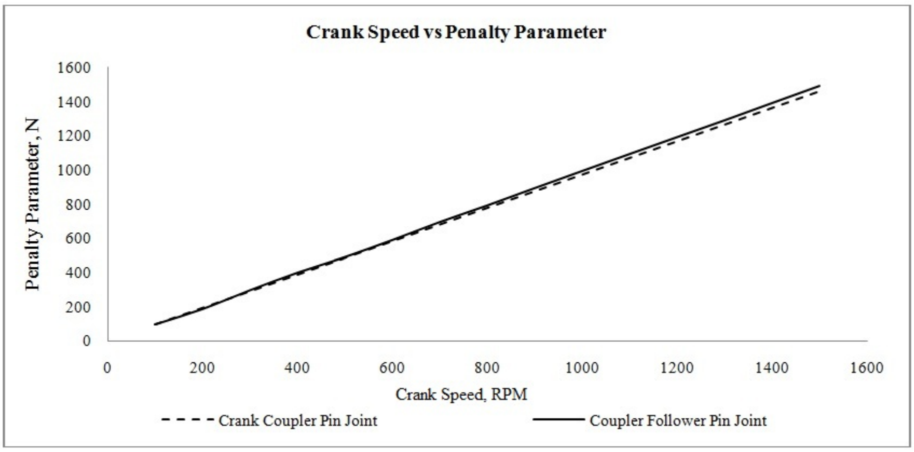

Fig.3 Graphical plot of crank speed vs. penalty parameter.

The mutual approach of colliding bodies is tabulated in Table III and graphically shown in Fig. 4. Polynomial trend of mutual approach is observed.

TABLE III. Mutual approach of colliding bodies.

Approach in Microns		
Input Speed in RPM	**Crank Coupler Pin**	**Coupler Follower Pin**
100	0.0090	0.0093
200	0.0227	0.0219
300	0.0390	0.0404
400	0.0572	0.0598
500	0.0770	0.0784
600	0.0982	0.1002
700	0.1206	0.1244
800	0.1441	0.1479
900	0.1686	0.1736
1000	0.1940	0.2000
1100	0.2203	0.2269
1200	0.2474	0.2546
1300	0.2753	0.2828
1400	0.3039	0.3126
1500	0.3332	0.3429

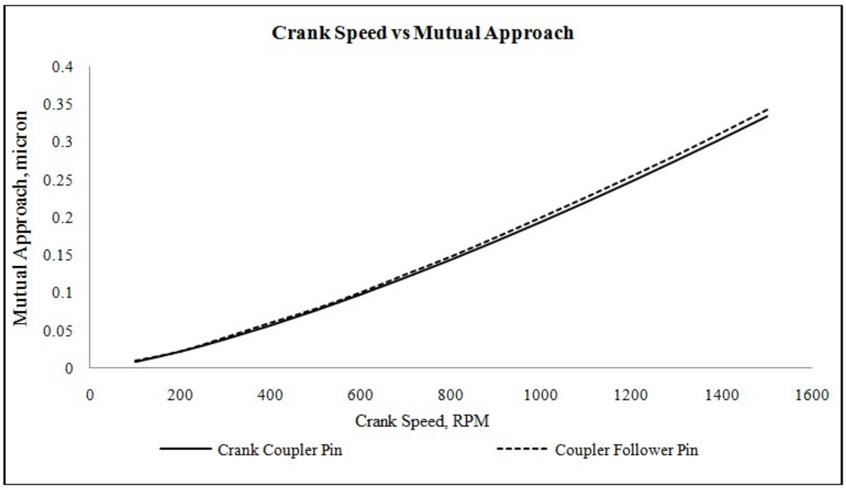

Fig.4 Graphical plot of crank speed vs. mutual approach

TABLE IV. Linkage Parameters with Maximum Stack up.

Tolerance Stack-Up For Four Bar Mechanism with Normal Tolerance Grade				
Linkage	**Basic Dimension**	**Tolerance**	**Max Link Dimension**	**Min Link Dimension**
Crank	108	± 0.3	108.3	107.7
Coupler	279.4	± 0.5	279.9	278.9
Follower	270.5	± 0.5	271	270
Fixed	254	± 0.5	254.5	253.5
Pin hole	10:H7	0.015	10.015	0
Pin	10:g6	0, -0.005	0	9.995
	10:g6	0, -0.014	0	9.986

IV. Analysis of Crank And Rocker Mechanism

A. Linkage Parameter

A CAD Model of crank rocker mechanism with geometric tolerance at the links and clearance at revolute joints is generated. Second model with nominal linkage dimension with ideal joints is generated and it is termed as ideal mechanism. Gravity force is considered in the analysis, which is acting vertically downward. Material for all linkages and pins is considered as steel with material properties given at Table IV. The detailed linkage parameters are given in Table V. Input joint, which is directly coupled to the drive, therefore considered as an ideal revolute joint, having single degree of freedom.

TABLE V. Material Properties.

Density, kg/mm^3	7.80E-06
Young's Modulus, N/mm^2	2.07E+05
Poisson's Ratio	0.29
Gravity Force, mm/sec^2	-9806.65
Input Speed (rad/sec)	1000

TABLE VI. Linkage Parameters.

Component	Length	I_{XX}	I_{YY}	I_{ZZ}	Mass
	mm	mm^4	mm^4	mm^4	kg
Crank	108.00	173.92	167.88	6.69	0.12
Coupler	279.40	2070.46	2056.16	15.49	0.29
Follower	270.50	1988.39	1974.43	15.28	0.28
Fixed	254.00	2103.45	2089.71	16.39	0.29
Pin	13.00	1.48	1.48	0.83	0.0226

Tolerances on the nominal dimensions are considered as per IS 2102:1993 and ISO 2768-1: 1989, details are given in the Table VI. The maximum clearance between the link hole and the pin is 42μ. Collared pins are used at the joints for the perfect contact of pin collar surface with link surface. Pin is used to connect the crank with the coupler and the joint further termed as clearance joint C_{ij}.

B. Kinematic Analysis

Crank and rocker mechanism with nominal link dimension and revolute joints with one degree of freedom, further termed as ideal crank and rocker mechanism. Another set of crank and rocker mechanism with tolerance stack is termed as stack mechanism. Kinematic analysis of both the mechanism was carried out and results are validated with analytical approach. Stack mechanism was analysed with two models i.e. impact function model as well as POSSION or restitution model. Fig.5 shows the angular velocity plot of ideal stack-up mechanism. The results are validated with the analytical approach.

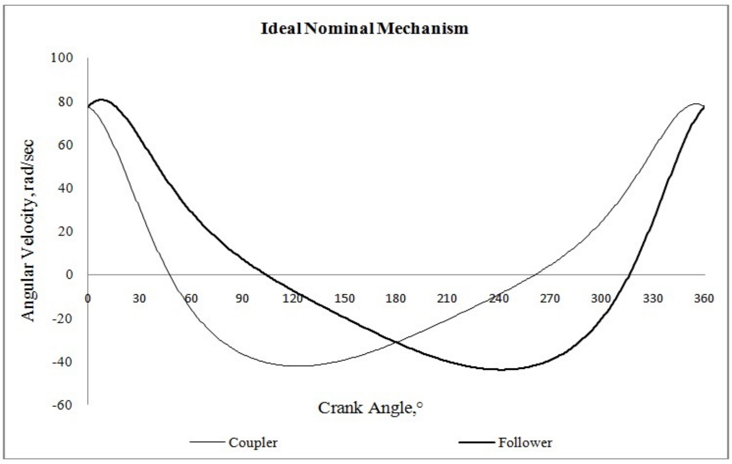

Fig.5 Velocity plot of coupler and follower for ideal stack-up mechanism case.

Maximum and minimum angular velocity results are tabulated in Table VII. 0.33 % and 0.15% variation in angular velocity of coupler and follower respectively, the variation is due to the change in the linkage dimensions due to the geometric tolerances. Angular velocity plot of stack-up mechanism with the restitution model is shown in Fig. 6. Increase in angular velocity of coupler and follower is observed at the initial stage of motion, at the initiation of the contact of pin with the bearing surface. Small spikes are observed between the crank angle 120° to 130° and 290° to 300°. Encircled regions where crank and coupler becomes normal to each other and coupler is perpendicular to the ground respectively. The angular velocity plot of coupler and follower for stack-up mechanism with impact model is given at the Fig.7.

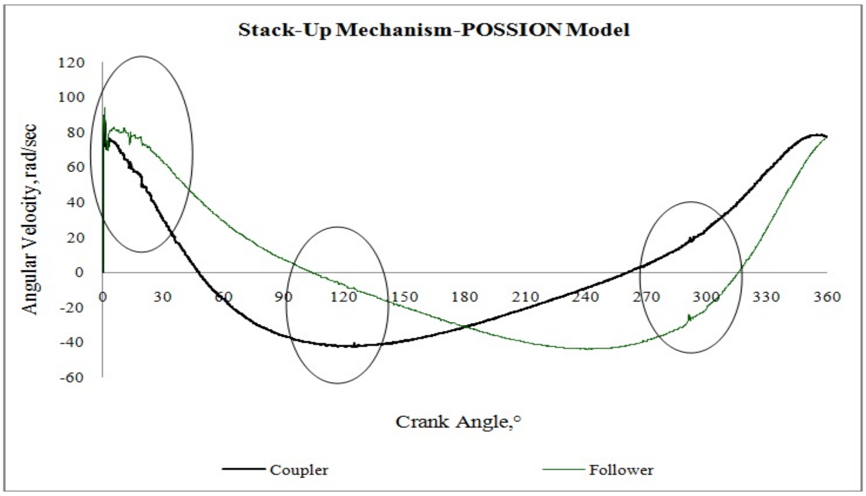

Fig.6 Velocity plot of coupler and follower for stack-up mechanism-POSSION Model.

Angular acceleration of coupler and follower for ideal stack-up mechanism is graphically shown in Fig. 8. Results of angular accelerations are tabulated in Table VIII.

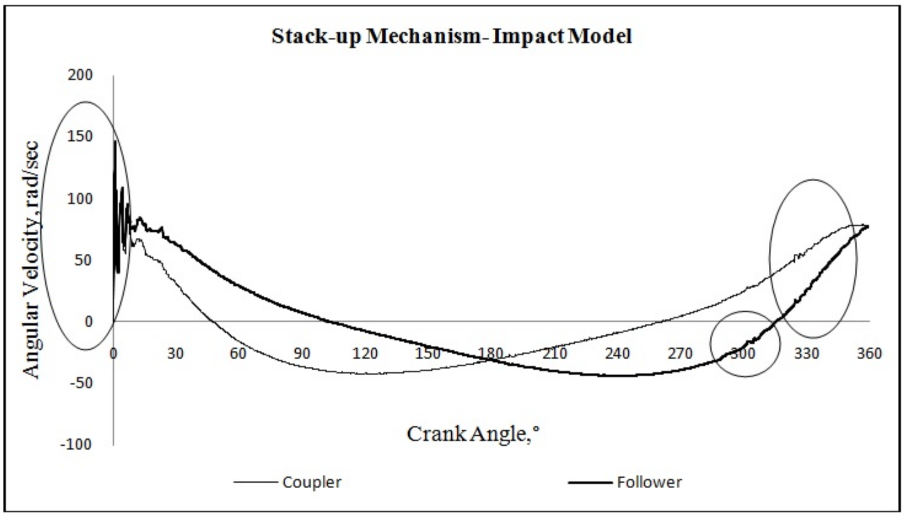

Fig.7 Velocity plot of coupler and follower for stack-up mechanism-impact model

TABLE VII. Summary of results (Angular Velocity).

Case	**Maximum Angular Velocity of Coupler, rad/sec**		**Maximum Angular Velocity of Follower, rad/sec**	
	Max	**Min**	**Max**	**Min**
Ideal Nominal Mechanism	78.696	-42.201	80.555	-43.819
Ideal Stack-up Mechanism	78.442	-42.138	80.289	-43.745
Stack-up Mechanism- POSSION Model	89.512	-42.677	94.018	-43.905
Stack-Up Mechanism - IMPACT Model	134.502	-42.461	146.082	-43.793
Ideal Analytical Model	78.697	-42.199	80.553	-43.818

TABLE VIII. Summary of results (Angular Acceleration)

Case	**Maximum Angular Acceleration of Coupler, rad/sec^2**		**Maximum Angular Acceleration of Follower, rad/sec^2**	
	Maximum	**Minimum**	**Maximum**	**Minimum**
Ideal Nominal Mechanism	7574	-11970	12484	-7512
Ideal Stack-up Mechanism	7537	-11921	12430	-7476
Stack-up Mechanism- POSSION Model	74069000	-9970500	7120800	-7086500
Stack-Up Mechanism - IMPACT Model	1719800	-1490300	1480000	-1383100
Ideal Analytical Model	7573	-11970	12485	-7511

In ideal stack-up mechanism, there is a variation in angular velocity and angular acceleration is 0.33% and 0.48% respectively with respect to analytical model. There are two modules impact force and restitution modules, for solving the stack-up mechanism with multi body dynamic software. Both the modules are contact driven. In restitution module 13.74% variation in maximum angular velocity of coupler with respect to the ideal or analytical case is recorded. 16.72% variation is observed in maximum angular acceleration of coupler.

Initial spikes are observed in both the model, which are due to the clearance present in the joint and the gravity effect. The nature of spikes is random in nature which shows that there is an impact between hole and pin of the connecting links. The impact method is more realistic than restitution method, the only limitation is the contact stiffness and depth of penetration should be real values.

Adopting the methodology for the estimation of penalty parameter path and function is generated [21]. Which shows that the approach used is appropriate for kinematic and dynamic simulation of stack-up mechanisms.

V. CONCLUSIONS

The estimated penalty regularization parameter form Eq. (9) is sensitive to the input speed and increasing linearly with respect to input speed. The penalty regularization parameter is one of the essential input data for the multi-body dynamic analysis. In case of colliding bodies due to the clearance at joint, the mutual approach is very small.

At the lower crank speed mutual approach is 0.02 micron for 100 RPM crank speed and increasing up to 0.4 microns at 1500 RPM i.e. higher speed. This shows that the estimated penalty regularization parameter gives very small violation of impenetrability constraint.

The combined effect of link dimension tolerance and joint clearance drastically deteriorate the performance of the mechanism. The error in an angular velocity and angular acceleration of coupler and follower of nominal dimension mechanism are negligible as compared with analytical method. The methodology adopted allows us to find the variations at the output even at the lower values of crank speed.

The output variations increase with increase in crank speed gives dynamic instability to the mechanism. In case of special purpose machines like sinuous loop spring-wire bending machine and stamping machine, where the follower feed is important, which deteriorates the performance of the machine due to tolerance stack-up.

REFERENCES

[1] Hamid M Lankarani and Parviz E Nikravesh, Continuous contact force models for impact analysis in multi-body systems, Nonlinear Dynamics, 5(2), 1994, 193–207.

[2] Hamid M Lankarani and Parviz E Nikravesh, Continuous contact force models for impact analysis in multi-body systems, Nonlinear Dynamics, 5(2), 1994, 193–207.

[3] Ali Azimi Olyaei, Mohammad Reza Ghazavi, Stabilizing slider-crank mechanism with clearance joints, Mechanism and Machine Theory, Vol. 53, 2012, 17–29.

[4] A L Schwab, J P Meijaard, P Meijers, A comparison of revolute joint clearance models in the dynamic analysis of rigid and elastic mechanical systems, Mechanism and Machine Theory, Vol. 37, 2002, 895–913.

[5] Paulo Flores and Jorge Ambrosio, On the contact detection for contact-impact analysis in multi-body systems, Multi-body System Dynamics, Vol. 24(1), 2010, 103–122.

[6] A M Vaidya, P.M. Padole, A performance evaluation of four bar mechanism considering flexibility of links and joint stiffness, The open mechanical engineering Journal, Volume 04, 2010, 16-28.

[7] C A Akhadkar, A B Deoghare, Deviation Analysis of a Clearance Joint Mechanism, 3rd International Conference on Recent Trends in Engineering and Technology (ICRTET 2014) at Chandwad, Nasik, India, Elsevier science and technology, 2014, 510-516.

[8] C A Akhadkar, A B Deoghare, A M Vaidya, Influence of joint clearance on kinematic and dynamic parameters of mechanism, IOSR Journal of Mechanical and Civil Engineering, Vol. 10, Issue 6, 2014 39-52.

[9] Basic ADAMS simulation package course V12.0, 2012.

[10] Juan C, Garcia Orden, Analysis of joints incorporating clearances in multi-body systems, European congress on computational methods in applied sciences and engineering, ECCOMAS, 2004, 1-17.

[11] Elisabet Lensa and Alberto Cardona, Dynamic analysis of constrained nonlinear multi-body systems with intermittent contact, Mecanica Computational, Vol XXVI, 2007, 3005-3018.

[12] D Dopico, A Luaces, U.Lugris, J Cuadrado, A general contact algorithm for multi-body system dynamics with complex non-conforming 3D geometry, 13th World Congress in Mechanism and Machine Science, Guanajuato, Mexico, 2011, 1-7.

[13] Laulusa A, Bauchau, O A, Review of Classical Approaches for Constraint Enforcement in Multi-body Systems, Journal of Computational and Nonlinear Dynamics, 3, No 1, 2008, 1– 8.

[14] Mihai Anitescu, A fixed time-step approach for multi-body dynamics with contact and friction, Report, 2003.

[15] Samer Mutawe, Yahia M. Al-Smadi, Rajpal.S. Sodhi, Planar four-bar path generation considering worst case joint tolerances, "Proceedings of the World Congress on Engineering and Computer Science, San Francisco, USA, 2011, pp 1-5.

[16] S M Varedi, H.M.Daniali, M.Dardel, , Dynamic synthesis of a planar slider–crank mechanism with clearances, Nonlinear Dynamics, Vol 14, 2014, 1-14.

[17] Galal A Hassaan , Mohammed A. Al-Gamil and Maha M Lashin, New Approach for the Synthesis of Planar 4-Bar Mechanisms for 2 Coupler-Positions Generation, New York Science Journal, Vol.5(10), 2012, 86-90.

[18] Arash Sardashti, H.M. Daniali, S M Varedi, Optimal free-defect function generation synthesis of four-bar linkage with joint clearance using PSO algorithm, Journal of Science and Engineering, Vo l.1(1), 2013, 67-78.

[19] Ming-June Tsai, Tien-Hsing Lai, Kinematic sensitivity analysis of linkage with joint clearance based on transmission quality, Mechanism and Machine Theory, Vol 39, 2004, 1189–1206.

[20] Xianzhen Huang, Yimin Zhang, Robust tolerance design for function generation mechanisms with joint clearances, Mechanism and Machine Theory, Vol 45, 2010, 1286–1297.

[21] C A Akhadkar, A B Deoghare, A M Vaidya, P V Walke, Influence of tolerance stack-up on dynamics of zero degree of freedom parallelogram configuration, Journal of Science and Engineering, Vol 06(01), 2015, 008-023.

[22] C A Akhadkar, A B Deoghare, A M Vaidya, Influence of tolerance stack-up on mechanism dynamics-Review, International Journal of Applied Engineering Research, Vol. 10, No. 11, 2015, pp 10580-10587.