Direction of Onset estimation using Multiple Signal Classification, Estimation of Signal Parameter by Revolving Invariance Techniques and Maximum-likelihood Algorithms for Antenna arrays

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Abstract— In this paper a comparison of the performance of three famous Eigen structure based Direction of arrival (DOA) algorithms known as the Multiple Signal Classification (MUSIC), the Estimation of Signal Parameter via Rotational Invariance Techniques (ESPRIT) and a non-subspace method maximum-likelihood estimation (MLE) has been extensively studied in this research work The performance of this DOA estimation algorithm based on Uniform Linear Array (ULA). We estimated various DOA using MATLAB, results shows that MUSIC algorithm is more accurate and stable compared to ESPRIT and MLE algorithms.

Keyword- Multiple Signal Classification, Estimation of Signal Parameter via Rotational Invariance Techniques, Maximum-Likelihood Estimation, Direction of arrival

I. INTRODUCTION

DOA estimation uses antenna arrays. It is known that antenna radiation main lobe beam width is inversely proportional to the number of elements in antenna. So, if we consider a single antenna then array pattern will be wider and the resolution cannot be good. Instead of using single antenna, an antenna array system is used in DOA estimation which will improve the resolution of the received signals (Resolution in DOA estimation is the ability to distinguish two signals arriving at different angles). An array system has a multiple elements distributed in space.

There are various methods to estimate the angle of arrival (DOA) of radio signals on the antenna array. DOA estimation techniques can be broadly divided into three different categories namely; conventional methods subspace based methods and maximum likelihood methods. Convolution methods are based on the concepts of beam forming and null steering, but it requires a large number of elements to provide high resolution. Examples of this method are delay and sum and Capon's minimum variance method [2].

One major limitation of this method is poor resolution that is its ability to separate closely spaced signals. Unlike conventional methods, subspace methods exploit the information of the received data resulting in high resolution. Two main subspace based algorithms are Multiple Signal Classification and Estimation of Signal Parameters via Rotational Invariance Techniques.

The DOA algorithms are classified as quadratic (non subspace) type and subspace type. The Barltett and Capon (Minimum Variance Distortion less Response) are quadratic type algorithms. Both methods are highly dependent on physical size of array aperture, which results in poor resolution and accuracy. Subspace based DOA estimation method is based on the Eigen decomposition. The subspace based DOA estimation algorithms MUSIC and ESPRIT provide high resolution; they are more accurate and not limited to physical size of array aperture [3].

These algorithms give information about number of incident signals and DOA of each signal. Maximum likelihood method is one of the first techniques to be investigated for DOA estimation but has the drawback of intensive computational complexity [4].

In this paper we present a DOA estimation procedure for M uncorrelated signals impinged on uniform linear array of N elements using high resolution 'MUSIC, ESPRIT' subspace methods and non-subspace method 'maximum-likelihood' . We observed the important parameters like number of antenna elements, number of snapshots and spacing between elements to take into consideration for better accuracy. Finally, we will conclude with an analysis of performance of algorithms.

II. MATERIALS & METHODS

An array antenna is an essential part of a communication system It can be used to exploit spatial and spectral characteristics of the incoming signals to provide highly accurate location information (Before implementing such system, a simulation step should be carried out in order to optimize its efficiency. In this study, we use an array antenna with a four element uniform linear array. Figure 1 shows the general configuration for an array antenna having N elements arranged along a straight line with a distance d between sensor elements. The angle of the incoming signal θ_M is determined relative to the antenna bore sight.

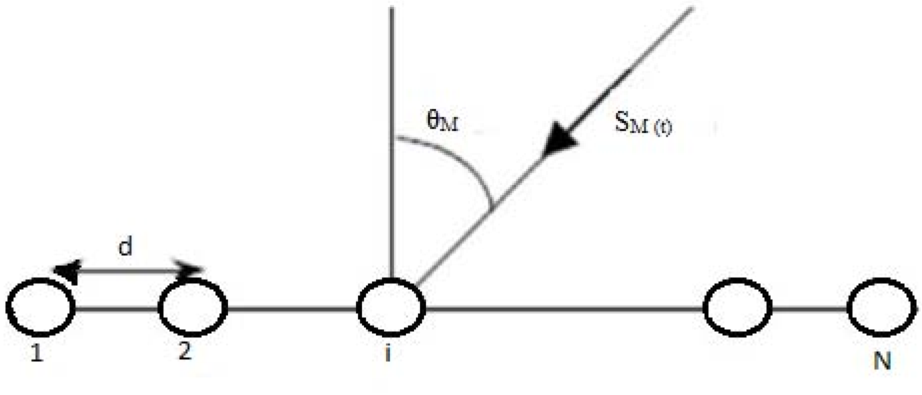

Fig.1.N Element linear array with M signals

A. Mathematical Model for MUSIC Algorithm

Multiple Signal Classification (MUSIC) method [5] is widely used in signal processing applications for DOA [6] estimation. It is applied to only narrow band signal sources i.e., frequencies of interest are narrowband [3], [7]. Consider M number of narrow band signal sources arriving from different angles θ_i =1, 2…M, impinging on a uniform linear array of N equal spaced array elements (Where N > M) as shown in figure1. At different instants of time $t, t = 1,2....K,$ where K being the number of snapshots, the array output will consist of signal along with noise components [5].

Let a signal source $S(t)_p = e^{jwt}$ impinges on the array with an angle θ. If the received signal at element 1 is, $X_1(t) = S(t)$ and then the delay at element i is:

$$\Delta i = \frac{(i-1)d \sin\theta}{c} \tag{1}$$

The received signal at sensor i is,

$$xi(t) = e^{-jw\Delta i} S_1(t) = e^{=jw\Delta i} S(t) = e^{-\frac{jw(i-1)d\sin\theta}{c}} S(t) \tag{2}$$

The received signal at N elements due to a single source is:

$$X(t) = \left[1, e^{-\frac{jwd\sin\theta}{c}}, e^{-\frac{jw2d\sin\theta}{c}}, ..., e^{-\frac{jw(N-1)d\sin\theta}{c}}\right] S(t) = a(\theta)S(t) \tag{3}$$

If there are M sources the signals received at the array is given by:

$$X = AS + W \tag{4}$$

$$A = [a(\theta_1), a_2(\theta_2), ...a(\theta_M)] \tag{5}$$

$$S = [S_1(t), S_2(t), ...S_M(t)]^T \tag{6}$$

Where $a(\theta)$ denotes a steering vectors, A denotes a matrix, j is defined as $\sqrt{-1}$, $[\ ]^T$ and $[\ ]^H$ denote Transpose a Hermitian of a matrix respectively. A is a N×M matrix of the M steering vectors, S is a signal source vector of order (M×N). The correlation matrix of received vector can be written as:

$$R = E\left[XX^H\right] \quad (7)$$

$$= E\left[ASS^H A^H\right] + E\left[WW^H\right] \quad (8)$$

$$= AVA^H + \sigma^2 \quad (9)$$

Where σ^2 is the variance of white Gaussian noise vector (W), V is covariance matrix of signal vector (S) which is a full rank matrix of order M×M given by,

$$V = E\left[SS^H\right] \quad (10)$$

$$= \begin{bmatrix} E[|S_1|^2] & .... & 0 \\ 0 & E[|S_2|^2] & 0 \\ 0 & 0 & E[|S_M|^2 \end{bmatrix} \quad (11)$$

Where the statistical expectation is denoted by E [ ],R_S is a signal covariance matrix of order (N×N) with rank M given by:

$$R_S = \begin{bmatrix} E[|S_1|^2] & .... & 0 & .... & 0 \\ 0 & E[|S_2|^2] & 0 & .... & 0 \\ 0 & 0 & E[|S_M|^2] & .... & 0 \\ 0 & 0 & 0 & .... & 0 \end{bmatrix} \quad (12)$$

So R_S , has N-M Eigen vectors corresponding to zero Eigen values. We know that steering vector $a(\theta_1)$ which is in the signal subspace is orthogonal to noise subspace let Q_n be such an eigenvector.

$$R_S Q_n = AVA^H Q_n = 0 \quad (13)$$

$$Q_n^H AVA^H Q_n = 0 \quad (14)$$

Since V is a positive definite matrix:

$$AVA^H Q_n = 0 \quad (15)$$

$$a^H(\theta_i) Q_n = 0 \quad (16)$$

This implies that signal steering vectors are orthogonal to eigenvector corresponding to noise subspace. So the MUSIC algorithm searches through all angles and plots the spatial spectrum.

$$P_{MUSIC}(\theta) = \frac{1}{a^H(\theta_i) Q_n} \quad (17)$$

Assuming the number of signals M is known. Given the data set X (k), k = 1, 2. . . K, the MUSIC algorithm proceeds as the following steps:

- Compute the sample covariance matrix R .
- Compute its Eigen values $\lambda_1 \geq \lambda_2 \geq .... \geq \lambda_n$ and the corresponding eigenvectors $Q_1, Q_2, ...Q_n$ then $Q_n = \left[Q_{M+1} ......Q_N\right]$
- We plot $P_{MUSIC}(\theta)$ for $-\pi/2 < \theta < \pi/2$
- Choose the M minimum of $P_{MUSIC}(\theta)$ to estimate (θ)

B. Mathematical Model for MUSIC Algorithm

ESPRIT's acronym stands for Estimation of Signal Parameter via Rotational Invariance Technique. This algorithm is more robust with respect to array imperfections than MUSIC [8][9][10]. Computation complexity and storage requirements are lower than MUSIC as it does not involve extensive search throughout all possible steering vectors. But, it explores the rotational invariance property in the signal subspace created by two sub arrays derived from original array with a translation invariance structure [11]. It is based on the array elements placed in identical displacement forming matched pairs, with N array elements, resulting in m=N/2 array pairs called "doublets" [12]. Computation of signal subspace for the two sub arrays, array-1 and array-2, are displaced by distance d. The signals induced on each of the arrays are given by:

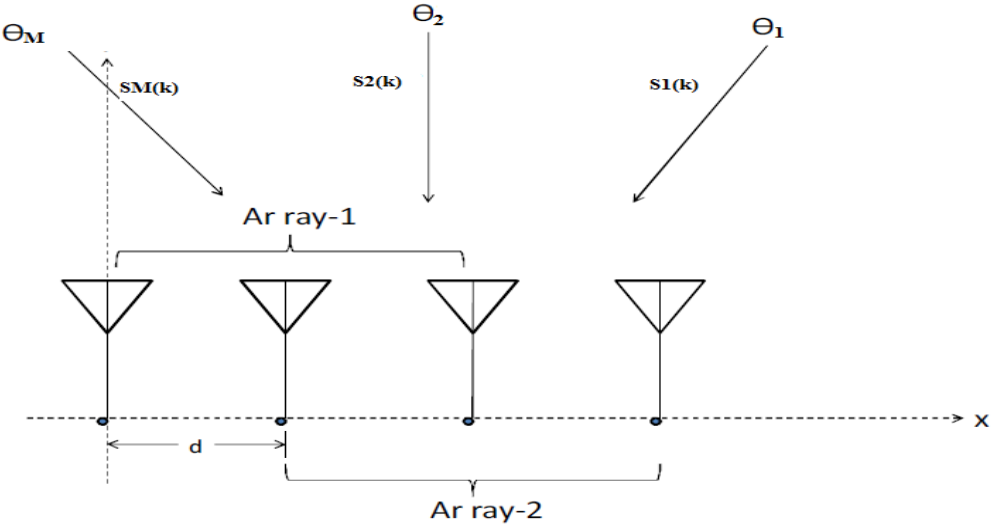

Fig. 2.Four element linear array with two doublets

$$X_1(k) = A_1 * S(k) + N_1(k) \tag{18}$$

$$X_2(k) = A_1 * \Lambda * S(k) + N_2(k) \tag{19}$$

Where $\Lambda = \text{diag}[e^{jkd\sin(\theta_1)}, e^{jkd\sin(\theta_2)}, \ldots, e^{jkd\sin(\theta_D)}]$ (D x D) diagonal unitary matrix with phase shifts between doublets for DOA.

Creating the signal subspace for the two sub-arrays results in two matrices V1 & V2. Since the arrays are translation ally related, the subspaces of eigenvectors are related by a unique non-singular transformation matrix φ such that [6]:

$$V_1 \varphi = V_2 \tag{20}$$

There must also exist a unique non-singular transformation matrix T such as:

$$V_1 = AT, V_2 = A\Lambda T \tag{21}$$

And finally we can derive

$$T\varphi T^{-1} = \Lambda \tag{22}$$

Thus, the Eigen values of φ must be equal to the diagonal elements of Λ such that

$$\lambda_1 = e^{jkd\sin(\theta_1)}, \lambda_2 = e^{jkd\sin(\theta_2)}, \ldots, \lambda D = e^{jkd\sin(\theta_1)}$$

Once the Eigen values of ϕ, $\lambda_1, \lambda_2, \ldots \lambda D$ are calculated, we can estimate the angles of arrivals as

$$\theta_i = \sin^{-1}\left(\frac{\arg(\lambda_i)}{kd}\right) \tag{23}$$

Clearly the ESPRIT eliminates the search procedure & produces the DOA estimation directly in terms of the Eigen values without much computational and storage requirements. This Eigen structure method has shown excellent accuracy and resolution in many experimental and theoretical studies.

C. Mathematical Model for Maximum Likelihood Estimation Algorithm

This method depends on spatial spectrum [13]; DOAs are obtained as locations of peaks in the spectrum. The concept of localisation is simple but offer modest or poor performance in terms of resolution [14]. One of the main advantages of these techniques is that it can be used in situations where we lack information about properties of signal [15].

The estimate is derived by finding the steering vector A which minimizes the beam energy AVA^H subject to the constraint $EA^H = 1$..

$$F = AVA^H + \alpha(EA^H - 1) \tag{24}$$

When the gradients of A and A^H are evaluated, they are found to be complex conjugates of each other. Setting one of them to zero results in the solution

$$A = \frac{-\alpha V^{-1}}{2} \tag{25}$$

The quantity α is determined from the constraint $EA^H = 1$.. Hence,

$$A = R^{-1}E(E^H V^{-1} E)^{-1} \tag{26}$$

Thus, the power spectrum in the beam is given by

$$P(\theta) = AVA^H \tag{27}$$

$$= (E^H V^{-1} E)^{-1} \tag{28}$$

As expected, the peaks of $P(\theta)$ correspond to the direction of arrival of the given signal. Hence the following algorithm steps:

- Collect the data samples X
- Estimate the correlation matrix R
- Estimate the number of signals
- Evaluate $P(\theta)$

D. Results & Discussion

A comparative study [16-18] has been made between MUSIC, ESPRIT and MLE algorithms for DOA estimation, using MATLAB software tool. We analyzed the performance of these algorithms by varying a number of parameters relating to antenna array such as number of array elements N, spacing between the array elements d and the number of snapshots taken at any time. In this simulation, we have considered M number of stationary signal sources impinging on number of uniform linear array elements which are equi-spaced with a separation of λ/2; we also consider the randomly generated symbols for each of the signal with equal magnitudes. The noise is assumed to be additive white Gaussian having unit variance. The simulation has been done for three signals arriving from different angles θ_1= -30º, θ_2= 30º and θ_3= 60º and our algorithm spatially searches through angles from -90º to 90º.

TABLE I. DOA estimation (k=1024, d= 0.5λ).

N	$\theta_{in}(^\circ)$	$\theta_{MUSIC}(^\circ)$	$\Delta_{MUSIC}(^\circ)$	$\theta_{ESPIRIT}(^\circ)$	$\Delta_{ESPIRIT}$	$\theta_{MLE}(^\circ)$	Δ_{MLE}
4	-30	-30.2	-0.2	-29.8	+0.2	-29.5	+0.5
	30	30.2	0.2	29.6	-0.4	31.6	+1.6
	60	59.9	-0.1	59.4	-0.8	55.4	-4.6
6	-30	-30.1	-0.1	-29.9	+0.1	-29.9	+0.1
	30	30	0	30.1	0.1	30	0
	60	59.8	-0.2	60.1	-0.2	60.4	+0.4
8	-30	-29.9	+0.1	-29.8	+0.2	-30.1	-0.1
	30	30	0	30.2	0.2	29.9	0.1
	60	60	0	60	0	60	0
10	-30	-30	0	-30	0	-30	0
	30	30	0	29.8	-0.2	30.2	+0.2
	60	60	0	59.9	-0.1	60	0
12	-30	-30	0	-29.8	+0.2	-30	0
	30	30	0	30.1	0.1	29.9	-0.1
	60	60	0	59.9	-0.1	60	0

TABLE III. DOA estimation (k=128, d$= 0.5\lambda$).

N	$\theta_{in}(^\circ)$	$\theta_{MUSIC}(^\circ)$	$\Delta_{MUSIC}(^\circ)$	$\theta_{ESPIRIT}(^\circ)$	$\Delta_{ESPIRIT}$	$\theta_{MLE}(^\circ)$	Δ_{MLE}
4	-30	-30.6	-0.6	-30.8	-0.8	-30.4	-0.4
	30	29.4	-0.6	31.5	+1.5	32.6	+2.6
	60	58.6	-1.4	55.1	-5.9	56	-4
6	-30	-29.9	+0.1	-30.9	-0.9	-30.1	-0.1
	30	30	0	28.8	-1.2	30.1	+0.1
	60	59.9	-0.1	61	+1	60.1	+0.1
8	-30	-30	0	-30.1	-0.1	-30.2	-0.2
	30	30.2	0.2	30.1	+0.1	30	0
	60	59.6	-0.4	59.7	-0.3	60.1	0.1
10	-30	-30	0	-29.9	+0.1	-29.9	+0.1
	30	30.1	0.1	30	0	30.1	+0.1
	60	59.9	-0.1	60.5	+0.5	60	0
12	-30	-30	0	-30.2	-0.2	-30.1	-0.1
	30	30	0	30	0	30	0
	60	59.9	-0.1	60.6	+0.6	59.9	-0.1

TABLE IIIII. DOA estimation (k=1024, d$= 0.25\lambda$).

N	$\theta_{in}(^\circ)$	$\theta_{MUSIC}(^\circ)$	$\Delta_{MUSIC}(^\circ)$	$\theta_{ESPIRIT}(^\circ)$	$\Delta_{ESPIRIT}$	$\theta_{MLE}(^\circ)$	Δ_{MLE}
4	-30	-30.6	-0.6	-31.9	-1.9	-29.5	+0.5
	30	31	1	40.2	+10.2	32.6	+2.6
	60	64.4	+4.4	55.1	-5.1	42.6	-12.6
6	-30	-29.8	+0.2	-29.6	+0.4	-30.3	-0.3
	30	30	0	30.4	+1.4	30.1	+0.1
	60	60.2	+0.2	61	+1	42.6	-12.6
8	-30	-30.1	-0.1	-30.2	-0.2	-30.1	-0.1
	30	30	0	30.1	+0.1	31.3	1.3
	60	59.9	-0.1	60.1	+0.1	56.7	-4.3
10	-30	-30	0	-29.9	+0.1	-30	0
	30	30	0	29.9	-0.1	30.4	+0.4
	60	59.9	-0.1	59.8	-0.2	59.4	-0.6
12	-30	-30	0	-30	0	-30.2	-0.2
	30	30	0	30.1	0.1	29.9	-0.1
	60	59.9	-0.1	60.5	+0.5	60.2	+0.2

TABLE IVV. DOA estimation (k=128, d$= 0.25\lambda$).

N	$\theta_{in}(^\circ)$	$\theta_{MUSIC}(^\circ)$	$\Delta_{MUSIC}(^\circ)$	$\theta_{ESPIRIT}(^\circ)$	$\Delta_{ESPIRIT}$	$\theta_{MLE}(^\circ)$	Δ_{MLE}
4	-30	-29.9	+0.1	-28.9	+1.1	-31.6	-1.6
	30	33.7	3.3	26.5	-4.5	22.6	-7.4
	60	60.4	+0.4	45.1	-15.9	44.5	-16.5
6	-30	-30.4	-0.4	-31.9	-1.9	-30.2	-0.2
	30	29.9	-0.1	32.8	2.8	30.1	+0.1
	60	59.1	-0.9	57.5	-3.5	42.2	-17.8
8	-30	-30	0.1	-29.8	0.5	-28.8	1.4
	30	30.1	0.1	30.7	0.7	30.4	0.4
	60	59.9	-0.1	61	0.1	60.1	0.1
10	-30	-29.7	0.3	-30.5	-0.5	-30	0.1
	30	27.7	-2.3	30.3	-30.3	30.6	0.6
	60	59.9	-0.1	60.5	0.5	58.7	-1.2
12	-30	-30	0	-30.9	-0.8	-29.7	0.3
	30	30	0	29.4	0.7	30.1	0.1
	60	59.9	-0.2	59.3	-0.8	60.1	0.1

TABLE V. DOA estimation (k=1024, d$= 0.75\lambda$).

N	$\theta_{in}(^\circ)$	$\theta_{MUSIC}(^\circ)$	$\Delta_{MUSIC}(^\circ)$	$\theta_{ESPIRIT}(^\circ)$	$\Delta_{ESPIRIT}$	$\theta_{MLE}(^\circ)$	Δ_{MLE}
4	-30	-28.7	1.3	-28.7	1.3	-28.9	1.1
	30	29.6	-0.4	30.3	0.3	30.6	0.6
	60	58.5	-1.5	58.6	-1.4	58.4	-1.6
6	-30	-29.1	0.9	-24.7	5.5	-28.9	1.2
	30	30.1	0.1	30	0	29.9	-0.1
	60	58	-2.1	56.7	-3.6	58.1	-1.9
8	-30	-29.8	0.5	-28.3	1.9	-30	1.1
	30	30	-0.2	30.2	0.2	30.1	0.1
	60	57.6	-2.9	60	-0.6	58.2	-1.9
10	-30	-29.1	0.8	-29.1	0.3	-29	1.3
	30	30	0	30	0	30.5	0.2
	60	56.8	-2.6	56.2	-3.3	58.4	-1.7
12	-30	-29.9	0.3	-29.1	0.12	-28.9	1.2
	30	30	0	30.3	0.13	30.3	0.2
	60	58.6	-1.3	60.6	0.5	59	-1.9

The simulation results of MUSIC, ESPRIT and MLE algorithms on 3 signals coming from different angles (-30, 30, 60), indicate clearly that if array size increases from 4 to 12 elements, the peak spectrum become sharp. The resolution capacity increases also if the number of snapshots increases (from 128 to 1024). The 3 signals are clearly identified .we observe also observed that if the spacing between the antenna array changes from 0.25 λ to 0.75 λ we get better resolution of estimated peaks, but we also observed some peak in the case of d=0.75 λ due to grating lobes.

The comparison between the 3 methods is in the errors the tables indicate that MUSIC presents less error then ESPRIT and MLE. The tables illustrate that for different number of array, values of snapshot and distance between the elements of array, MUSIC present a maximal error 11% and minimal error 0.16% contrary then

ESPRIT with a maximal and minimal errors 33.3%, 0.33 % respectively and MLE with a maximal error 29.66% and minimal error 0.33%.

We resolve that MUSIC algorithm provided great resolution and accuracy. In the previous studies [16], the authors show that the spectrum does not contain side lobes unlike other techniques it's true if the distance between elements of antenna do not exceed 0.6 λ otherwise we get side lobes. Computation complexity and storage requirements for ESPRIT are lower than MUSIC as it does not involve extensive search throughout all possible steering as he was presented in the work [17].

Finally MLE present a performance degrades by changing the parameters N, K and d, the author in [18] show that wideband MUSIC yields poor estimation results (especially in range) for a finite number of data samples (limited due to moving source) and low SNR it's true at the level of power of the signals, it is degraded but comparing the result of the MLE and MUSIC notes that the first present more errors at the level of angles then MUSIC presents a good estimate of angles of arrival . For MUSIC, The ideal value of number of snapshots is 1024 and d=0.5 λ.

III. CONCLUSION

This paper presents the results of direction of arrival estimation using MUSIC, ESPRIT and MLE algorithms. MUSIC, ESPRIT methods have greater resolution and accuracy than MLE and hence they are investigated much in detail. The simulation results show that performance of MUSIC, ESPRIT and MLE improves with more elements in the array, with higher number of snapshots of signals. These improvements are analysed in the form of sharper peaks in MUSIC spectrum and smaller errors in angle detection. Results indicate that if the number of snapshots, distance and number of element of array increases the errors of angle of arrival decreases.

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