

Evaluation of Synthetic Communication Data Traces obtained from Multifractal Algorithms using Queueing Theory

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Abstract—Multifractal studies have applications in several science and engineering fields with a tendency to increase nowadays. In this paper, it is evaluated a synthetic multifractal trace set generated by a novel algorithm in which the multifractal spectrum width of these signals is manipulated. This algorithm is also developed by the same authors and it is based on multifractal wavelet model (MWM) approach. This evaluation is performed by comparing the synthetic traces with some real traces acquired from Bellcore laboratories database. A timing parameter from queueing theory was measured. The results are promising, the synthetic traces exhibit results with margin errors lower than 5%.

Keyword—multifractal signals, queueing theory, multifractal spectrum, Hurst exponent, multifractal wavelet model.

I. INTRODUCTION

Multifractal theory has several applications such as modeling the temperature variations [1], measure of geoelectrical signals [2], characterization of telecommunication traffic [3] and others. Such is the importance of multifractal datasets that the main objective is to evaluate an algorithm to generate and simulate datasets with similar features to real-life signals. Therefore, an algorithm based on previous developments is proposed, which allows synthesizing new signals and then the queue theory was used to evaluate the traces created. Finally, these synthetic signals with some real datasets extracted from Bellcore database were compared.

When new traces are created, the algorithm proposed by López [4] is used, which allows setting up the values of the Hurst exponent, the mean and the length of the signal. 1000 synthetic traces were generated for each real trace; altogether, 4000 synthetic traces (four real traces for performance comparison and evaluation were used). By each group of one thousand synthetic traces, a histogram measuring the waiting time of the customer for each signal was plotted; with these four histograms, the mean and the standard deviation were measured and thereby the evaluation of the artificial signal behavior.

Comparing the results obtained with the aforementioned steps with a new group of 1000 thousand synthetic traces, which had a bigger error in their multifractal spectrum width, it was concluded the direct dependence of the signal behavior with its multifractal structure. It was also achieved the main goal at finding some synthetic traces with queueing modeling results near to the real signals; the lowest error was under a percentage of 5%. A finer manipulation of multifractal features is proposed in order to get a smaller standard deviation in the waiting time parameter, $E[T_w]$.

II. THEORETICAL FRAMEWORK

A. Multifractal Signals

In the last few years, a plenty set of studies approaching information traffic modeling in the network had been developed for several LAN/WAN environments. One of the most important types of mathematical modeling is the multifractal behavior approach. The theory of self-similarity in signals evolution was adopted by Mandelbrot [5] at fluid mechanics, specifically, in turbulence phenomena. A signal has self-similarity properties or fractal behavior if its structure and properties do not change as a consequence of scale changes in observation. The fractal signals have similar mean along their evolution.

Some signals cannot be evaluated only by one statistical moment (in this case, mean), therefore, Halsey et al. [6] proposed a novel method to the characterization of strange sets with long self-similarity. Halsey evaluated the datasets using different statistical moments of the signal and representing the results using the Legendre spatial domain. As a result, this new type of signals do not belong to fractal type but instead, to multifractal type. This new mathematical tool developed by Halsey for multifractal analysis is called multifractal spectra [6].

The behavior of a fractal signal can be represented by the eq. (1), in this math modeling, the fluctuations of the trace are represented by the function F_q , and these fluctuations are proportional to the different scales s used in the measures and powered by a singularity exponent h which describes the singularities and behavior of the signal and is dependent of the statistical moment q evaluated [7].

$$F_q(s) \sim s^{h(q)} \tag{1}$$

1) *Multifractal Spectrum:* The multifractal representation allows an easy observation and evaluation of the multifractal signals behavior among different scales or statistical moments. There are several algorithms proposed for multifractal spectrum generation [3]. In this case, the algorithm used is based on wavelet modeling; the signal is segmented in different lengths for its evaluation, several scales imply a set of tinier segmentation for each statistical evaluation. The statistical moments are located in the abscissa axis, whereas the results of the parameters evaluation translated into the Legendre's space are located along the ordinate axis [7].

The Fig. 1 shows three examples of multifractal spectra for three different signals; the smallest spectra belong to a noise signal, the medium sized spectra appertain to monofractal serial data and the biggest spectra models belong to the multifractal dataset. Fig. 2 shows these signals; it is noticeable the visual differences between the three serial datasets. The noisy signal with uniform distribution has regular boundaries, and a similar behavior with regard to monofractal signal; this can be observed in their multifractal spectra alike sizes. On the other hand, the multifractal serial dataset has a notorious irregular shape, with high peaks together with low tendencies.

The multifractal spectra plot the singularity exponent of q-order called as α and the singularity dimension of q-order called as $F(\alpha)$. These two parameters obtained through Legendre's transform are nonlinear and its shape can be modeled as a polynomial approximation [8]. In order to find these two Legendre's values, there is a third parameter which must be calculated, called the mass exponent of q-order or τ . The eq. (2)-(4) are used to find these parameters.

$$\tau = qH - 1 \tag{2}$$

$$\alpha = \frac{d\tau}{dq} \tag{3}$$

$$F(\alpha) = q\alpha - \tau \tag{4}$$

The mass exponent can be easily disposed using the López algorithm [4], which is based on the wavelet transform method [9]; furthermore, the order q of the terms is directly related with the statistical moments.

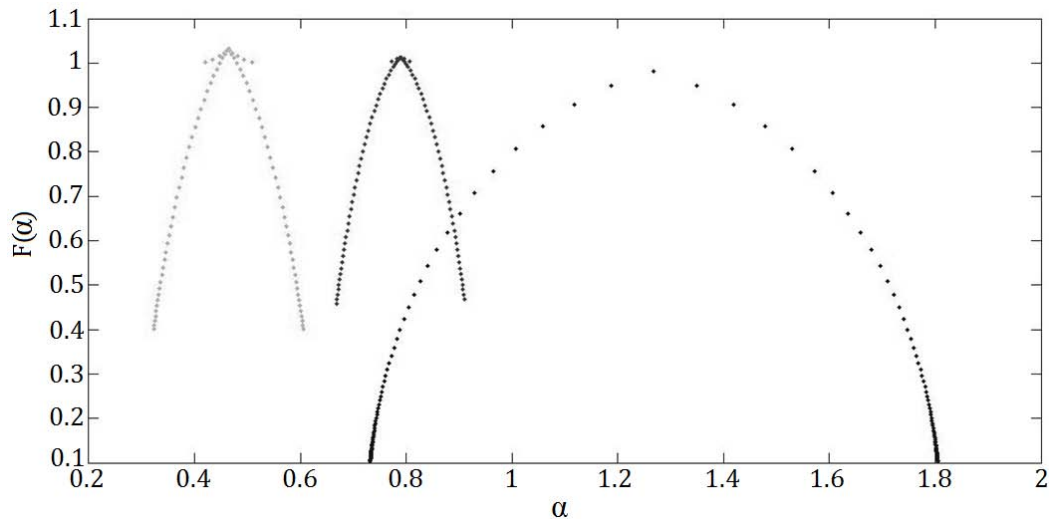


Fig. 1. Multifractal spectra for three different signals: noise (center), monofractal serial dataset (left) and multifractal serial dataset (right). Source: The authors.

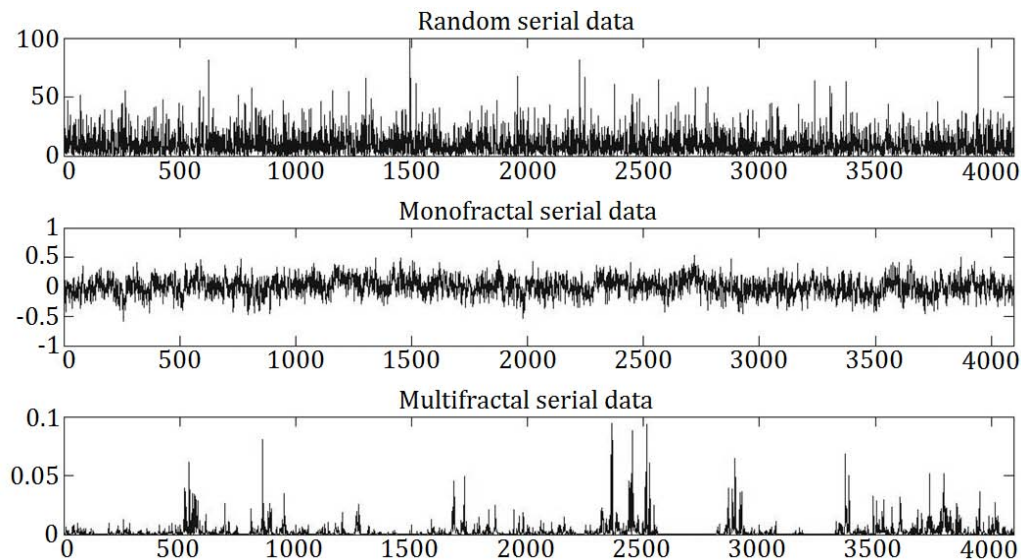


Fig. 2. Plot of three different signals: noise with uniform distribution and parameters a and b equal to 0 and 1 respectively, monofractal serial dataset and multifractal serial dataset. Source: The authors.

Multifractal spectra present different parameters which allow a signal characterization [1]. The first one is the spectra width (W) and it is equal to the length between the cutoff points of the spectra on the abscissa axis. As wider the spectrum, the signal will have more randomness; it means, the signal has an evolution with small variations, as well as big peaks. The second parameter is the statistical moment for which the spectrum has its maximum amplitude (the biggest amplitude for any spectrum is always equal to unity); this parameter is known as α , and it behaves in a similar way than the spectrum width, the higher it gets, the more random the signal is. The last important parameter of the multifractal spectrum is its asymmetry, a right-skewed spectrum indicates a signal with a more regular or smooth structure [10].

2) *Hurst exponent*: The Hurst exponent (H) is a self similarity measure for dataset signals; if a signal has fractal properties, its Hurst exponent will be less than unity and greater than 0.5. When the value of H is almost 1, the behavior of the signal presents more inertia, then, it tends to keep its evolution (the signal's behavior could be incrementing or decreasing). In conclusion, as the Hurst exponent signal gets closer to the unity; the signal has more self similarity behavior [4].

The Hurst exponent is used for the creation of synthetic network datasets or traces, based on the algorithm proposed by López et al. [4]. In this algorithm, the Hurst exponent determinates the wavelet components through the required resources for meeting the traffic demand (k) in the binomial cascade, according to the eq. (5) [9, 11, 12].

$$k = \frac{2^{2H-1} - 1}{2 - 2^{2H-1}} \quad 0.5 < H < 1 \quad (5)$$

3) *Discrete wavelet decomposition*: Multifractal wavelet modeling (MWM) allows the simplified analysis of multifractal signals. This model allows the signal to be decomposed in several random components which take values between $[0, 1]$, these components are symmetric with respect to 0.5 and possess a high variability; on these grounds, the MWM can be used to generate artificial n -length traces, with computational complexity $O(N)$ [11].

On the other hand, the wavelet decomposition T is defined in eq. (6) [13], and it is equivalent to the orthogonal projection of an incoming signal X in a dilated and shifted set of mother wavelet signals Ψ . One of the main features of this decomposition is its redundant information; to eliminate this redundancy, the discrete wavelet decomposition or transformation d defined in eq. (7), is commonly employed [14]; this transformation can be achieved through a pyramidal algorithm based on low-pass filters and high-pass filters. The computational complexity to the discrete wavelet decomposition estimation in an N -term sequence is $O(N)$ thanks to its recursive structure [13]. The discrete wavelet components allow the computation of Hurst exponent using the linear approximation described in [4] and [15].

$$T_x(a, b) := \frac{1}{\sqrt{a}} \int_{t_1}^{t_2} X(t) \Psi\left(\frac{t-b}{a}\right) dt \quad (6)$$

$$d_x(j, k) = T_x(2^j, 2^j k) \quad (7)$$

4) *Queuing theory*: The queuing theory is a set of mathematical models which describe the behavior of particular waiting lines systems. This queue could range from the line at the bank to a digital information networking system. The main objective of this modeling is to find the equilibrium point between the quantitative factors or the server costs and the qualitative factors or the customer satisfaction. In this context, the customer is a service order in the network traffic system and the server is the station supporting the request demanded by the customer [16].

Fig. 3 shows the schematic model of a basic queueing system for just one queue and one available server. In order to represent this model, Kendall-Lee notation is used, A/B/C. The value of A is the probability distribution in the system arrivals; B is the probability distribution for the server attendance and C is the number of presented servers in this system. The easiest model to represent a queueing system is M/M/1, where M is an exponential distribution describing the behavior of arrival orders, and there is just one server available. Nevertheless, in the case of network communication systems like WAN, LAN and others, the exponential distribution math modeling for probability distribution cannot represent appropriately its behavior inasmuch as the traces in these systems present multifractal characteristics [17].

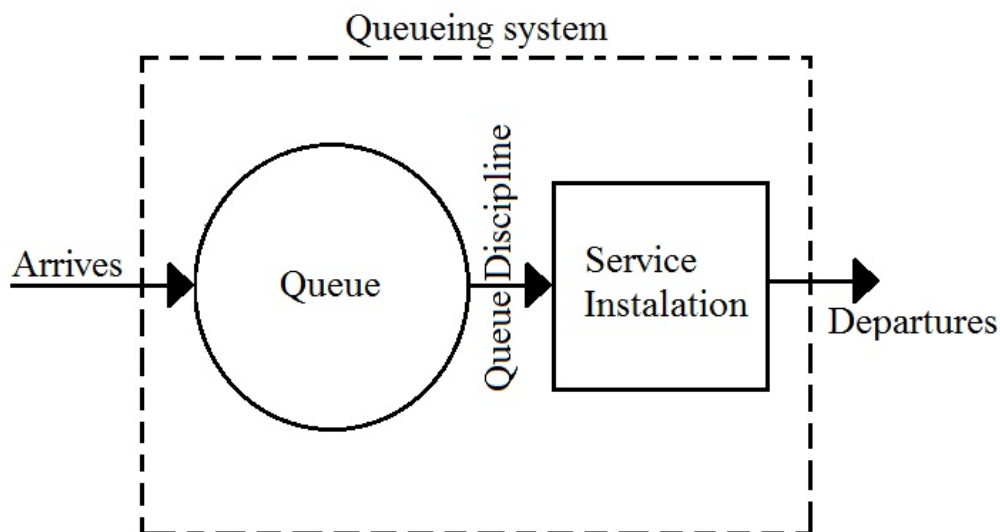


Fig. 3. Schematic model for queueing system with one available server and one queue. Source: The authors.

An M/M/1 queue model performance can be evaluated using several parameters. One of these parameters is the mean time employed by a client waiting in the line, and it could be found using eq. (8). The μ and λ parameters represent the mean rate of departures and the mean rate of arrivals respectively, and they are measured in customers/minutes, where a customer is equal to an arrival petition in the income of the system. Moreover, the system must accomplish the general rule $\lambda < \mu$, otherwise, the system will be saturated by customers, pushing the queue to a constant and infinite growing [16]. The modeling described in eq. (8) can only be applied when the queueing system is M/M/1 type and the exponential distribution of M-type systems is $1/\lambda$ for arrivals and $1/\mu$ for departures.

$$E[T_w] = \frac{\lambda}{\mu(\mu - \lambda)} \quad (8)$$

When the probabilistic distribution is not distributed as an exponential modeling, i.e. a multifractal system [4]; the mathematical modeling in eq. (8) does not work appropriately, ergo, this approaching do not predict a multifractal system behavior.

III. MATERIAL AND METHODS

A trace is a dataset containing measures of the communication network timing. With the purpose of evaluating the performance of the synthetic traces; four real signals obtained from Bellcore database were used [18]. These datasets represent the information arrival time between communication systems. Thus, these traces are directly related with the mean waiting time spent by a customer in an M/M/1 type queue system, therefore, the $E[T_w]$ measure of these signals do not require the use of eq. (8); instead, the mean waiting time was acquired directly from the genuine traces. Besides, this procedure was adopted since the real signals have multifractal behavior [15].

The main objective of this paper is to determine the performances of the artificial traces generated with the proposed algorithm, and thus, establishing if these traces present a similar behavior than the real traces in a simulated environment using queueing theory. Hence, the first step is to generate the synthetic traces with mean, length, Hurst exponent and multifractal spectrum width values equals to the real traces values obtained through the public databases [18]. Thus, the algorithm described by López et al. was used [4] called multifractal Hurst, MFH; this method allows three essential parameters setting: The Hurst exponent (H), the signal mean (m) and the length of the trace (n). This algorithm uses the eq. (7) to find the beta symmetric distribution parameter (k). Once calculated k , a set of random numbers must be found, this set is the new artificial signal. These random numbers present beta distribution, its parameters are both equal to the k value; albeit, initially there is just one random number in the range of (0, 1). In the next step, a statistical complementary set of numbers is calculated; the components belonging to the first wavelet level cascade are obtained after combining the first number and its complement, in the first level there are just these two results. This procedure is repeated for these two values, creating other four new values and the second level of the cascade, and then it repeats n -times, whereby the length of the trace is 2^{n-1} . The computational cost of this algorithm is $O(N)$ [4].

As an additional adjustment, a new algorithm in this paper was included by us; with this novel method, the multifractal spectrum width (W) can be set through a Hurst exponent lineal transformation, see eq. (9), which also varies the wavelet components until the sixteenth level of the cascade (all the synthetic traces have a total amount of twenty levels). In other words, the Hurst exponent was changed from the first cascade level until the sixteenth cascade level using the eq. (9); the original Hurst parameter defined in López algorithm is used to calculate the last four levels of the cascade. With this new adjust; the multifractal spectrum width of a signal can be set changing the parameters of the first cascade levels as long as the width remains inside the established error margin. Using the last four levels, the desired value for the Hurst exponent could be kept. In eq. (9), H_m belongs to the modified Hurst parameter used in the first levels; the parameter W is the chosen width, so in this paper, it is equal to the real traces multifractal spectrum width.

$$H_m = -0,0547W + 1,0355 \quad (9)$$

The Hurst exponent on each trace is estimated using the procedure described in section *Discrete wavelet decomposition*. The value for this parameter is equal to the linear approximation slope in discrete wavelet decomposition procedure. After finding Hurst exponent, the multifractal spectrum was calculated using eq. (2)-(4); then, the spectrum was approximated using a two-order polynomial function. With the aim of finding the width of the spectra, the two cutoffs in abscissa axis of polynomial approximation are subtracted, thus the distance between these two points could be found.

In order to guarantee the error margin, an iterative algorithm was used, searching in a set of synthetic traces for the signal with the same requirements specified in eq. (7) and eq. (9); thus, the results are not instantaneous, instead, with this algorithm, the time spent generating the synthetic trace is large (with a mean of 129.6 s for each artificial dataset). The margin error allowed for the Hurst exponent is 5%, whilst the margin error for the multifractal spectrum width was much less restrictive, equal to 25% for time reduction and considering the linear approximation low precision in eq. (9).

1000 synthetic traces were created for each real trace; altogether, 4000 synthetic traces were generated. For each synthetic trace, it was measured the waiting time in the queue, and for each group of one thousand synthetic traces representing one real trace; a histogram was used in order to facilitate the visualization of the mean and the standard deviation in the behavior of these signals regarding the waiting time as an evaluation parameter. The four histograms represent the performance of the synthetic traces in a simulated real environment. It was measured the waiting time of queueing theory in real traces and exponential distribution randomly generated traces with the purpose of evaluating the results obtained in the histograms. The random signals have a mean equal to the real signal mean; and because of its exponential distribution nature, it could be valued using eq. (8), whereby, it is possible to compare this simple kind of signals with synthetic traces crated in this paper.

Finally, another 1000 traces were generated, with the same features of BCpAug89 [18]. With these new traces the margin error of the Hurst exponent and the multifractal spectrum width was changed. In the case of H value, the employed margin error was equal to 0.5%, whereas for W the used margin error was 50%. This means that for these new traces, H has a nearer real value to the desired value, whilst, the real value of W is probably farther than the desired value. With this change, it was desired to observe the behavior in waiting time parameter histogram mean and standard deviation for the new traces.

IV. RESULTS

The features of the real traces obtained from Bellcore database [18] are shown in Table I. All traces have the same length of 220 elements.

TABLE I. Features of real traces obtained from Bellcore database.

Name	Mean (m)	Hurst exponent (H)	Multifractal spectrum width (W)
BCpAug89	0.0031	0.7118	1.3066
BCpOct89	0.0018	0.6704	1.5604
BCpOct89Ext	0.1228	0.7735	1.4477
BCpOct89Ext4	0.0759	0.7891	1.2696

It can be appreciated the results of the waiting time histograms for the synthetic traces on the Fig. (4)- (7) set.

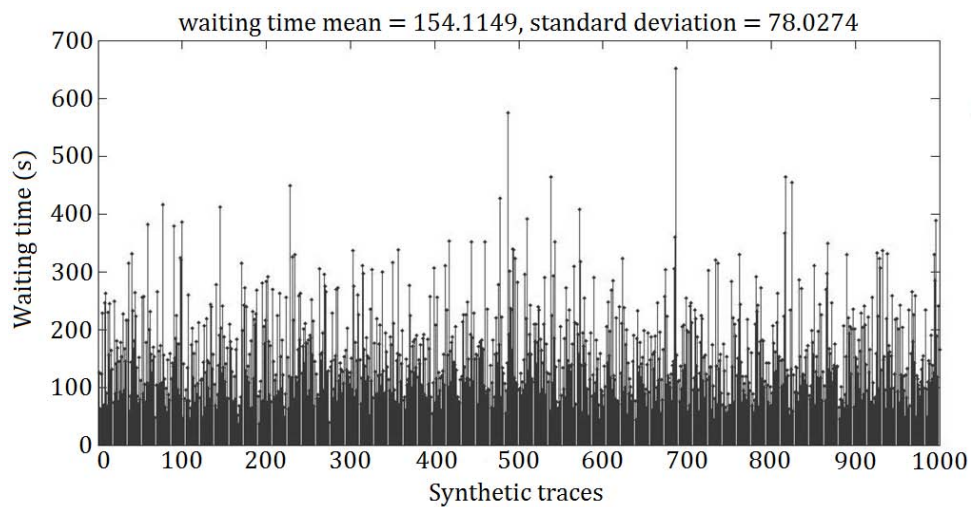


Fig. 4. Waiting time $E[Tw]$ histogram for one thousand synthetic traces setting its parameters (Hurst exponent, mean, length and multifractal spectrum width) according to BCpAug89 real trace from Bellcore database [18]. Notice the mean and standard deviation value for this histogram in the top of the image. Source: The authors.

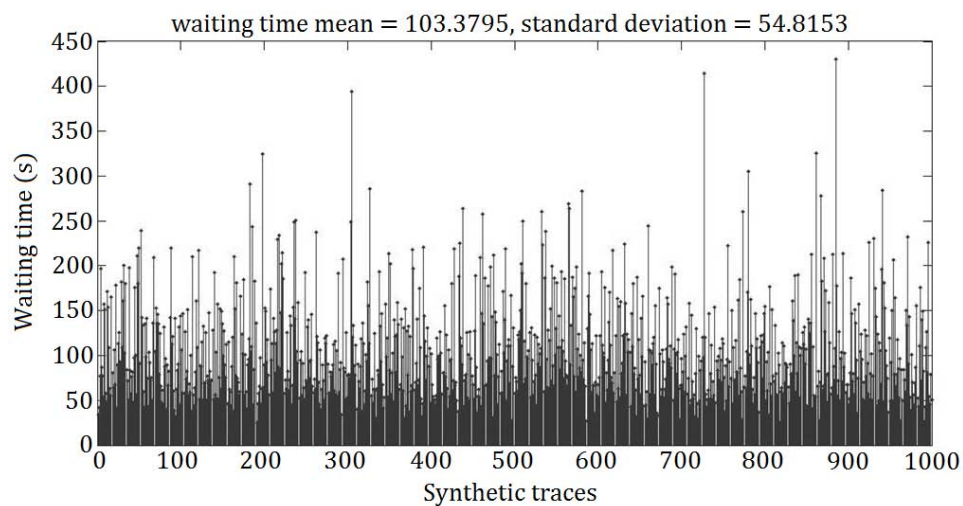


Fig. 5. Waiting time $E[Tw]$ histogram for one thousand synthetic traces setting its parameters (Hurst exponent, mean, length and multifractal spectrum width) according to BCpOct89 real trace from Bellcore database [18]. Notice the mean and standard deviation value for this histogram in the top of the image. Source: The authors.

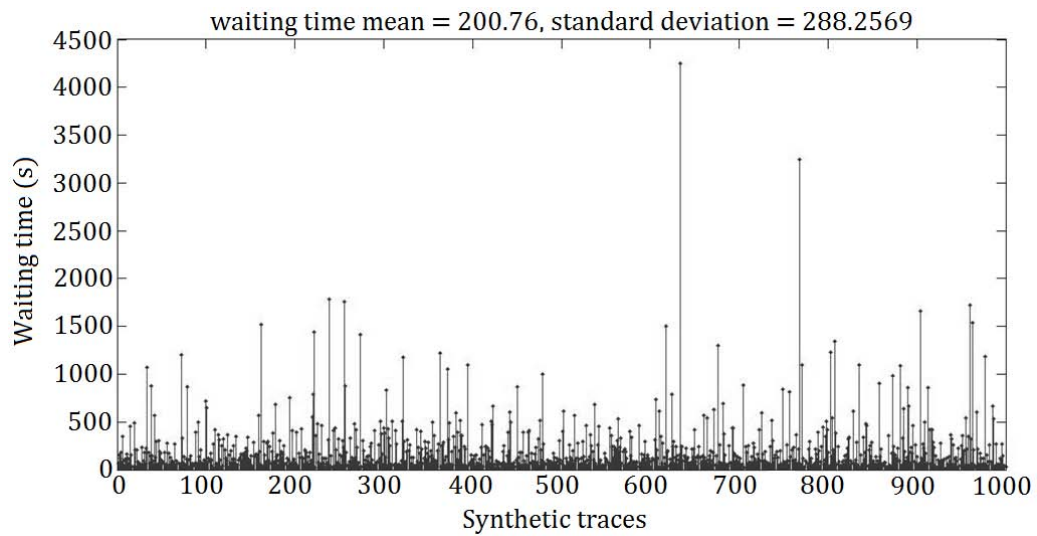


Fig. 6. Waiting time $E[T_w]$ histogram for one thousand synthetic traces setting its parameters (Hurst exponent, mean, length and multifractal spectrum width) according to BCpOct89Ext real trace from Bellcore database [18]. Notice the mean and standard deviation value for this histogram in the top of the image. Source: The authors.

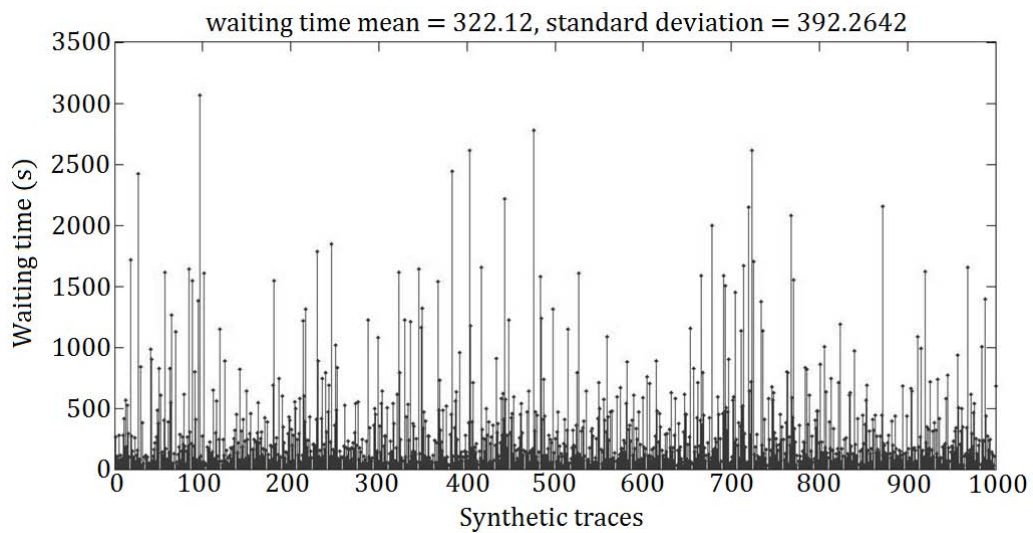


Fig. 7. Waiting time $E[T_w]$ histogram for one thousand synthetic traces setting its parameters (Hurst exponent, mean, length and multifractal spectrum width) according to BCpOct89Ext4 real trace from Bellcore database [18]. Notice the mean and standard deviation value for this histogram in the top of the image. Source: The authors.

Table II shows the summary of the results obtained in the four histograms.

TABLE II. Mean and standard deviation in waiting time histograms for each group of one thousand synthetic traces

Name	Mean	Standard deviation
BCpAug89	154.1149	78.0274
BCpOct89	103.3795	54.8153
BCpOct89Ext	200.76	288.2569
BCpOct89Ext4	322.12	392.2642

Comparison of synthetic traces, real traces and exponential distribution random traces using the queueing theory parameter, waiting time $E[T_w]$, are shown in Table III.

TABLE III. Comparison between three different kind of traces: Synthetic traces, real traces and random traces using waiting time parameter.

Waiting time $E[T_w]$ [s]			
	Synthetic traces	Real traces	Random traces
BCpAug89	154.11 ± 78.02	42.0228	0.0974 ± 0.0068
BCpOct89	103.37 ± 54.81	38.6007	0.0985 ± 0.0114
BCpOct89Ext	200.76 ± 288.25	765.412	0.0449 ± 0.0002
BCpOct89Ext4	322.12 ± 392.26	3508.48	0.0568 ± 0.0003

The results obtained at changing the margin errors for the Hurst exponent and the multifractal spectrum width to 0.5% and 50% respectively for the trace BCpAug89, are shown in Fig. (8).

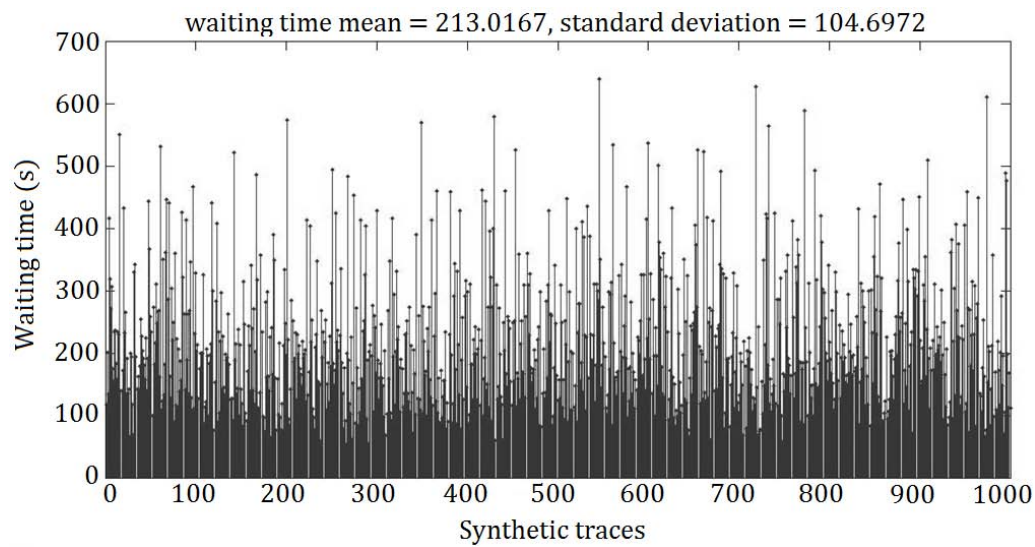


Fig. 8. Waiting time $E[T_w]$ histogram for one thousand synthetic traces setting its parameters (Hurst exponent, mean, length and multifractal spectrum width) according to BCpAug89 real trace from Bellcore database [18]. In this case, it was changed the margin error allowed in the iterative algorithm for the Hurst exponent value to 0.5% and the multifractal spectrum width to 50%. Source: The authors.

V. DISCUSSION AND FUTURE WORK

The multifractal traffic signals are an investigation field extensively studied. In this paper, the main objective was to establish if synthetic traces with features alike real traces, could generate similar results as its genuine predecessors.

The results were promising since some of the synthetic traces obtained waiting time values very close to the real traces. In fact, some of them had an error in this parameter less than 5%. An example of this result can be observed in Fig. 9, where the original trace is BCpOct89; the accurate synthetic trace took 36.58 s, that is to say, a 4.8% error. Nevertheless, the distant synthetic trace got a waiting time of 380 s and an error greater than 800%. These results can be observed in the shapes of the traces in Fig. 9, where the accurate trace has a smoother shape, similar to the original signal.

The setting of the multifractal spectrum width approximates the results of synthetic traces in queueing systems better than regular random traces with exponential distribution. The results observed in Table 3 show a similar error in means for both, random and synthetic traces; however, the standard deviation for synthetic traces is greater than random traces; this result means that a finer manipulation of multifractal spectrum properties allows a better approximation of synthetic traces. An interesting result in Table 3 is the proportional relation between variations in waiting times of real traces and waiting times means and standard deviations of synthetic traces; it seems to effectively demonstrate that multifractal spectrum width controls the similarity between two traces behavior.

In Fig. 8, it is clear that the mean and standard deviation grew up when the margin error for the multifractal spectrum width increased; this also suggests that the value of W controls the evolution of the multifractal trace in a real case.

A final result of these evaluations shows an inverse relation between processing times and error margins in iterative algorithms. It seems necessary to reduce the iterations and thus, the processing time required for these algorithms.

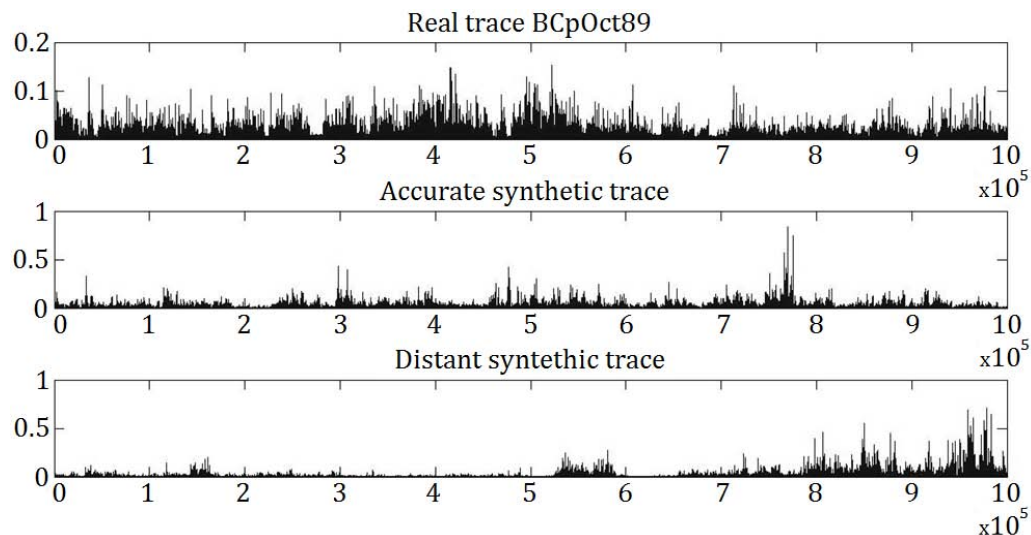


Fig. 9. Comparison between three different traces: The first one shows the original trace BCpOct89, the second one is the most accurate in waiting time measure, unlike the third one, which has the worst result. Observe the shape of the two synthetic traces; the most accurate has a smoother shape, similar to the original trace. Source: The authors.

The future approaches to solve this problem may include the evaluation of other parameters in queueing theory like number of customers or the total waiting time of the customer in the whole system. It could also be a good idea to use this algorithm in several queueing models and not just M/M/1 systems. The greater complexity of bigger queueing systems could lead to manipulate other multifractal spectrum parameters like α or its asymmetry and this is another development that could enhance the results obtained in this paper.

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