Investigation of PAPR in Discrete Wavelet Transform based Multi-carrier Systems

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Abstract— The objective of the paper is to formulate a measure to reduce PAPR problem in Orthogonal Frequency Division Multiplexing. To mitigate the problem of PAPR, a Discrete Wavelet Transform based system is employed instead of conventional OFDM. For the comparative study, the PAPR in conventional OFDM is analyzed for varying number of subcarriers and for different channel taps. The result of conventional OFDM is compared with wavelet based OFDM, employing wavelets namely - 'Haar', 'Daubechies', 'Symlets' and 'Biorthogonal' wavelets. Further the PAPR is analyzed for varying levels and different length of channel impulse response. The simulation results show that wavelet based OFDM has less PAPR than conventional OFDM. With the increase in the number level, the PAPR at the demodulator side decreases in the wavelet based OFDM.

Keywords—Orthogonal Frequency Division Multiplexing; Peak to Average Power Ratio; Nonlinear power amplifier; Wavelet based Orthogonal Frequency Division Multiplexing

I. INTRODUCTION

In order to accomplish the data rate requirements and increasing number of users in the limited bandwidth, the OFDM technology is employed. Orthogonal Frequency Division Multiplexing is a multi-carrier modulation technique that has high spectral efficiency and is less sensitive to frequency selective channel. The applications of OFDM includes- wireless MAN –WiMAX IEEE802.16,wireless LAN (WLAN) radio interfaces IEEE 802.11a,g,n,ac and HIPERLAN/2, cellular telecommunications standard LTE / LTE-A and terrestrial digital TV systems DVB-T to name a few [1] and [2].

Despite the advantages of OFDM, it suffers from challenges such as high PAPR value (Peak to Average Power Ratio), out of band emission, sensitivity to carrier and frequency offset and Inter-carrier interference between subcarriers. In this paper we focus on the PAPR problem of OFDM. PAPR is defined as the ratio of the peak power to the average power of an OFDM symbol [3]. OFDM, being a multi-carrier system has multiple sub-carrier and it may so happen that the sub-carriers may have same phase at any instant, causing a shoot in the peak power, thereby leading to a higher PAPR value [4]. High PAPR causes the signal to get into the non-linear region of the RF amplifier employed, thereby leading to severe non-linear distortion in power amplifier implementation [5]. Thus the increase in overhead and inefficient amplification are the consequences. High PAPR limits the deployment of OFDM in uplink of mobile communication standard. Thus PAPR problem is a serious challenge worth addressing. The PAPR in an OFDM system is found to increase with the increase in the number of sub-carriers [6]. Several methods can be employed to reduce the PAPR in an OFDM system, but at the expense of increase in the hardware complexity. Various PAPR reduction techniques in OFDM systems include: Tone Rejection, Tone Injection, Clipping and filtering, Partial Transmit Sequence, and Selective mapping [7].

In this paper, PAPR is reduced by employing a DWT based OFDM system and results observed. The FFT/IFFT block employed in the conventional OFDM system is replaced by DWT/IDWT block. The different types of wavelet employed for the same include - 'Haar', 'Daubechies', 'Symlets' and 'Orthogonal' wavelets. The PAPR is analysed for various number of subcarriers in conventional OFDM and for different levels in 'Haar', 'Daubechies', 'Symlets' and 'Biorthogonal' wavelets in Wavelet-based OFDM for varying channel impulse response. We anticipate a reduction in PAPR for the proposed system by exploiting the wavelet property.

The rest of the paper is organized as follows: a brief review of FFT based conventional OFDM is given in Section II. Section III gives a brief review of DWT based OFDM system. A brief in-sight on the various wavelets used in our work is given in Section IV. Simulation results are discussed in Section V and finally Section VI concludes this article.

II. REVIEW OF CONVENTIONAL OFDM SYSTEM

Orthogonal Frequency Division Multiplexing is a multi-carrier modulation technique that makes use of orthogonal subcarrier in order to carry data over the wireless channel. OFDM can be efficiently implemented using Inverse Fast Fourier Transform (IFFT) - Fast Fourier Transform (FFT) pair. Fast Fourier transform helps for its faster and easier implementation with less complexity. The input data is modulated using M-ary QAM or M-PSK modulation techniques and mapped into symbols. It is then passed to the IFFT block and the discrete time OFDM symbol is generated. The discrete-time complex OFDM symbol , $x(n)$, is represented as:

$$x(n) = \frac{1}{\sqrt{N}}\sum_{K=1}^{N-1}(X_K)e^{\frac{j2\pi Kn}{N}}, \quad (1)$$

Where X_K is the symbol carried by the k^{th} sub-carrier.

Cyclic Prefix is then added before transmission in order to compensate the effect of ISI. At the receiver side, the reverse operation is carried out. The cyclic prefix is removed first and then passed through the FFT block. FFT is implemented to carry out subcarrier demodulation. The parallel to serial conversion is then carried out and then M-ary QAM or M-PSK signal demapping is performed to reconstruct the data [8].

III. REVIEW OF DWT BASED OFDM

This section gives a brief insight into the DWT based OFDM system. Wavelet is a wavelike oscillation which begins with zero amplitude and collapse to zero effectively in a small time assuring an average value of 0. Wavelets, more specifically daughter wavelets, $\psi^{a,b}(x)$, are formed from a mother wavelet $\psi(x)$, scaling by a factor $'a'$ and translating it in time by factor $'b'$. In wavelet analysis, a long duration signal is transformed into shifted and scaled version of the mother wavelet. Wavelet transform is a tool, which have several advantages over Fourier transform. Wavelet transform is used for the analysis of both stationary and non-stationary signals while Fourier transform is applicable only to stationary signals. Fourier transform does not provide any temporal information about the transformed signal. Wavelet transform consist of small duration signals called wavelets, which provides both frequency and temporal information upon transformation [9] .

The potential advantages of wavelet transform over Fourier transform is enough to replace FFT- OFDM by DWT-OFDM.OFDM system based on wavelet transformation can decrease hardware complexity and provide better orthogonality between the carriers in the OFDM system. Reduced complexity is accomplished since DWT-OFDM systems do not require cyclic prefixes, which in turn reduces bandwidth wastage and transmission power and increases data rate. The basic functions of the Wavelet transform have different resolution in both time and frequency domain which makes it a powerful tool to find applications in data compression, image compression, radar, computer graphics [10], [11] and [12].

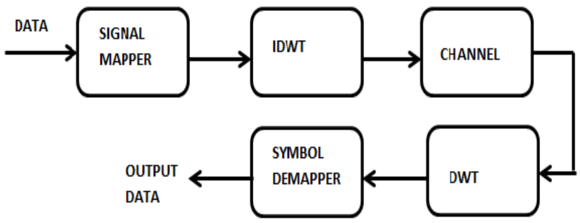

Fig. 1. Block diagram of DWT based OFDM

The block diagram of DWT based OFDM system is given in Fig. 1.Here the input data is fed into a signal mapper where the modulation is carried out and is mapped into symbols. It is then passed through IDWT block to generate OFDM symbols. OFDM symbol is then sent through a wireless channel followed by the receiver section. In this section the DWT is applied on the symbols to provide OFDM demodulation and then demapped to reconstruct the data.

IV. INSIGHT ON WAVELET USED

In this work we investigate multicarrier DWT system with four wavelets namely Haar, Daubechies, Symlets, and Biorthogonal for preliminary studies and analyze the PAPR values obtained.

A. Haar Wavelets:

Haar wavelets are recognized as the first known wavelets and are popular for their simplicity. Haar wavelet is a sequence of ones forming a bipolar square shape, whose rescaled and translated versions together forms the wavelet family. The length of the filter of Haar wavelet used here is ‘2’.

The Haar scaling function is defined as

$$\phi(x)=\begin{cases}1, 0\le x<1\\ 0, otherwise\end{cases}. \tag{2}$$

Haar mother wavelet function is defines as

$$\varphi(x)=\begin{cases}1, 0\le x<.5\\ -1, .5\le x<1\\ 0, otherwise\end{cases}. \tag{3}$$

B. Daubechies Wavelets:

Daubechies wavelets extend the Haar wavelets by using longer filters, that produce smoother scaling functions and wavelets. Basically it is a family of orthogonal wavelets defining a discrete wavelet transforms and are compactly supported orthogonal wavelets with a preassigned degree of smoothness. They are generally represented as 'dbN' where N specifies the order or level. The support length of ψ and Φ is $2N-1$. The number of vanishing moments of ψ is N, as N increases the regularity increases. The length of the filter of Daubechies wavelet used here is '2'.

C. Symlet Wavelets:

They are a modified version of Daubechies wavelets with increased symmetry and the properties of the two wavelet families are similar. The symlets have great level of simplicity which is obtained by the symmetrycity introduced to Daubechies. A symlet wavelet is denoted as 'symN', where N is the number of vanishing moments. The Daubechies wavelets 'dbN' has been generated by choosing minimum phase filter, such that the modulus of all its roots is less than 1. More symmetrical filters called symlets are obtained by making another choice. The length of the filter of Symlet wavelet used here is '8'.

D. Biorthogonal Wavelets:

Biorthogonal wavelet systems have two scaling functions Φ and $\tilde{\Phi}$ with two wavelet functions ψ and $\tilde{\psi}$ which helps for analysis and it demands more degree of freedom. A Biorthogonal wavelet transform is invertible and there is an additional freedom construct symmetric wavelet functions. The Biorthogonal wavelet that is used here has a filter length of '12'.

V. Numerical results

The PAPR for FFT based OFDM and DWT based OFDM was computed. The FFT based OFDM was evaluated for N=8, 16, 32 and 64 subcarriers. Similarly for DWT based OFDM, the PAPR was analyzed with different wavelets namely – Haar, Daubechies, Symlets and Biorthogonal wavelets. Further, PAPR with 1, 2, 3 and 4 levels of wavelet decomposition was also computed. The graph was plotted for different length of channel impulse response, i.e., L=3, 4, 5 and 6.The PAPR at transmitter or OFDM modulation (IFFT/IDWT), at receiver and after OFDM demodulation (FFT/DWT) was found out and represented in bar graph by blue, green and red bar respectively.

Fig. 2(a) shows the PAPR value for FFT based OFDM at N=8 subcarriers. Here we observe that the PAPR at the transmitter is 2.8. At the receiver, the PAPR value is found to increase than that of transmitter, in most of the cases, in the four channel taps under consideration. Further, at the demodulator output the PAPR values goes as high as 5.5 and follows the similar trend, i.e., it increases than that of the transmitter side. Fig. 2(b) shows the PAPR value for FFT based OFDM at N=16 subcarriers. The PAPR value in the transmitter is found to be 3.8. Also, the PAPR value at the receiver and the demodulated output is found to be greater than that at the transmitter side. The maximum PAPR value at the demodulator output is found to be 11.3.

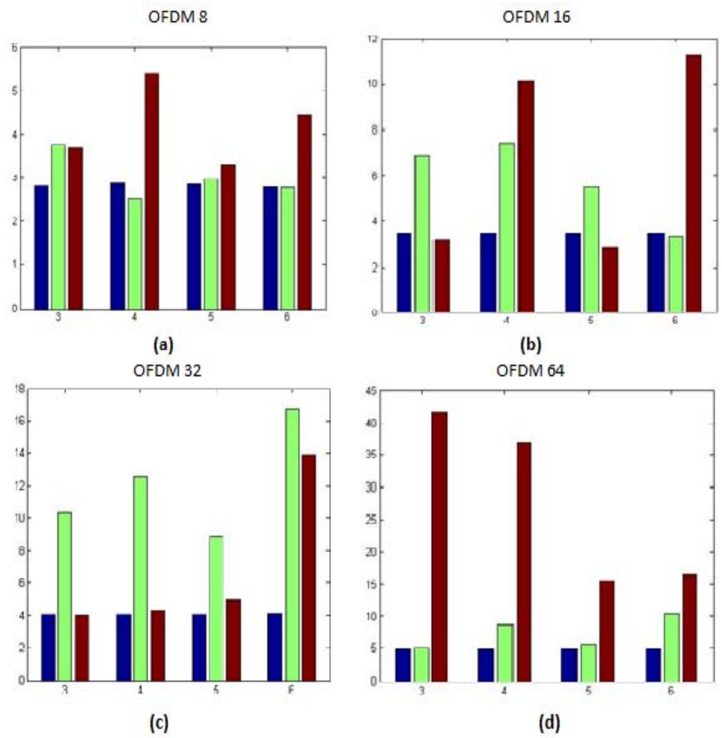

Fig. 2. PAPR for FFT based OFDM for (a) N=8, (b) N=16, (c) N=32, (d) N=64.

Fig. 2(c) shows the PAPR value for FFT based OFDM at N=32 subcarriers. The PAPR at the transmitter has a value 4. The PAPR at the receiver and demodulator output is greater than that, at the transmitter in most of the cases. Here the PAPR value at the demodulator output can go as high as 14.Fig. 2(d) shows the PAPR value for FFT based OFDM at N=64 subcarriers. The PAPR at the transmitter is found to be 5. Similar trends in increase in the PAPR value at receiver and demodulator output compared to the transmitter can be observed. The maximum value that can be attained by PAPR at the receiver is 42. Thus it can be observed that the PAPR increases with an increase in number of sub-carriers. The increase in PAPR at transmitter is gradual, while at the demodulator output the PAPR increase is found to be drastic. Further an interesting result observed is that the PAPR at the transmitter is lesser than that at the demodulator output.

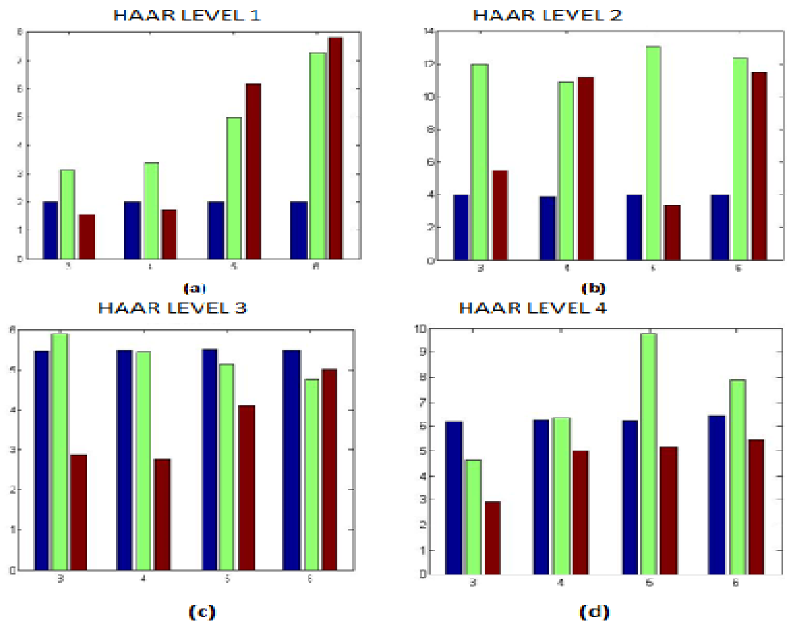

Fig. 3. PAPR for DWT based OFDM ('HAAR') for (a) level=1, (b) level=2, (c) level=3, (d) level=4.

Fig. 3(a) shows the PAPR value for DWT based OFDM that uses ‘Haar Wavelet’ for level-1 decomposition. The PAPR value at the transmitter is found to be 2. PAPR at the receiver is found to be greater than that of transmitter. The PAPR at the demodulator output can go up to 8.8. Fig. 3(b) shows the PAPR value for DWT based OFDM that uses ‘Haar Wavelet’ for level-2 decomposition. Here PAPR at the transmitter is having a value 4. As in the previous case PAPR at the receiver is greater than that of transmitter side. The maximum value of PAPR at demodulator output is 11. Fig. 3(c) shows the PAPR value for DWT based OFDM that uses ‘Haar Wavelet’ for level-3 decomposition. Here it is observed that the PAPR at the transmitter is 5.4. Interestingly, the PAPR at the demodulator output decreases compared to the transmitter and the maximum value is 5. Fig. 3(d) shows the PAPR value for DWT based OFDM that uses ‘Haar Wavelet’ for level-4 decomposition. PAPR value at the transmitter is 6.2. Like in the previous case, PAPR decreases at the demodulator output compared to that of transmitter side. Maximum value of PAPR at this side is found to be 4.9. Hence as the number of levels increases, the PAPR value increases. With the increase in number of levels, it is found that PAPR value at the demodulator is lesser than that of transmitter side. Thus, PAPR reduction efficiency increases at higher levels.

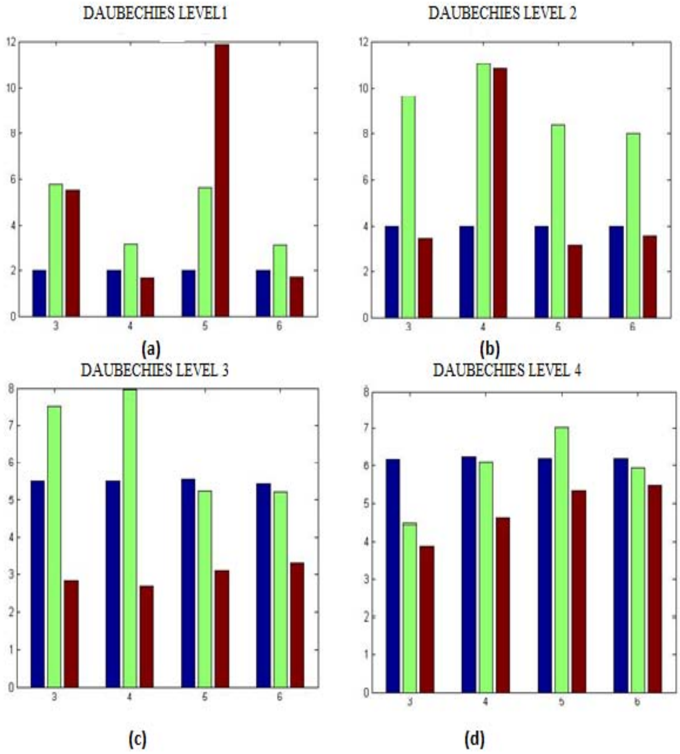

Fig. 4. PAPR for DWT based OFDM (‘DAUBECHIES’) for (a) level=1, (b) level=2, (c) level=3, (d) level=4.

Fig. 4(a) shows the PAPR value for DWT based OFDM that uses ‘Daubechies Wavelet’ for level-1 decomposition. In this graph, PAPR value at the transmitter side is 2. The PAPR value at the transmitter is less than that of the PAPR at the receiver. Maximum value of the demodulator output goes up to 12. Fig. 4(b) shows the PAPR value for DWT based OFDM that uses ‘Daubechies Wavelet’ for level-2 decomposition. Here it is noted that the PAPR at the transmitter side is 4. PAPR at the receiver side is greater than that of the transmitter side where as the PAPR at the demodulator output is found to be less compared to the transmitter in almost all cases. At the demodulator side the PAPR can rise up to a value of 11.Fig. 4(c) shows the PAPR value for DWT based OFDM that uses ‘Daubechies Wavelet’ for level-3 decomposition. It is found that the PAPR value at the transmitter is 5.5. Maximum value of PAPR at the demodulator output is 3. The PAPR value at the demodulator output is less compared to that of the transmitter side. Fig. 4(d) shows the PAPR value for DWT based OFDM that uses ‘Daubechies Wavelet’ for level-4 decomposition. The PAPR value at the transmitter side is 6. Here also the value of PAPR at the demodulator output is less compared to that of the transmitter side with a maximum value of 5. Hence, from all the above graph we can conclude that the PAPR value increases as the level of decomposition increases. As the level of decomposition increases, PAPR at the demodulator achieve a lesser value than that of the corresponding PAPR at the transmitter.

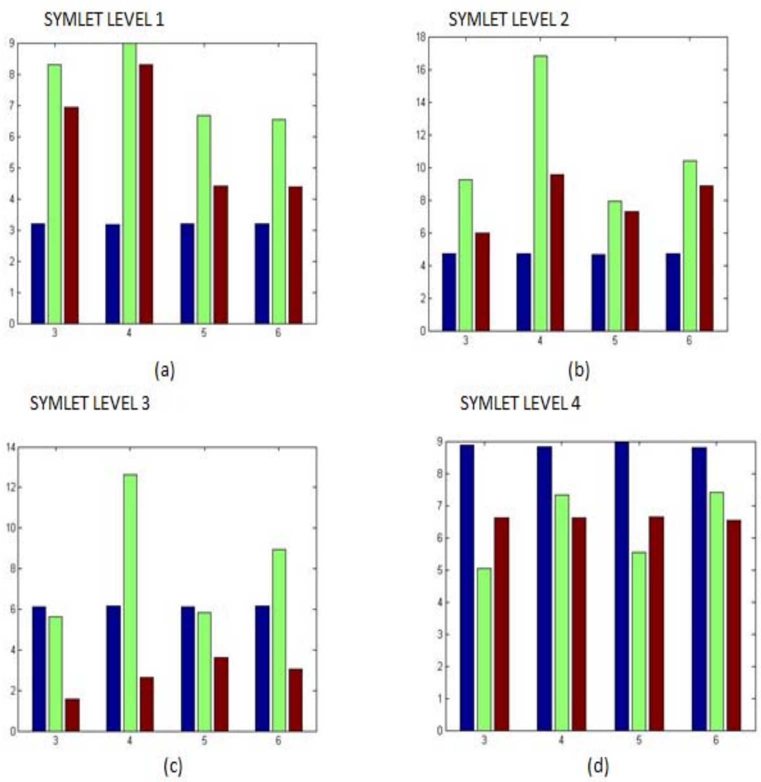

Fig. 5. PAPR for DWT based OFDM (‘SYMLETS’) for (a) level=1, (b) level=2, (c) level=3, (d) level=4.

Fig. 5(a) gives the PAPR value for DWT based OFDM with ‘symlet wavelet’ for level-1 decomposition. It can be observed that the PAPR is 3.2 at the transmitter side. Also the PAPR at the demodulator output is greater than that at the transmitter side. The maximum value of PAPR observed at demodulator output is 8.1. Fig. 5(b) gives the PAPR value for DWT based OFDM with ‘symlet wavelet’ for level-2 decomposition. Here the PAPR at the transmitter is found to be 4.4. It can be further noted that the PAPR at the transmitter side is smaller compared to the PAPR at receiver and demodulator output. The maximum value of PAPR observed at demodulator output is 9 in this case. Fig. 5(c) gives the PAPR value for DWT based OFDM with ‘symlet wavelet’ for level-3 decomposition. Here it may be noted that the PAPR value at the transmitter is 6.2. Also the PAPR at the transmitter side is greater as compared to that at demodulator output. Here the PAPR at the demodulator output is between ranges 1 to 3.8. Fig. 5(d) gives the PAPR value for DWT based OFDM with ‘symlet wavelet’ for level-4 decomposition. This graph shows an interesting trend; the PAPR at the receiver and demodulator output is always lesser than that at the transmitter, which can be used as an advantage. The PAPR at the demodulator output is found to be 6.5. Thus for symlet wavelet as the number of level increases the PAPR value increases at the transmitter. At the demodulator output, there is a decrease in PAPR value as the number of levels increases.

Fig. 6(a) gives the PAPR value for DWT based OFDM with ‘biorthogonal wavelet’ for level-1 decomposition. The PAPR value at the transmitter side is found to be 5. Here it is observed that the PAPR at the receiver is greater to that at transmitter. Also the PAPR at demodulated output has a maximum value of 6. Fig. 6(b) gives the PAPR value for DWT based OFDM with ‘biorthogonal wavelet’ for level-2 decomposition. It is observed that the PAPR value at the transmitter side is 8. The maximum PAPR value at the receiver side is found to be 19. Fig. 6(c) gives the PAPR value for DWT based OFDM with ‘biorthogonal wavelet’ for level-3 decomposition. In this the PAPR at the transmitter is attaining a value of 12. The maximum value of the demodulated output is observed to be 7, which is very much less that the transmitter side PAPR. Fig. 6(d) gives the PAPR value for DWT based OFDM with ‘biorthogonal wavelet’ for level-4 decomposition. Transmitter side has a PAPR of 18. Maximum value of PAPR at the demodulator side is 7, which is very much less compared to the transmitter side. Like in the previous wavelet, here also PAPR value increases as the level of decomposition increases. With higher levels it is observed that the PAPR at the demodulator output reduced drastically compared to the transmitter side.

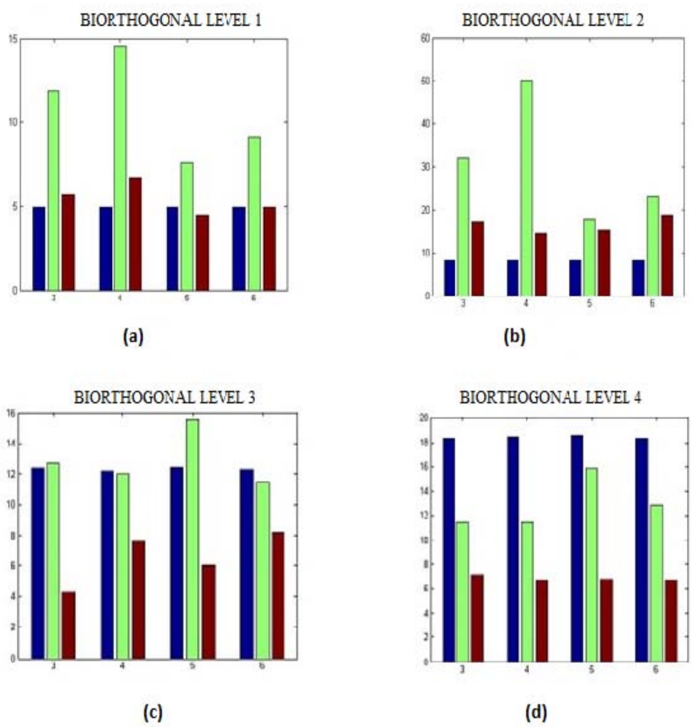

Fig. 6. PAPR for DWT based OFDM (‘BIORTHOGONAL’) for (a) level=1, (b) level=2, (c) level=3, (d) level=4.

Thus the PAPR for FFT based OFDM and for DWT based OFDM was computed. The PAPR value at transmitter in FFT based OFDM is in a range of 2 to 5, in DWT based OFDM – Haar and Daubechies it ranges from 4 to 6, in Symlet from 3 to 8 and in Biorthogonal it is 4 to 18. Thus Haar and Daubechies wavelet can be employed in transmitter side of DWT based OFDM, to tackle the problem of power amplifier. Also in FFT based OFDM the value of PAPR at demodulator output ranges from 3 to 42. In Haar based OFDM, PAPR ranges from 1.5 to 11 and in Daubechies based OFDM, it ranges from 1.6 to 10.8. The PAPR in Symlet based OFDM ranges from 1.5 to 9.5 while PAPR in Biorthogonal based OFDM is from 4.5 to 18. Thus to have a lower PAPR at demodulator output, Symlet wavelet can be preferred as it mitigates the problem due to time varying nature of the channel. A significant observation is that in the case of FFT based OFDM, the PAPR at the demodulator output is always greater than that at the transmitter, while in DWT based OFDM, and the PAPR in the demodulator output is lesser than that of the transmitter, which is advantageous. Also it can be observed that DWT based OFDM system provides better reduction in PAPR compared to conventional OFDM.

VI. Conclusion

PAPR is a serious challenge faced by the OFDM system, causing the power amplifier to operate in non-linear region. The issue of high PAPR was hence addressed in this paper. Further, it can be concluded that PAPR in OFDM system is reduced by employing DWT based OFDM instead of conventional OFDM. It is noticed that in DWT based OFDM, PAPR at the demodulator output is less than that at the transmitter side. The fact that no additional complexity is added, is the perk of DWT based OFDM system. Thus it can be concluded that DWT based OFDM can be employed as it eliminates the need of serial/ parallel cyclic conversion prefixing and brings down the value of PAPR. As a future scope for the work, DWT based OFDM can be implemented using the wavelets namely – coiflets, meyer, reverse biorthogonal and complex Gaussian wavelets, wherein the PAPR reduction can be studied.

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