FIR Filter Design Using An Adjustable Novel Window and Its Applications

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***Abstract*—In this paper, a new class of adjustable window function, based on combination of tangent hyperbolic function and a weighted cosine series, is proposed to design an FIR filter. The proposed window is adjustable since the spectral characteristics of the window vary with the change of a simple window's controlling parameter. The spectral characteristic of the proposed window is studied and its performance is compared with Hamming and Kaiser windows. Simulation results show that the proposed window yields better ripple and side-lobe roll-off ratios compare to the mentioned windows. Moreover, the paper represents the application of the proposed window in finite impulse response (FIR) filter design. The results confirm that the filter designed by the proposed window provides 11.5607 dB better ripple ratio than Kaiser windows. In side-lobe roll-off ratio measurement, the proposed window based FIR low-pass filter attains 95.75 dB and 14.4534 dB better result than Hamming and Kaiser windows respectively. Moreover, the filter designed using the proposed window method reduces additive white Gaussian noise from the ECG signal more precisely than Kaiser window.**

Keyword- FIR Filter, Kaiser window, Hamming window, Tan hyperbolic function, Ripple ratio, Side-lobe roll-off ratio.

I. INTRODUCTION

In signal processing, the window function is a mathematical function that provides zero-valued outside of certain interval. In engineering term, a window is a finite array, consists of coefficients to satisfy the desirable requirements [1].Window functions are widely used in digital signal processing for the applications in signal analysis and estimation, digital filter design and speech processing [2]. Windows can be categorized as fixed or adjustable [3]. Fixed windows have only one independent parameter which is the window length. In the fixed windows, the length of the windows control the spectral characteristics. Among the all fixed windows, Hamming window is widely used for various applications. Adjustable windows have two or more independent parameters. Of them, one is the length of the window, as in the fixed windows and one or two additional parameter or parameters that can control the spectral characteristics the windows. The important spectral characteristics of a window function are i) main-lobe width ii) ripple ratio and iii) its side-lobe roll-off ratio. There are two desirable specifications for any window function. They are narrower main-lobe width and the smaller ripple ratio. However these two requirements are contradictory to one another [4]. The Kaiser window has the two parameters to adjust the spectral characteristics of the window.

In this paper, a novel adjustable window is proposed which has only two parameters (one is length of the window and another is the additional parameter "r") that control the spectral characteristics of the window. The proposed window is very simple as it does not involve any Bessel function like the Kaiser window. The filter designed with the proposed window attains 11.5607dB less ripple ratio than Kaiser window based FIR low-pass filter. The side-lobe roll-off ratio is also better than the Kaiser window. When the proposed window based FIR low-pass filter is applied to the noisy ECG signal, the filter precisely eliminate the white noise from the ECG signal. In term of spectral analysis of the ECG signal, the proposed window based FIR filter achieves 10.709 dB less ripple ratio than the Kaiser window. The side-lobe roll-off ratio of the filtered ECG signal with the proposed window based FIR filter is also 7.8084 dB better than the Kaiser window.

II. SPECTRAL PROPERTIES OF WINDOWS

A window, $w(nT)$, with a length of N is a time domain function which is nonzero co-efficient for $n <= |(N-1)/2|$ and zero for otherwise. A typical window has a normalized amplitude spectrum in dB as shown in Fig 1.

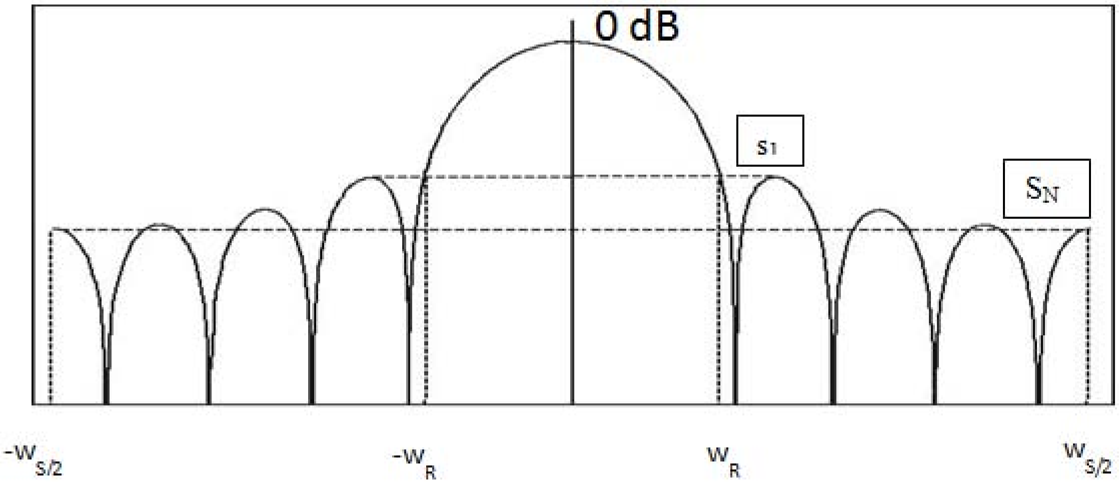

Fig 1: A typical window's normalized amplitude spectrum.

Normalized spectrum in Fig 1 is obtained by following equation

$$|W_n(e^{j\omega T})| = 20\log_{10}\left(\frac{|A(w)|}{|A(w)|_{\max}}\right) \tag{1}$$

The spectral characteristics which determine the windows performance are the main-lobe width(w_M), the ripple ratio (R) and the side-lobe roll-off ratio(S). From the Fig. 1, these parameters can be defined as

W_M = Two times of the half of the main-lobe width=$2w_R$.

R = (Maximum side-lobe amplitude in dB) – (Main-lobe amplitude in dB) = S1.

S = (Maximum side-lobe amplitude in dB) – (Minimum side-lobe amplitude in dB) = S_1-S_N.

III. PROPOSED WINDOW

The new proposed window is a combination of tan hyperbolic function and a weighted cosine series. The tan hyperbolic function is given by the following equation

$$y_1 = \left[\tanh\left\{ \frac{n - \frac{(N-1)}{2} + \cosh^2(\alpha)}{B} \right\} - \tanh\left\{ \frac{n - \frac{(N-1)}{2} - \cosh^2(\alpha)}{B} \right\} \right] \tag{2}$$

The weighted cosine function is expressed as follows

$$y_2 = 0.375 - 0.5\cos\left\{ \frac{2\pi(0:N-1)}{N-1} \right\} + 0.125\cos\left\{ \frac{4\pi(0:N-1)}{N-1} \right\} \tag{3}$$

Where N is the length of the window, α and B are the constants. Here n is the number of samples. Here the symbol ":" indicates upto.

These two equations are combined in the following way.

$$w(n) = (y_1 * y_2)^{\gamma}, \quad where, \gamma = r^{(\frac{1}{5})} \quad \text{and “*” means multiplication.} \tag{4}$$

Here r is a variable which controls the shapes and frequency response of the window. So "r" can be named as controlling parameter of the window. In mathematical term, the window can be expressed a

$$w(n) = \left| \begin{array}{ll} (y_1 * y_2)^{\gamma}, & n \le 0 \le N \\ 0, & otherwise \end{array} \right. \tag{5}$$

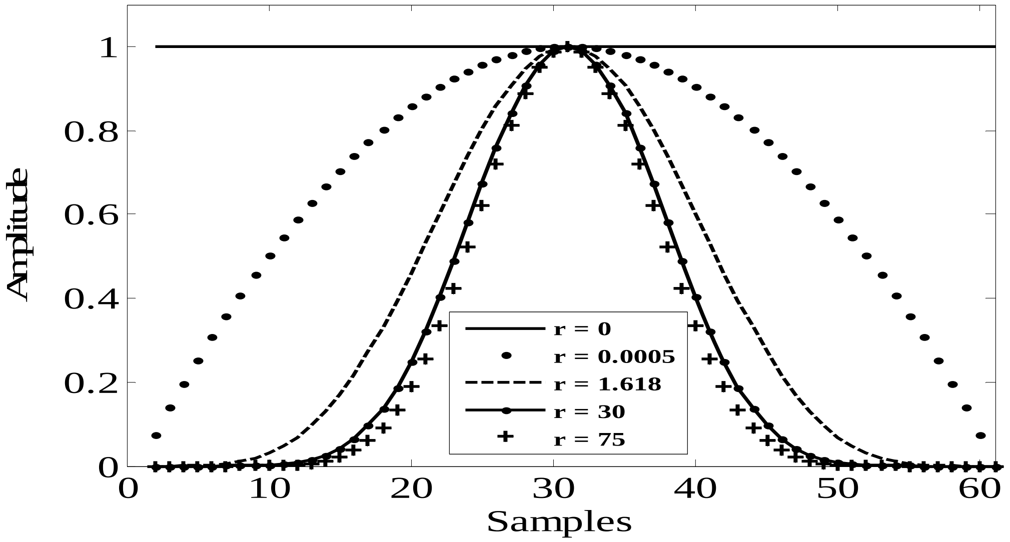

Fig 2: Shapes of proposed window with different values of r.

From the Fig 2 it has been seen that the shape of the window varies as the value of the adjusting parameter r changes. With r = 0, the proposed window becomes rectangular shape. Fig 2 also shows that the width of the window gets narrower as the value of the controlling parameter gets bigger.

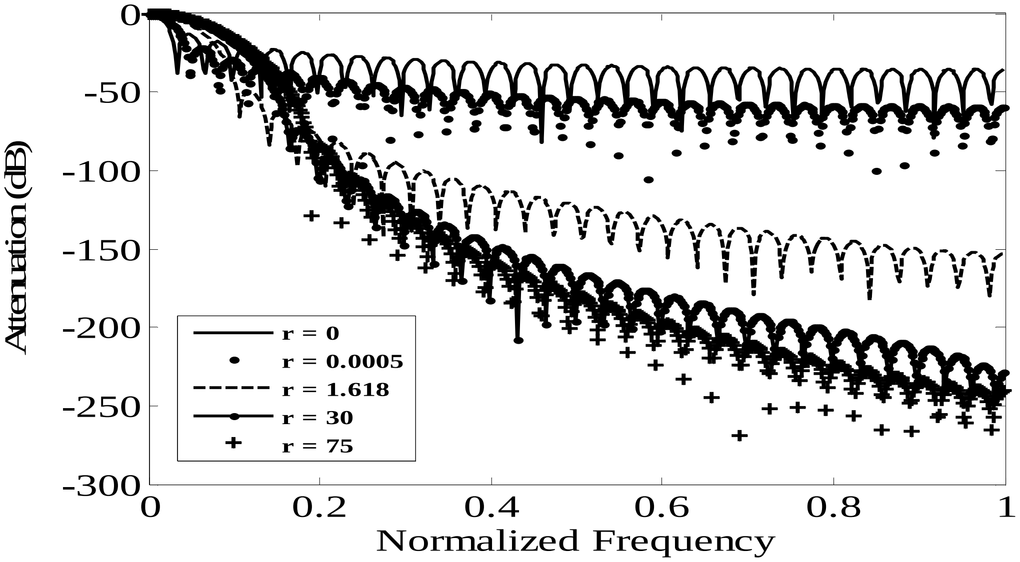

Fig 3: The normalized frequency response of the proposed window with different values of r.

From the Fig 3, it has been observed that frequency response of the proposed window with r=0, is exactly same to the frequency response of the rectangular window. With r = 0, the main-lobe width is 2π×0.0332 rad/sample, the ripple ratio is -9.139 dB and the side-lobe roll-off ratio is 26.561 dB. When r = 1.618, the main-lobe width becomes 2π×0.1055 rad/sample, the ripple ratio and the side-lobe roll-off ratio are -45.4704 dB and 113.5493 dB respectively. If the r =75, the main-lobe width, the ripple ratio and the side-lobe roll-off ratio are 2π×0.1914 rad/sample, -81.1869 dB and 175.7724 dB respectively. At the same time, Fig 3 shows that as r increases the main-lobe width and the ripple ratio become wider and smaller respectively.

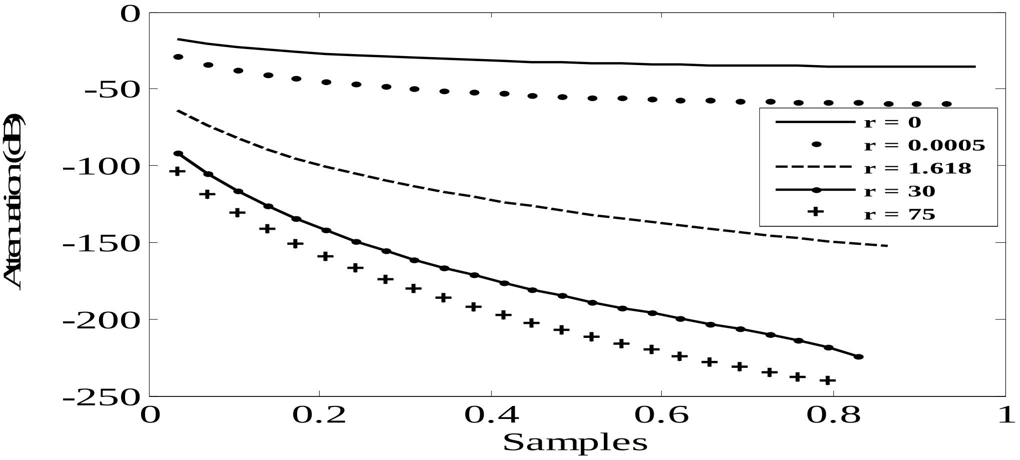

Fig 4: Ripple ratio and side-lobe roll-off ratio measurement curves

Fig 4 shows that as the r increases it improves side-lobe roll-off ratio. The curves become sharper with the increment of r. The more sharper curve means that it can reject the far end interference better [5].

IV. PERFORMANCE ANALYSIS:

In this section, we compare the shape and the spectral characteristics of the proposed window with several commonly used windows.

A. Hamming window:

Hamming window is the most popular and most commonly used window function. This window function also belongs to cosine family. Hamming window function can be expressed as follows

$$w(n) = 0.54 - 0.46\cos(2\pi\frac{n}{N}), \quad 0 \le n \le N \tag{6}$$

where the window length is N+1 .

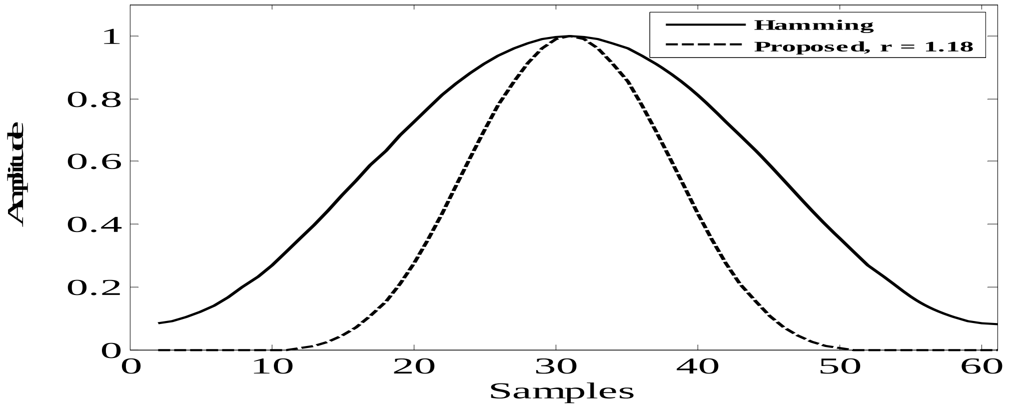

Fig 5: The comparison of Hamming window and the Proposed window in time domain representation.

The starting and ending points of Hamming window do not touch the X-axis. This means that the coefficients of the Hamming window are always greater than zero. The proposed window touches X-axis symmetrically. It has been seen that the Proposed window is narrower than the Hamming window with its controlling parameter r =1.18.

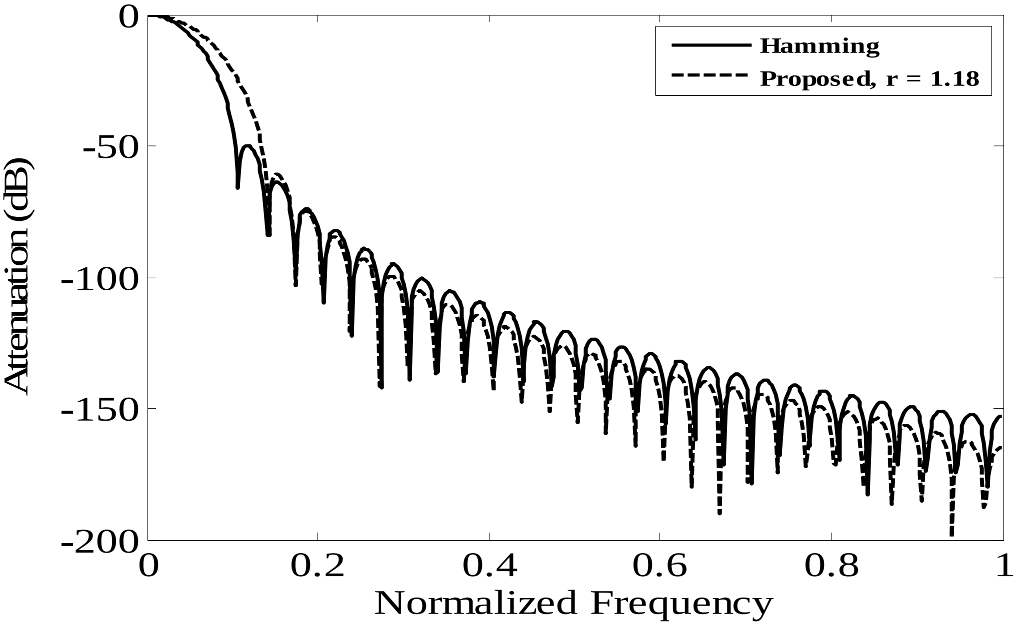

Fig 6: Normalized frequency response comparison of Hamming and the Proposed windows.

From the Fig 6 it has been observed that the main lobe width of Hamming window is smaller than the proposed window with the adjustable parameter r =1.18. The ripple ratio of the proposed window with r=1.18 is 61.07 dB which is 11.39 dB less than the Hamming window. The side-lobe roll-off ratio of the proposed window is slightly better than Hamming window. Table I shows the numeric comparison between Hamming window and the proposed window.

TABLE I. Comparison of spectral parameters between Hamming and the Proposed windows

Window	**Main-lobe Width (rad/sample)**	**Ripple Ratio (dB)**	**Side-lobe Roll-off Ratio(dB)**
Hamming	2π×0.1074	-49.68	102.42
Proposed, r=1.2	2π×0.1406	-61.07	103.83

B. *Kaiser window:*

The Kaiser window is also known as Kaiser-Bessel window function. The Kaiser window function can be expressed as follows [6][7].

$$w(n) = \frac{I_0(\pi\alpha\sqrt{1-(\frac{2n}{N-1}-1)^2})}{I_0(\pi\alpha)} \, where, 0 \le n \le N \tag{7}$$

where I_0 is the zero-th order modified Bessel function of the first kind [6]. Kaiser window is another very popular adjustable window. The variable parameter α determines the shape of the window in time domain representation. In frequency domain representation, it settles the tradeoff between the main-lobe width and the side lobe attenuation. So, α is the controlling or adjustable parameter of the Kaiser window function. The default value of α is 0.5.

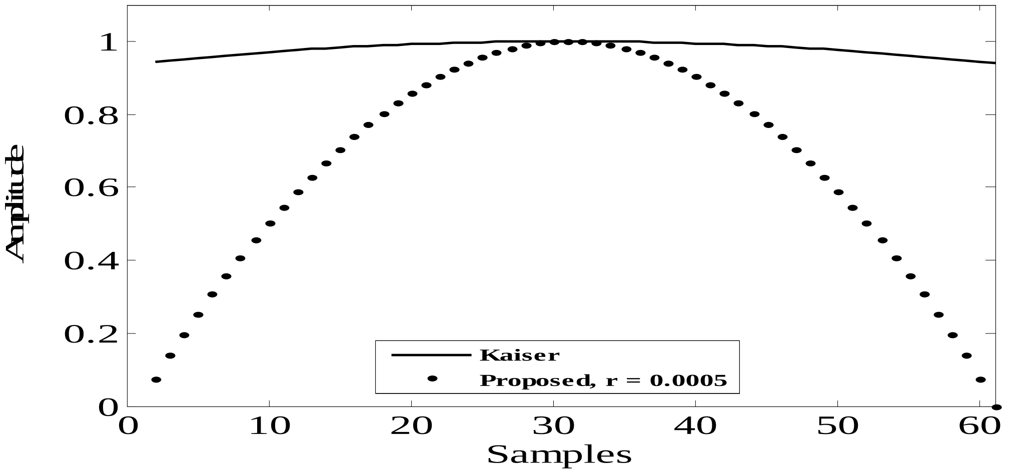

Fig 7: The comparison of Kaiser window and the Proposed window in time domain representation

It has been seen from the Fig 7 that Kaiser window is almost like a rectangular shape with its controlling parameter α =0.5. On the other-hand, the proposed window looks like a bell shaped window, with its adjustable parameter r=0.0005.

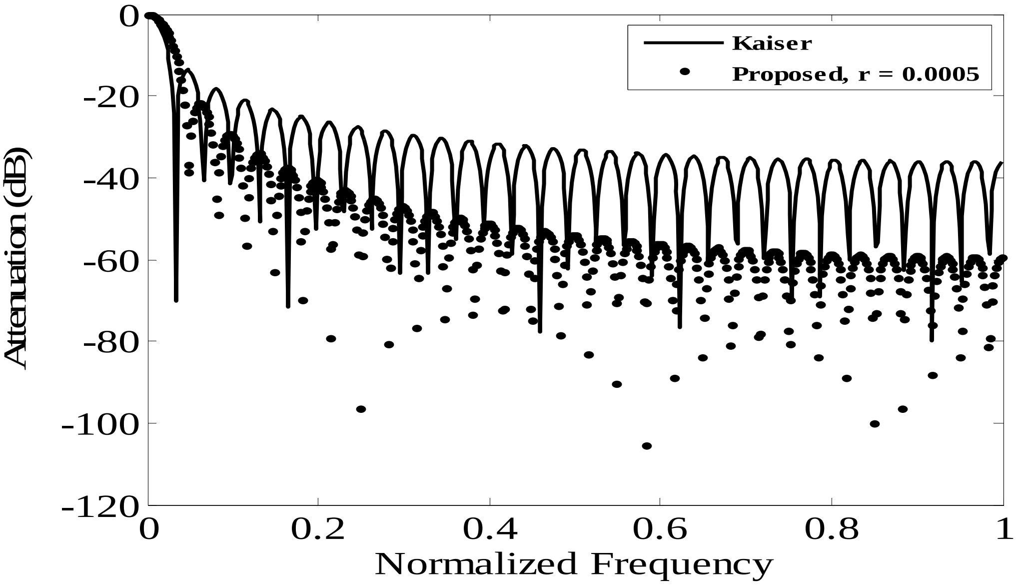

Fig 8: Normalized frequency response comparison of Kaiser and the Proposed windows.

It has been seen that in main-lobe width calculation, Kaiser window is slightly better than the proposed window with controlling parameter r=0.0005. On the other hand, in terms of ripple ratio, the proposed window performs finer performance than the Kaiser window. Because the ripple ratio of Kaiser window and the proposed window are -13.0716 dB and -22.0168 dB respectively. So the proposed window yields 8.9452 dB better ripple ratio than the Kaiser window. Moreover side-lobe roll-off ratio of the proposed window is 12.8582 dB better than the Kaiser window. So it is more frequency selective. Table II shows the comparative performance of the Kaiser window and the proposed window.

TABLE II. Comparison of spectral parameters between Kaiser and the Proposed windows

Window	**Main-lobe Width (rad/sample)**	**Ripple Ration (dB)**	**Side-lobe Roff-off Ratio(dB)**
Kaiser	2π×0.0332	-13.0716	26.0264
Proposed, r=0.0005	2π×0.0469	-22.0168	38.8846

V. APPLICATION

In this section we will discuss how the proposed window helps us to design FIR low pass filter and also compare the simulation result with the other adjustable windows.

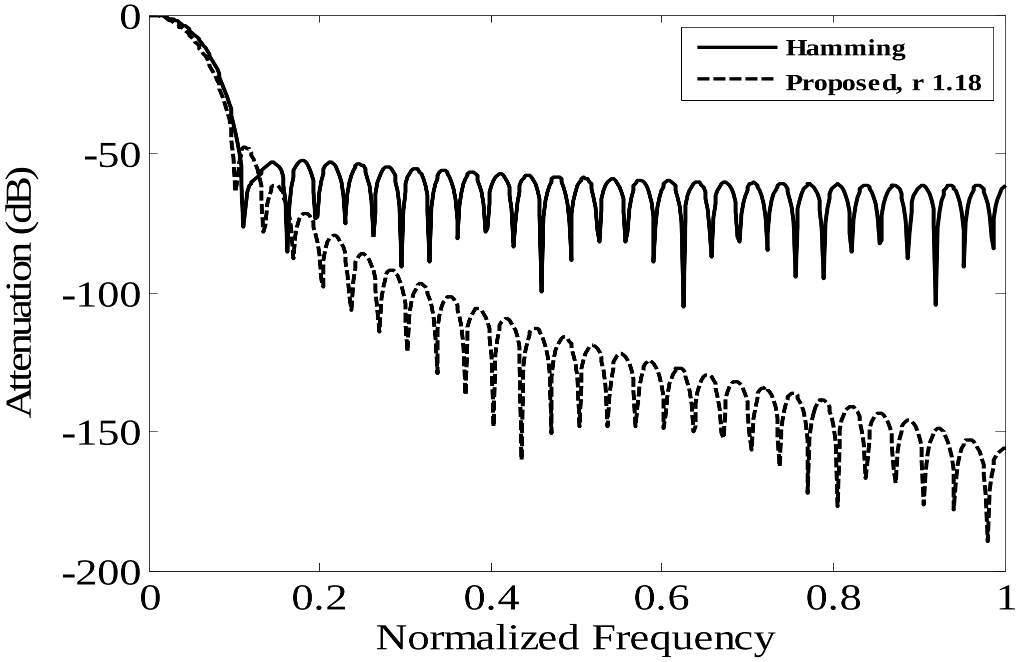

Fig 9(a): Performance analysis of FIR low pass filter between Hamming and the Proposed windows.

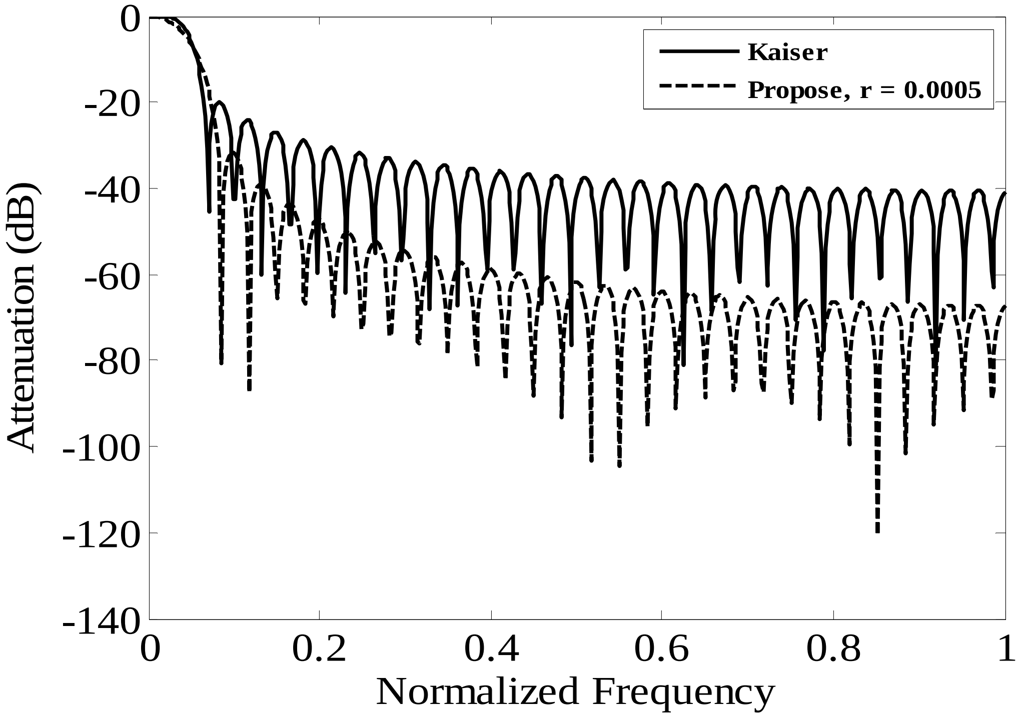

Fig 9(b): Performance analysis of FIR low pass filter between Kaiser and the Proposed windows.

Having a cut off frequency of ω_c, the impulse response of an ideal low pass filter is given by

$$h_{LP,ideal}[n] = \frac{\sin(\omega_c n)}{\pi n} \tag{8}$$

By windowing this IIR filter with the windows discussed in this paper, different FIR filter can be obtained. For $\omega_c = 0.1\pi$ Fig 9(a)-9(b) show the normalized frequency response of the FIR low-pass filters designed by applying different windows of length 61. Fig 9(a) shows that the ripple ratio of Hamming window and the proposed window are -52.98 dB and -48.54 dB respectively. The ripple ratio of Kaiser window and the proposed window are -17.0831 dB and -28.6438 dB respectively which has been seen from the Fig 9(b). The side-lobe roll-off ratio of Hamming window and the proposed window are 8.41 dB and 104.16 dB respectively. So the side-lobe roll-off ratio of the proposed is 12.3853 times better than Hamming window in FIR low-pass filter design. Now if we compare the side-lobe roll-off ratio of Kaiser window and the proposed window from the Fig 9(b), we have seen that the values are 26.4604 dB and 40.9138 dB respectively. Hence, the proposed window based FIR low-pass filter gives 14.4534 dB better performance than the Kaiser window. The following table summarized the total simulation result.

TABLE III. Data analysis of FIR filter obtained by windowing with different windows

Comparing Windows	**Main-lobe Width (rad/sample)**	**Ripple Ratio (dB)**	**Side-lobe Roll-off Ratio(dB)**
Proposed Vs Hamming	$2\pi\times0.1018$	-48.54	104.16
	$2\pi\times0.0.1115$	-52.98	8.41
Proposed Vs Kaiser	$2\pi\times0.0861$	-28.6438	40.9138
	$2\pi\times0.0841$	-17.0831	26.4604

The above table shows that the filter designed using the Proposed window yields much more better spectral response than Hamming and Kaiser windows.

VI. NOISE ELIMINATION FROM ECG SIGNAL

The electrical activity of heart is called ECG signal which is generated by repolarization and depolarization of the atria and ventricles [8]. The ECG signals are very small in amplitude. The frequency range of the ECG signals lies in between 0.05 to 100 Hz. ECG signals are consist of P-waves, QRS complex and T-waves. The depolarization of atria results P-waves [8]. The QRS complex consists of three waves, sequentially Q, R and S waves. The rapid depolarization of both the ventricles yields the QRS complex. Ventricular repolarization generates T-waves [9]. The standard ECG signal of one period is shown in the Fig 10.

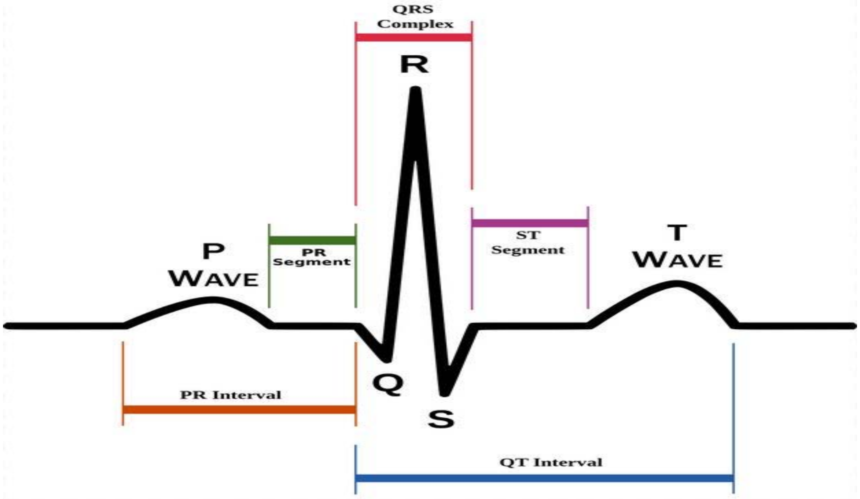

Fig 10: Typical ECG trace.

Arti-factual signals are generated from various internal and external sources and mixed with ECG signal. The noise generally caused by the signal from muscle contractions, which can be expressed as zero mean Gaussian noise [10]. A corrupted ECG signal is given below.

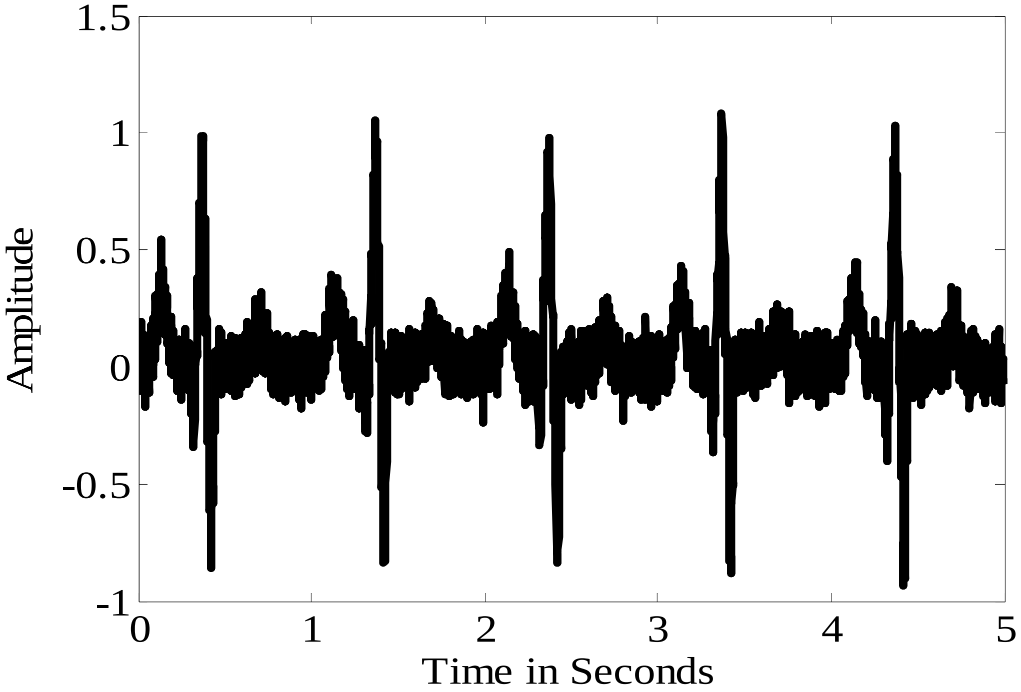

Fig 11: Corrupted ECG signal due to Additive White Gaussian Noise

This corrupted ECG signal is then passed through a low pass FIR filter which is designed using Kaiser window function. The result is shown in the Fig 12.

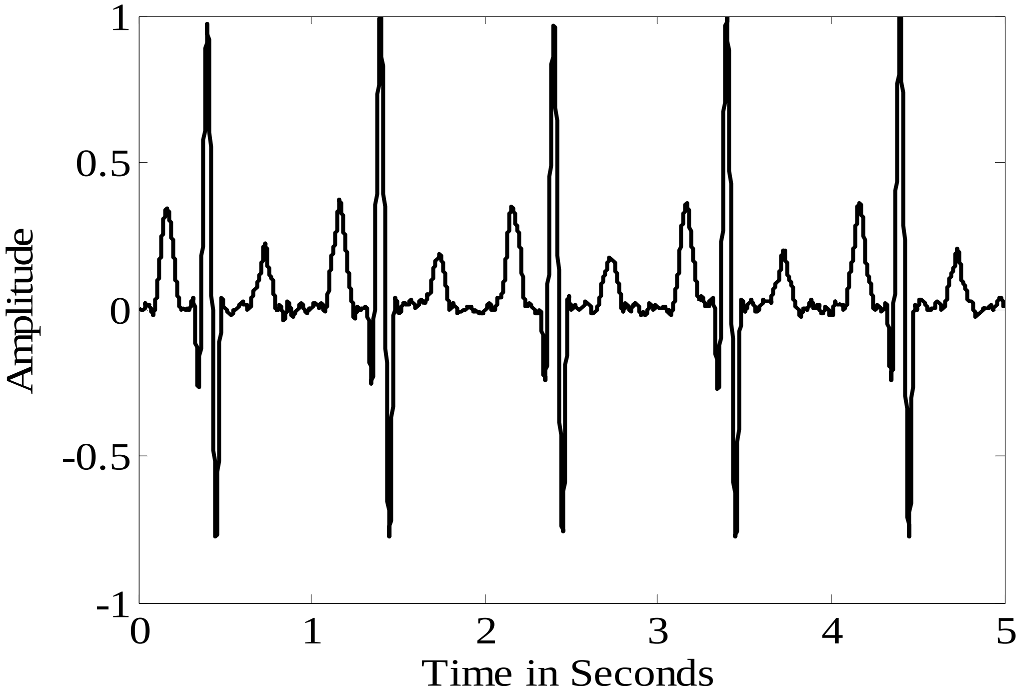

Fig 12: Filtered ECG signal using Kaiser window based FIR low pass filter

The same corrupted ECG signal is then passed through another low pass FIR filter which is designed using the Proposed window function. The result is shown in the Fig 13.

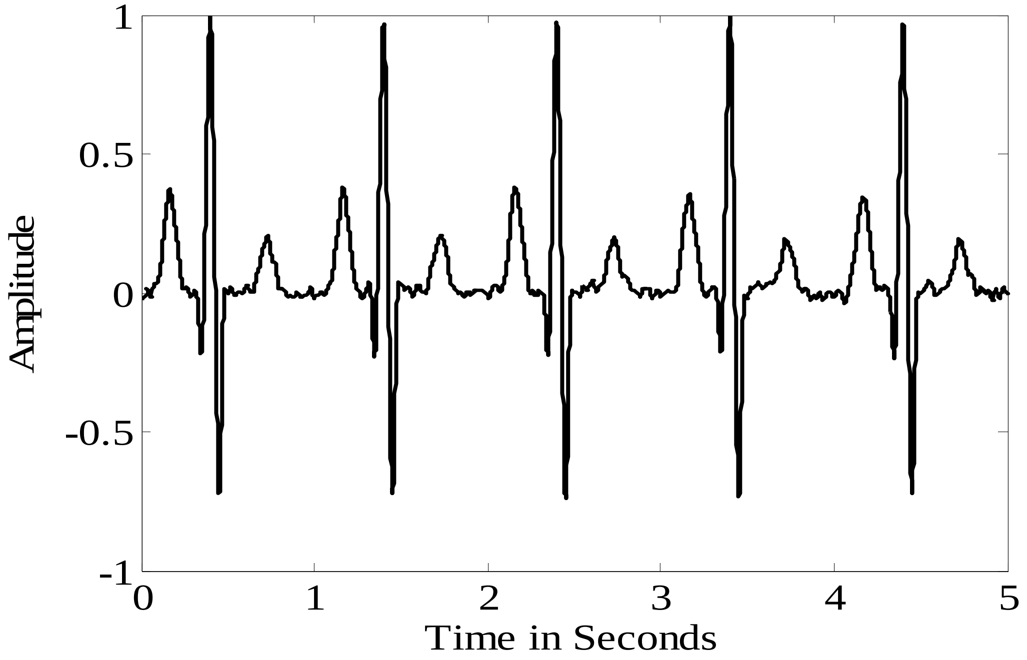

Fig 13: Filtered ECG signal using the Proposed window based FIR low pass filter

From the Fig12 and Fig13, it can be seen that the P and the R peaks are sharper in fig13 than in fig12. That means that the proposed window based FIR low-pass filter is more efficient than the Kaiser window in order to reduce the additive white Gaussian noise from the noisy ECG signal.

VII. NORMALIZED FREQUENCY RESPONSE OF ECG SIGNAL

After obtaining the filtered version of the ECG signal using Kaiser and the proposed windows based FIR low-pass filters, their respective normalized frequency response has been analysed. Fig14 and Fig15 show the normalized frequency response of the ECG signal using Kaiser window and the proposed window based FIR filter respectively.

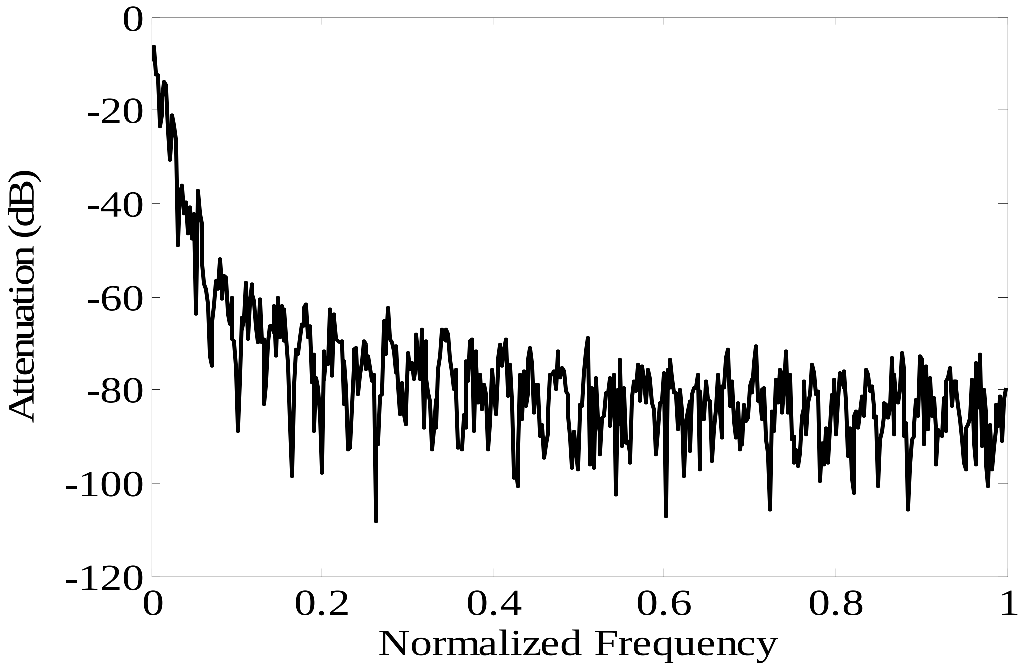

Fig 14: : Normalized frequency response of the filtered ECG signal using Kaiser window based FIR low-pass filter.

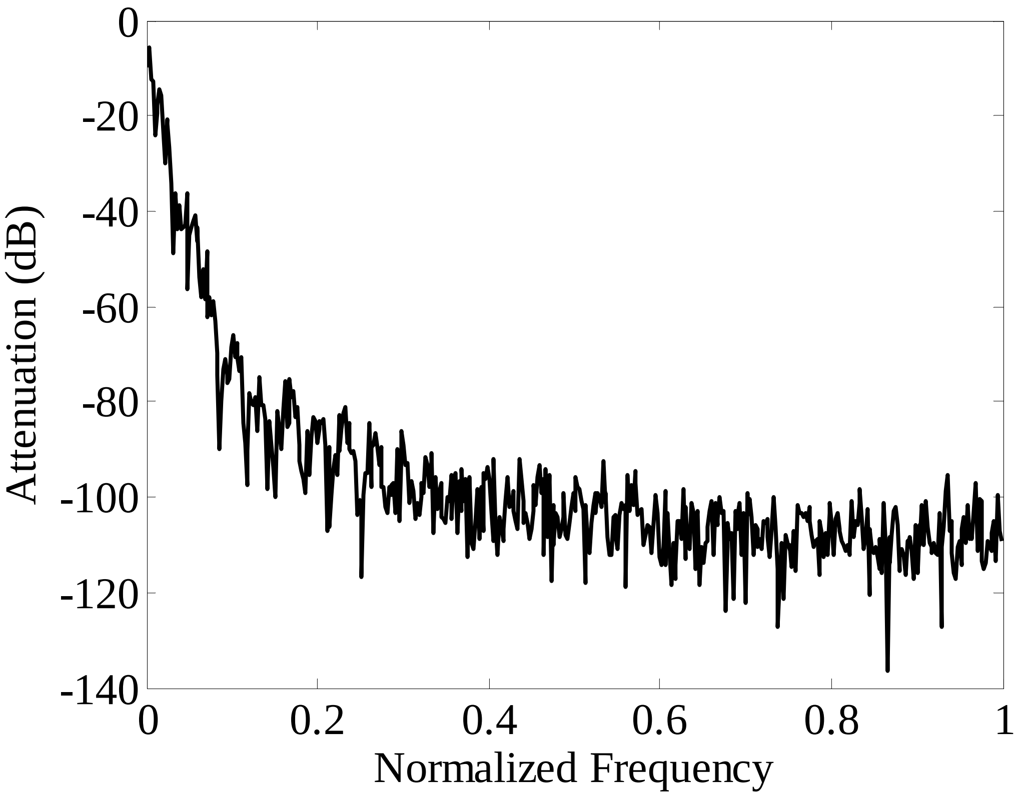

Fig 15: Normalized frequency response of the filtered ECG signal using the proposed window based FIR low-pass filter.

From the Fig 14, it has been seen that the ripple ratio of the filtered ECG signal using Kaiser window based FIR low-pass filter is about -52.5317 dB. On the other-hand, the value of the ripple ratio is approximately -63.2407 dB for the proposed window based FIR low-pass filter after filtration of the ECG signal. Moreover as the frequency increases the side-lobe peaks of the filtered ECG signal using the proposed window method decreases sharply and the last side-lobe peak is almost -100dB. On the contrary, the last side-lobe peak of the filtered ECG signal using Kaiser window function is -81.4828dB. So it is observed that the proposed window base FIR filter reduces additive white Gaussian noise more precisely than the Kaiser window based FIR filter. The following table summarized the simulation result.

TABLE IV. Spectral parameters of filtered ECG signal after filtering with Kaiser and the Proposed windows based FIR low-pass filters

Window	**Ripple Ration (dB)**	**Side-lobe Roff-off Ratio(dB)**
Kaiser	-52.5317	28.9511
Proposed, r=0.0005	-63.2407	36.7593

VIII. CONCLUSION

A novel highly adjustable window function has been proposed which has adaptive spectral characteristics. The proposed window has very high side-lobe roll-off ratio compared to commonly used windows named as Hamming and Kaiser windows. The filter designed by the proposed window achieves 11.5607dB less ripple ratio than Kaiser window based FIR low-pass filters. The proposed window based FIR low-pass filter also gives 95.75 dB and 14.4534 dB better side-lobe roll-off ratio than Hamming and Kaiser windows respectively. Moreover, the proposed window based filter yields more efficient to reduce noise from ECG signal. Another advantages of the proposed windows is that it has only one adjustable parameter. So it is very much simple one.

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