PSO Based Multiuser Detection over GK Fading Channels with MRC Receive Diversity in Impulsive Noise

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Abstract—The direct sequence-code division multiple accesses (DS-CDMA) signals are transmitted over multipath channels that introduce fading and shadowing. Combined effect of multipath fading and shadowing along with multiple access interference (MAI) and inter-symbol interference (ISI) worsens the system performance. Further, experimental results have confirmed the presence of impulsive non-Gaussian noise in wireless mobile communication channels. Hence, this paper presents a particle swarm optimization (PSO) based multiuser detection technique for DS-CDMA systems over generalized-K (GK) fading channels in presence of impulsive noise. Maximal ratio combining (MRC) receive diversity is also incorporated to mitigate the effects of fading and shadowing. An approximate closed-form expression for average error rate of BPSK signals over GK fading channels is derived. Performance of proposed M-estimator based detector is also studied by evaluating average error rate. Simulation results reveal that the proposed M-estimator based detector performs better in the presence of fading, shadowing and heavy-tailed impulsive noise when compared to least squares, Huber and Hampel M-estimator based detectors.

Keywords—diversity combining, fading channel, GK distribution, impulsive noise, multiuser detection, probability of error, particle swarm optimization, shadowing

I. INTRODUCTION

Recent research has explored the potential benefits of multiuser detection for code division multiple access (CDMA) communication systems with present multiple access interference (MAI) [1]. These optimal multiuser detectors have led to the developments of the various linear multiuser detectors with Gaussian noise though various experimental results have confirmed that many realistic channels are impulsive in nature [2], [3]. Lately, the problem of robust multiuser detection in non-Gaussian channels has been addressed in the literature [4], [5], [6], which were developed based on the Huber, Hampel, and a new M-estimator (modified Hampel), respectively. Since CDMA transmissions are frequently made over channels that exhibit fading, it is of interest to design receivers that take this fading behavior of the channel in to account [3]. Robust multiuser detection for synchronous DS-CDMA system with maximal ratio combiner (MRC) receive diversity over Nakagami-m fading channels is presented in [7] by assuming that the modulation is binary PSK (BPSK). But, the simultaneous presence of multipath fading and shadowing leads to worsening the wireless channels [8]. Recently, the performance of M-decorrelator in simultaneous presence of fading and shadowing with impulsive noise is presented in [9] and by incorporating MRC diversity in [10]. Implementation of particle swarm optimization (PSO) based M-decorrelator in Nakagami-m fading channels for the demodulation of DPSK signals is presented in [11]. Hence, in this paper, the PSO based multiuser detection for DS-CDMA system over generalized-K (GK) fading channels with impulsive noise is considered by incorporating MRC diversity.

The remaining portion of the paper is organized as follows: DS-CDMA system over multipath fading channel in non-Gaussian impulsive noise environment is presented in Section II. Section III presents the GK fading channel with MRC receive diversity. Influence function of the proposed M-estimator, derivation of an approximate closed-form expression for average probability of error of an M-decorrelator and PSO based M-decorrelator is presented in Section IV. Section V presents the simulation results to study the performance of proposed M-decorrelator in fading and shadowing with heavy-tailed impulsive noise. Finally, conclusions are drawn in Section VI.

II. SYSTEM MODEL

An L-user CDMA system, where each user transmits information by modulating a signature sequence over a single-path Nakagami-m fading channel, is considered in this paper. The received signal over one symbol duration can be modeled as [3]
\[ r(t) = \Re \left\{ \sum_{i=0}^{M-1} \sum_{l=1}^{L} b_l(i) \alpha_l(t) e^{j \phi_l(t)} s_l(t - i T_s - \tau_l) \right\} + n(t) \] 

where \( \Re \{ \} \) denotes the real part, \( M \) is the number of data symbols per user in the data frame of interest, \( T_s \) is the symbol interval, \( \alpha_l(t) \) is the time-varying fading gain of the \( l \)-th user’s channel, \( \phi_l(t) \) is the time-varying phase of the \( l \)-th user’s channel, \( b_l(i) \) is the \( i \)-th bit of the \( l \)-th user, \( s_l(t) \) is the normalized signaling waveform of the \( l \)-th user and \( n(t) \) is assumed as a zero-mean complex non-Gaussian noise. It is assumed that the signaling constellation consists of non-orthogonal signals given by [3]

\[ s_l(t) = \begin{cases} \sqrt{\frac{2}{T_s}} \alpha_l(t) e^{j (\omega t + \phi_l)} & \text{for } t \in [0, T_s] \\ 0 & \text{for } t \notin [0, T_s] \end{cases} \] 

where \( j = \sqrt{-1} \), \( \omega_l \) is the phase of the \( l \)-th user relative to some reference, \( \alpha_c \) is the common carrier frequency, and the spreading waveforms, \( a_l(t) \), are of the form

\[ a_l(t) = \sum_{n=1}^{N} a_n^l u_{T_c} (t - (n-1)T_c) \] 

where \( \{a_n^l\}_{n=1}^{N} \) is a signature sequence of +1s and -1s assigned to the \( l \)-th user user, and \( u_{T_c}(t) \) is a unit-amplitude pulse of duration \( T_c \) with \( T_s = N T_c \). At the receiver, the received signal \( r(t) \) is first filtered by a chip-matched filter and then sampled at the chip rate, \( 1/T_c \). The resulting discrete-time signal sample corresponding to the \( n \)-th chip of the \( l \)-th symbol is given by [3]

\[ r_n^l(i) = \frac{2}{T_c} \int_{i T_c}^{i+1 T_c} r(t) e^{-j \omega_l t} dt, n = 1 \ldots N. \] 

For synchronous case (i.e., \( \tau_1 = \tau_2 = \ldots = \tau_l = 0 \)), assuming that the fading process for each user varies at a slower rate that the magnitude and phase can taken to be constant over the duration of a bit, (4) can be expressed in matrix notation as [3]

\[ \mathbf{r}(i) = \mathbf{A} \mathbf{\theta}(i) + \mathbf{w}(i) \] 

where

\[ \mathbf{\xi}(i) \triangleq [r_l(i),...,r_N(i)]^T \] 

\[ \mathbf{w}(i) \triangleq [w_l(i),...,w_N(i)]^T \] 

\[ \mathbf{\theta}(i) \triangleq \frac{1}{\sqrt{N}} [b_l(i) \mathbf{g}_1(i),...,b_l(i) \mathbf{g}_L(i)]^T \] 

with \( \mathbf{w}(i) \) is a sequence of independent and identically distributed (i.i.d.) complex random variables whose in-phase and quadrature components are independent non-Gaussian random variables with probability density function (PDF) of this noise model has the form

\[ f = (1-\nu) \mathbb{K}(0,\nu^2) + \nu \mathbb{K}(0,\nu^2) \] 

with \( \nu > 0 \), \( 0 \leq \nu \leq 1 \), and \( \kappa \geq 1 \). Here \( \mathbb{K}(0,\nu^2) \) represents the nominal background noise and the \( \mathbb{K}(0,\nu^2) \) represents an impulsive component, with \( \nu \) representing the probability that impulses occur. It is assumed that the \( l \)-th user employs binary phase shift keying (BPSK) modulation to transmit the data bits \( b_l \in [-1,1] \) with equal probability and a symbol rate \( 1/T_s \).

\[ \text{III. GK FADING CHANNEL WITH MRC DIVERSITY} \]

It is assumed that the signal of each user arrives at the receiver through an independent, single-path fading channel. For the shadowed fading channels, \( \alpha_l(i) \) are i.i.d. random variables with GK distribution given by [8]
\[ p_{\alpha}(\alpha_l) = \frac{2}{\Gamma(m)/\Gamma(\mu)} \left( \frac{m\mu}{\Omega_\alpha} \right)^{m+\mu} \alpha_l^{\frac{m+\mu}{2}-1} K_{m-\mu} \left( 2 \frac{m\mu}{\Omega_\alpha} \right) \]  

(10)

where, \( m \) is the Nakagami fading parameter that determines the severity of the fading, \( \mu \) represents the shadowing levels, \( \Omega_\alpha \) is the average SNR in a shadowed fading channel, \( K_\xi(\cdot) \) is the modified Bessel function and \( \Gamma(\cdot) \) is the Gamma function [10]. Assuming that the fading is mitigated through \( D \)-branch MRC, the output of maximal ratio combiner can be written as [8, 11]

\[ R = \sum_{j=1}^{D} \alpha_j \]  

(11)

where \( \alpha \) are i.i.d. GK distributed random variables each having a PDF of the form (10). The PDF of \( R \), with multipath fading and shadowing from branch to branch are distinct, is given by [8]

\[ p_R(r) = \frac{2}{\Gamma(m)/\Gamma(\mu)} \left( \frac{m\mu m\mu}{\Omega_\alpha} \right)^{m+\mu} r^{\frac{m+\mu}{2}-1} \times K_{m-\mu} \left( 2 \frac{m\mu}{\Omega_\alpha} \right) \]  

(12)

where

\[ m_m = Dm + (D - 1) \left[ -0.127 - 0.95m - 0.0058\mu \right] \]  

(13)

and

\[ \mu_m = D\mu \]  

(14)

**IV. AVERAGE PROBABILITY OF ERROR OF **\( M \)-**DECORRELATOR**

In this section, the proposed \( M \)-estimator is presented and the average probability of error is derived for a single-path shadowed fading channel.

**A. \( M \)-estimator**

An \( M \)-estimator is, a generalization of usual maximum likelihood estimates, used to estimate the unknown parameters \( \theta_1, \theta_2, ... \theta_L \) (where \( \theta = Ab \)) by minimizing a sum of function \( \rho(\cdot) \) of the residuals [4]

\[ \hat{\theta} = \arg\min_{\theta \in \mathbb{R}^L} \sum_{j=1}^{N} \rho \left( r_j - \sum_{l=1}^{L} s_j \theta_l \right) \]  

(15)

where \( \rho \) is a symmetric, positive-definite function with a unique minimum at zero, and is chosen to be less increasing than square and \( N \) is the processing gain. An influence function \( \psi(\cdot) = \rho(\cdot) \) is proposed (as shown in Fig. 1 along with other influence functions), such that it yields a solution that is not sensitive to outlying measurements, as [6]

\[ \psi_{PRO}(x) = \begin{cases} 
  x & \text{for} \ |x| \leq a \\
  \text{asgn}(x) & \text{for} \ a < |x| \leq b \\
  \frac{a}{b} x \exp \left( \frac{1-x^2}{b^2} \right) & \text{for} \ |x| > b 
\end{cases} \]  

(16)

where the choice of the constants \( a \) and \( b \) depends on the robustness measures.
Fig. 1. Influence functions of (a) the linear decorrelating detector, (b) Huber estimator, (c) Hampel estimator, and (d) the proposed estimator.

B. Average probability of error

The asymptotic probability of error for the class of decorrelating detectors, for large processing gain $N$, is given by [4]

$$P_e^d = \Pr(\hat{\theta}_i < 0 | \theta_i > 0) = Q \left( \frac{A_i}{V \sqrt{R^{-1}_{ll}}} \right)$$

(17)

where $Q(x)$ is the Gaussian $Q$-function [11],

$$V^2 = \int \left[ \int f(u) du \right]^2$$

(18)

and $R = S^T S$ with $S$ is an $N \times L$ matrix of columns $a_i$. Average probability of error can be obtained by averaging the conditional probability of error (17) over the PDF, (12), of maximal ratio combiner output as

$$\overline{P_e} = F \int_0^\infty x^{(m + \mu_x - 1)/2} Q \left( \frac{x}{V \sqrt{R^{-1}_{ll}}} \right) K_{m_x - \mu_x} \left( 2 \sqrt{\frac{m_x \mu_x}{\Omega_0}} x \right) dx$$

(19)

where $F = \frac{2}{\Gamma(m) \Gamma(\mu_x)} \left( \frac{m_x \mu_x}{\Omega_0} \right)^{m_x + \mu_x} \sqrt{\frac{m_x \mu_x}{\Omega_0}}$. Using the well known upper-bound approximation, called Chernoff bound, to $Q(x)$, given by

$$Q(x) \leq \frac{1}{2} e^{-x^2/2}$$

(20)

the integral of (19) can be expressed as

$$\int_0^\infty x^{(m + \mu_x - 1)/2} \exp \left( \frac{-x^2}{V^2} \right) K_{m_x - \mu_x} \left( 2 \sqrt{\frac{m_x \mu_x}{\Omega_0}} x \right) dx$$

(21)

Now, by using [Eq. 6.313.10] to evaluate the integral (21), the average probability of error can be derived as [9]

$$\overline{P_e} = F \frac{1}{2} e^{-0.5l \beta^3 l \Gamma \left( \frac{1+d+l}{2} \right) \Gamma \left( \frac{1-d+l}{2} \right) \exp \left( \frac{\beta^3}{8\xi} \right) W_{-0.5l, \beta^3} \left( \frac{\beta^3}{4\xi} \right)}$$

(22)
where \( d = m_m - \mu_m \), \( l = \frac{m_m + \mu_m}{2} - 1 \), \( \xi = \frac{1}{\nu^2 R_{11}^{-1}} \) and \( \beta = 2 \sqrt{\frac{m_m \mu_m}{\Omega_0}} \), and \( W_{\lambda, \nu}(\cdot) \) is the Whittaker function [9].

C. PSO based M-decorrelator

PSO is a swarm intelligence method for global optimization modeled after the social behavior of bird flocking and fish schooling [13]. In PSO algorithm the solution search is conducted using a population of individual particles, where each particle represents a candidate solution to the optimization problem (15). Each particle keeps track of the position of its individual best solution (called as pbest), \( p_{d}^{\text{best}} = [p_1^{\text{best}}, \ldots, p_L^{\text{best}}] \) and the overall global best solution (called as gbest), \( g^{\text{best}} = [g_1^{\text{best}}, \ldots, g_L^{\text{best}}] \) among pbests of all the particles in the population represented as \( p_d^{\text{best}} = [p_1^{\text{best}}, \ldots, p_d^{\text{best}}] \), where \( p_d^{\text{best}} \) is the \( d^{\text{th}} \) particle in the \( i^{\text{th}} \) iteration, \( d = 1, 2, \ldots, N_p \), \( i = 1, 2, \ldots, N_{\text{max}} \). \( N_p \) is the number of particles, and \( N_{\text{max}} \) is the maximum number of iterations. Corresponding to each position, the particle velocity is \( v_d^{i} = [v_1^{i}, \ldots, v_L^{i}] \) [13], [14]. The steps involved in the PSO based M-decorrelating detector’s implementation are [11], [13], [15]:

Step 1: Compute the decorrelating detector output, \( \theta^{0} = R^{-1} A^T \xi \). Here, \( R(\frac{1}{2} A^T A/\nu) \) is the normalized cross-correlation matrix of signature waveforms of all users.

Step 2: Initialization: The output of decorrelating detector is taken as input first particle \( d_1 = \theta^{0} \).

Step 3: Fitness evaluation: The objective function (3) is used to find the fitness vector by substituting residuals. Local best position \( p_d^{\text{best}} \) is recorded by looking at the history of each particle and the particle with lowest fitness is taken as \( g^{\text{best}} \) of the population.

Step 4: Update the inertia weight by using the decrement function \( w^i = \beta w^{i-1} \), where \( \beta < 1 \) is the decrement constant.

Step 5: Update the particle velocity by using the relations
\[
\nu_d^{i+1} = w^i \nu_d^i + c_1 \times \mu_1 \times (p_d^{\text{best}} - p_d^i) + c_2 \times \mu_2 \times (g^{\text{best}} - p_d^i) \\
p_d^{i+1} = p_d^i + \nu_d^{i+1}
\]  

(23)  

(24)

where \( c_1 \) and \( c_2 \) are the acceleration constants representing the weighting of the stochastic acceleration terms to pull the particle to pbest and gbest. \( \mu_1 \) and \( \mu_2 \) are random numbers that are uniformly distributed between 0 and 1. Particle position is updated according to (6). Particle velocity is limited by the maximum velocity \( v_{\text{max}} = [v_1^{\text{max}}, \ldots, v_L^{\text{max}}] \).

Step 6: The individual best particle position is updated by following rule:
\[
\text{if} \quad F(p_d^i) \leq F(p_d^{\text{best}}) \quad \text{then} \quad p_d^{\text{best}} = p_d^i
\]  

(25)

Step 7: \( g^{\text{best}} \) is the global best particle position among all the individual best particle positions \( p_d^i \) at the \( i^{\text{th}} \) iteration such that \( F(g^{\text{best}}) \leq F(p_d^i) \).

Step 8: The above steps are repeated until the maximum number of iterations has been reached.

The computed \( g^{\text{best}} \) value is used to evaluate average probability of error (22).
TABLE I
PSO Parameters used for Simulation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of particles, (N_p)</td>
<td>20</td>
</tr>
<tr>
<td>Maximum number of iterations, (N_{\text{max}})</td>
<td>100</td>
</tr>
<tr>
<td>Acceleration constants (c_1) and (c_2)</td>
<td>2</td>
</tr>
<tr>
<td>Maximum velocity of the particles (v^\text{max}_i) for all users</td>
<td>2</td>
</tr>
<tr>
<td>Initial inertial weight, (w)</td>
<td>1</td>
</tr>
<tr>
<td>Decrement constant, (\beta)</td>
<td>0.99</td>
</tr>
</tbody>
</table>

V. SIMULATION RESULTS

In this section, the performance of \(M\)-decorrelator is presented by computing (22) for different values of order of diversity. It is assumed that \(m = 1.4\), \(\mu = 2\) and \(\Omega = 10\) dB in the simulations.

Evolutionary behavior of \(M\)-decorrelator is presented in Fig. 2, Fig. 4 and Fig. 6. Performance of \(M\)-decorrelator with different influence functions is shown in Fig. 3, Fig. 5 and Fig. 7. In Fig. 3, the average probability of error versus the signal-to-noise ratio (SNR) corresponding to the user 1 under perfect power control of a synchronous DS-CDMA system with six users \((L = 6)\) and a processing gain, \(N = 31\) is plotted for Gaussian noise \((\epsilon = 0)\). In Fig. 5, the average probability of error is plotted for moderate impulsive noise \((\epsilon = 0.01)\). Similarly, in Fig. 7, the average probability of error is plotted for highly impulsive noise \((\epsilon = 0.1)\). Simulation results show that the proposed \(M\)-estimator based detector performs well compared to linear multiuser detector, minimax detector with Huber and Hampel estimator based detectors.

![Fig. 2. Evolution of objective function (15) with respect to number of iterations with LDD (least squares), MD with Huber, Hampel and Proposed \(M\)-estimator in Gaussian noise \((\epsilon = 0)\).](image-url)
Fig. 3. Average probability of error versus SNR for user 1 for LDD (Least Squares), MD with Huber, Hampel and proposed $M$-estimator in synchronous DS-CDMA channel with Gaussian noise, $N = 31$.

Fig. 4. Evolution of objective function (15) with respect to number of iterations with LDD (least squares), MD with Huber, Hampel and Proposed $M$-estimator in moderate impulsive noise ($\epsilon = 0.01$).

Fig. 5. Average probability of error versus SNR for user 1 for LDD (Least Squares), MD with Huber, Hampel and proposed $M$-estimator in synchronous DS-CDMA channel with $\epsilon = 0.01$, $N = 31$. 
VI. CONCLUDING REMARKS

The PSO based multiuser detection technique for DS-CDMA systems over GK fading channels under impulsive noise environment is presented. An approximate closed-form expression for average probability of error of the $M$-decorrelator to detect BPSK signals is derived by incorporating MRC receive diversity. An $M$-estimator based multiuser detector is proposed and its performance is analyzed by computing average probability of error using the expression derived. Simulation results show that the proposed multiuser detector offers significant performance gain over the linear multiuser detector and the minimax decorrelating detector with Huber and Hampel $M$-estimators, in heavy-tailed impulsive noise.

REFERENCES

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