Monovariable H_∞ Robust Controller for a Permanent Magnet Synchronous Machine (PMSM)

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Abstract—This paper deals with the modelling, analysis design and experimental validation of a robust control method for a permanent magnet synchronous machine (PMSM) supplied with a PWM inverter based on a monovariable H infinity standard controller. Under the influence of uncertainties and external disturbances, by a variation of ±10% of all motor parameters from the nominal values, the robust performance control problem is formulated as a monovariable H infinity standard scheme and solved by a suboptimal iterative H infinity strategy. This new design method is able to ensure the stabilization of the augmented system formed of the perturbed system and weighting filters with improved performance in face of parameter variation and external disturbances. A Simulation and experimental study was carried out to illustrate the effectiveness of the proposed method.

Keyword- Robust Performance, Monovariable Standard H infinity controller, Uncertainties, PMSM

I. INTRODUCTION

In recent years, growth in magnetic power device, and control theory have made the PMSM play a crucial important role in high performance industrial drives in the low-to-medium- power range. The desirable idiosyncrasy of the PMSM is its compact simple structure, high-energy efficiency, high air-gap flux density, reliable operation, high torque capability and high power density. Otherwise, compared with induction motors, PMSM have the advantage of higher efficiency, due to the absence of rotor losses and lower vacuum current below the rated speed and its decoupling control performance is much less sensitive to the parameter variations of the motor [1], [2]. Nevertheless the control performance of the PMSM model is still influenced by the uncertainties of the plant, which all along are composed of unpredictable plant parameter variations and external disturbances. Thus, many approaches such as optimal control [3], adaptative control [4], [5], nonlinear control [6] and robust control [7] have been developed for the PMSM to deal with uncertainties. During the past decade the H_∞ control problem have been extensively celebrated for its robustness in counter-acting parameters variations and external disturbances. The main objective of the H_∞ control is to synthesize a controller making the closed-loop system to satisfy a prescribed H_∞ norm constraint which representing desired stability or tracking requirements. However .to ensure good performance robustness under uncertainty perturbations, the H_∞ approach give usually a solution with high control gain. Motivated by this idea, our main contribution in this paper, is to develop an efficient robust method called monovariable H_∞ control that is insensitive to these variations. The organization of the paper is as follows. First the problem notification is presented and the nominal and uncertain plant of the PMSM is swiftly described .Second a robust monovariable H_∞ controller is developed. Besides filters weightings incorporated in the system to ensure H_∞ performance are mentioned. Finally the control law design is carried out to the regulation of the speed and d-axis current of the PMSM.

The experimental set-up assembled in the LAII laboratory is made of a Dspace 1104 numerical prototype card with its interface, a three-phase IGBT based inverter using a vector PWM technique and a permanent magnet synchronous servo-motor (PMSM) equipped with position sensors [8].

II. PROBLEM FORMULATION

Before conceiving the robust monovariable H_∞ controller, let's note that the process has to be surrogated is under the standard configuration below where the state space approach is adopted for resolution [9], [10], [11], [12], and [13]:

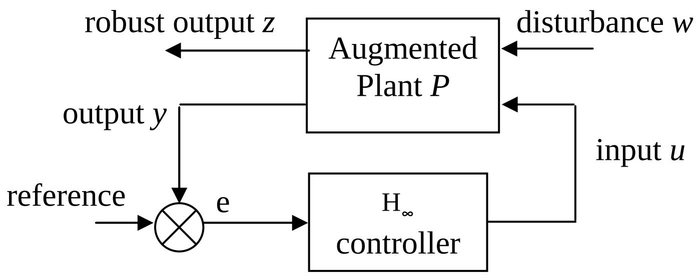


Fig 1 Standard robust feedback control system

Where w is the disturbance input, u is the regulated variable, z is the robust output that we want to minimize and y is the measured variable that we use to monitor the system. *P* is the augmented plant that includes weighting functions which will be defined later.

Let's note that the H_∞ controller is referred by *K*.

Thus we can write:

$$\begin{bmatrix} z \\ y \end{bmatrix} = P\begin{bmatrix} w \\ u \end{bmatrix} = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}\begin{bmatrix} w \\ u \end{bmatrix} \tag{1}$$

Consequently, we can express the dependency of z on w as:

$$z = F(P,K)w \tag{2}$$

As mentioned above, the objective of H_∞ control is to find a controller *K* stabilizing such that the H_∞ norm of $F(P,K)$ is bounded by a constant γ which represent the desired performance level of the closed loop system:

$$\|F(P,K)\|_\infty < \gamma \tag{3}$$

Let's give the H_∞ structure of the control plant as shown in Fig (2). This Fig shows the interconnection of the augmented plant which includes the nominal model of the system G_{nom}, the controller *K*, the input reference *r*, the unmeasured disturbances *p* on the control *u*, the output reference *e*, and the noise measure *b* on the output *y*. inputs and outputs are weighted by weighting functions W_1, W_2 and W_3

W_1 is known as the tracking error performance, W_2 *is* the robust performance and W_3 is chosen to set uncertainties norm that the closed-loop system must allow.

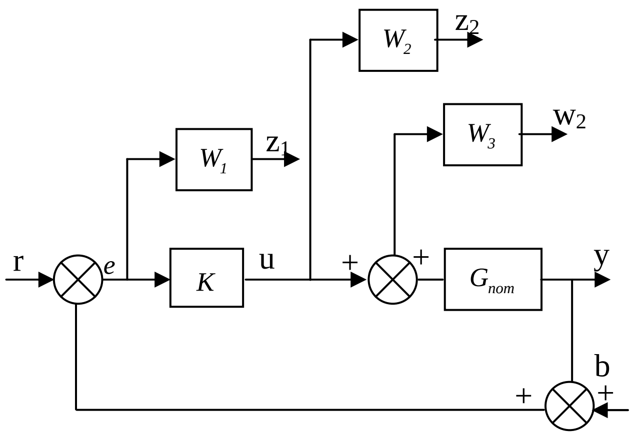


Fig 2 H_∞ standard configuration

Due to the control low structure u=*K*y, one confuse the input b and r. Thus we can adopt the new H_∞ structure:

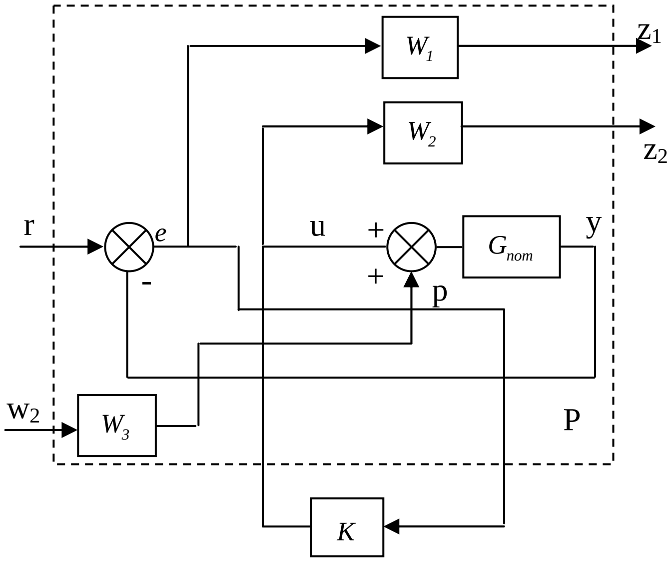


Fig 3 H_∞ configuration of the control plant

On referring to the Fig (3), the inequality (3) can be expressed in terms of weighting functions as follow:

$$\left\| \begin{matrix} W_1 S & W_1 G S W_3 \\ W_2 K S & W_2 T W_3 \end{matrix} \right\| < \gamma \qquad (4)$$

Where the sensitivity function $S = \dfrac{1}{1+GK}$ and it complementary $T = \dfrac{GK}{1+GK}$ are the ratio of output to the disturbance and the ratio of output to input respectively

State Model of the Permanent Magnet Synchronous Machine (PMSM)

Take into account the classical simplifying hypothesis, the nominal model of the permanent magnet synchronous machine, in the Park landmark [14], can be given by the following set of equations [15], [16], [17]:

$$\begin{aligned}
\frac{dI_d}{dt} &= -\frac{R}{L_d} I_d + \frac{L_q}{L_d} w_r I_q + \frac{1}{L_d} v_d \\
\frac{dI_q}{dt} &= -\frac{R}{L_q} I_q - \frac{L_d}{L_q} w_r I_d - \frac{\Phi}{L_q} w_r + \frac{1}{L_q} v_q \\
\frac{dw_r}{dt} &= p\frac{(c_e - c_r)}{J} - \frac{f_c}{J} w_r \\
c_e &= p[(L_d - L_q) I_d + \Phi] I_q \\
\frac{dw_r}{dt} &= \frac{p^2}{J}[(L_d - L_q) I_d + \Phi] I_q - \frac{f_c}{J} w_r - \frac{p}{J} c_r
\end{aligned} \qquad (5)$$

Where

$v_{d\text{-}q}$: d-q axis stator voltages

I_{dq} : d-q axis stator currents

$L_{d\text{-}q}$: d-q axis inductances

R : the stator resistance

Φ : the flux linkage of the permanent magnets

w_r : the rotor speed

f_c : the friction coefficient

J : the moment of inertia

c_{e-r} : the electromagnetic and the load torques

p : the number of pole .

To conceive such control law, we should linearise the nominal model around an operating point, x_s^0, u^0 [18]:

$$x_s^0 = \begin{bmatrix} I_{d0} & I_{q0} & w_{r0} \end{bmatrix}^T, u^0 = \begin{bmatrix} v_{d_0} & v_{q_0} \end{bmatrix}^T \tag{6}$$

Therefore, the linearized nominal model is as follow:

$$\begin{aligned} \delta\dot{x}_s &= A_{sl}\delta x_s + B_{sl}\delta u + D_{sl}v \\ y &= C_{sl}\delta x_s \end{aligned} \tag{7}$$

where

$$\delta x_s = \begin{bmatrix} \delta I_d & \delta I_q & \delta w_r \end{bmatrix}^T, \delta u = \begin{bmatrix} \delta v_d & \delta v_q \end{bmatrix}^T \tag{8}$$

$$A_{sl} = \begin{bmatrix} -\frac{R}{L_d} & \frac{L_q}{L_d}w_{r0} & \frac{L_q}{L_d}I_{q0} \\ -\frac{L_d}{L_q}w_{r0} & -\frac{R}{L_q} & -\frac{\Phi}{L_q}-\frac{L_d}{L_q}I_{d0} \\ \frac{p^2}{J}(L_d - L_q)I_{q0} & \frac{p^2}{J}[\Phi + (L_d - L_q)I_{d0}] & -\frac{fc}{J} \end{bmatrix}, B_{sl} = \begin{bmatrix} \frac{1}{L_d} & 0 \\ 0 & \frac{1}{L_q} \\ 0 & 0 \end{bmatrix},$$

$$C_{sl} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{and } D_{sl} = \begin{bmatrix} 0 \\ 0 \\ -\frac{p}{J} \end{bmatrix}$$

III. ROBUST MONOVARIABLE H∞ DESIGN CONTROLLER

As mentioned above, the control law is designed for the purpose of regulating the speed w_r and the direct current I_d . But by referring to equation (5), we see a strong coupling between the speed and the d axis current. In this regard, to obtain totally decoupled variables we adopted the following representation where the nominal control model include the following transfer matrix of variable we want to control that is to say $H_{idnom}(s)$ and $H_{wrnom}(s)$.

$$H_{sl}(s) = \begin{bmatrix} H_{idnom}(s) & 0 \\ 0 & H_{wrnom}(s) \end{bmatrix} \tag{9}$$

where:

$$\begin{aligned} H_{idnom}(s) &= \frac{1}{R + L_d s} \\ H_{wrnom}(s) &= \frac{p^2\Phi}{JL_q s^2 + (L_q f_c + JR)s + Rf_c + p^2\Phi^2} \end{aligned} \tag{10}$$

As mentioned in [19], parameters most probable to change, under several conditions, are the electrical and mechanical time constants (τ_{elc} and τ_{mec}). Indeed R influenced by temperature can vary, J increased and Φ decrease after an extended utilization.

Taking in consideration these variations pushes us to adopt a variation of $\pm 10\%$ of all machine parameters from its nominal values.

For the selection of weighting functions W_1, W_2 and W_3, is based on the choice of templates. Which being imposed by desired specifications. It is noted that the introduction of the weights on the different signals take the form of filters that allowing to the signal to which they apply, to focus a particular area of frequencies.

We can summarize these desired specifications by imposing:

- Frequency template
- Temporal template

In our application

$$W_1(s) = \frac{ks + \alpha}{s}$$

Where k is taken equal to 1400 for a module margin less than 6db, $\alpha = 1398$ which settle the rise time. A way that α is greater, the rise time is shorter, and the closed loop response is fast. In our case, the rise time is equal to 0.15s and the overshoot of the closed loop system is almost null.

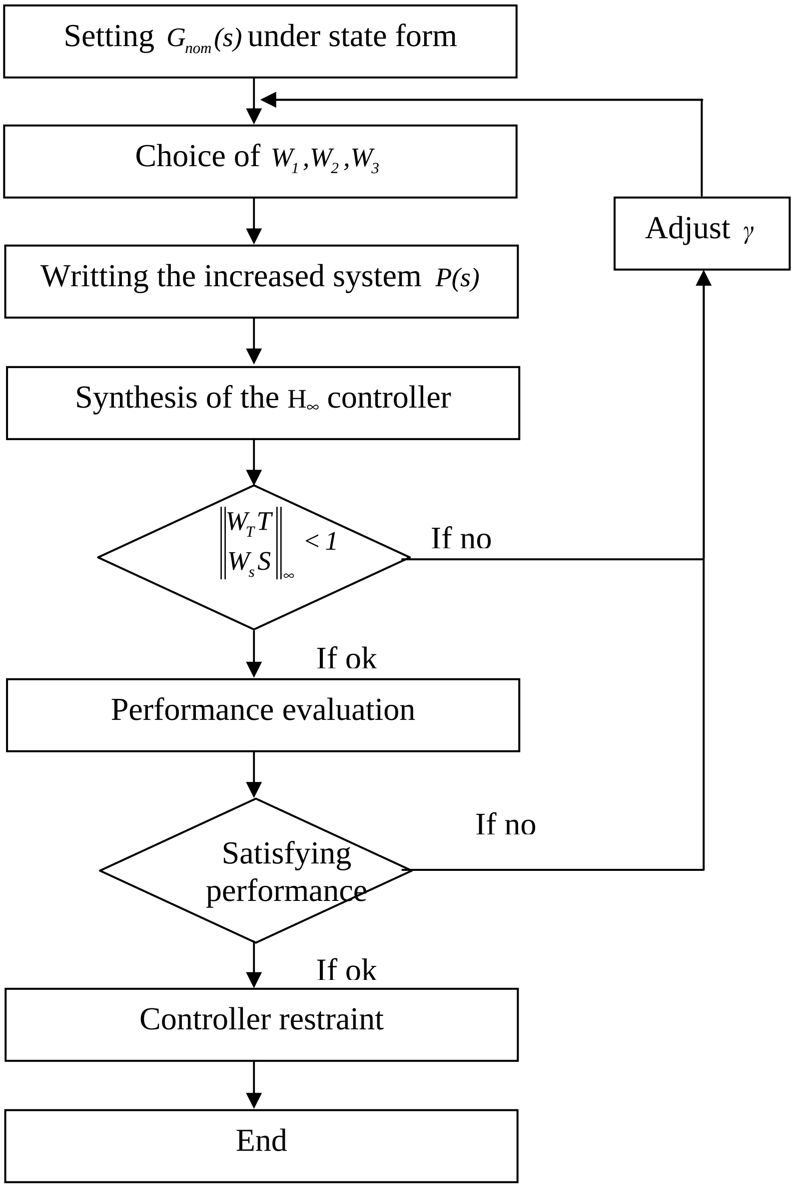
$W_u(s)$, which acts explicitly on the control signal u, is selected in order to limit its amplitude and reduce the order of the controller K. $W_u(s)$ is chosen a positive scalar

$W_u(s) = 0.01$ for o standard H_∞ of (1) less then $\gamma = 0.056$

The weight $W_T(s)$ is chosen based on uncertainties norm that the closed-loop system must accept [15].

$$W_T(s) = \frac{as + b}{cs + d} = \frac{0.1s + 1}{78e^{-5}s + 0.1}$$

The steps of calculating a robust controller by H_∞ synthesis can be summarized in the following flow chart:



Notice: Control of the d-axis current I_d can be realized by a classic PI corrector. This kind of control allows the PMSM to operate in oriented flow while ensuring at machine model, perfect decoupling. In our study $I_d = 0$ .because as we have a smooth poles machine, the best selection for its operation is obtained for a value where the internal angle of the machine is equal to $\frac{\pi}{2}$.

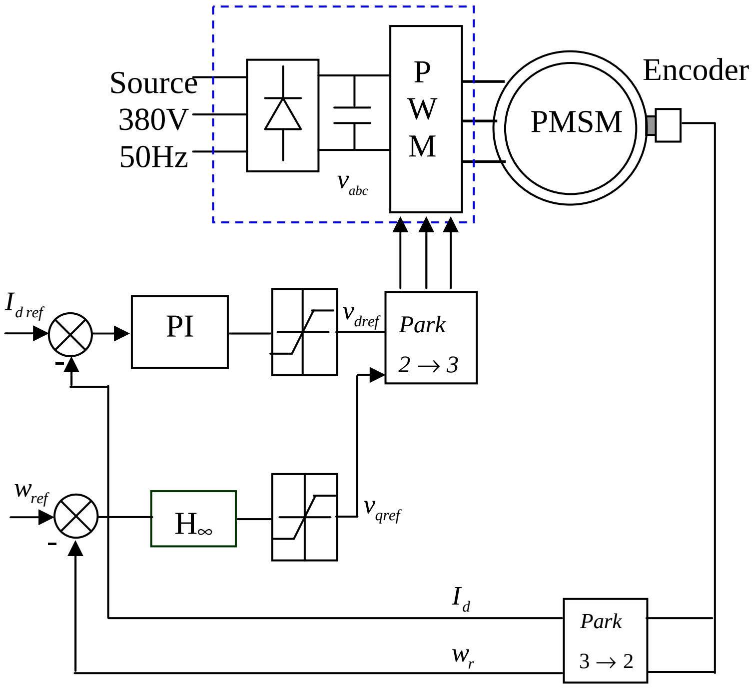


Fig 4 Overall scheme of the monovariable H∞ robust control of the PMSM

The robust controller K deduced from the algorithm described above and satisfying (1) takes the form:

$$K(s) = \frac{2.3e^5s^3 + 6.79e^7s^2 + 1.27e^9s + 3.06e^9}{s^4 + 1.72e^4s^3 + 1.48e^8s^2 + 3.9e^8s + 0.54e^{-3}}$$

IV. EXPERIMENTAL RESULTS AND INTERPRETATIONS

The photo given in Fig.5 present the test bench used

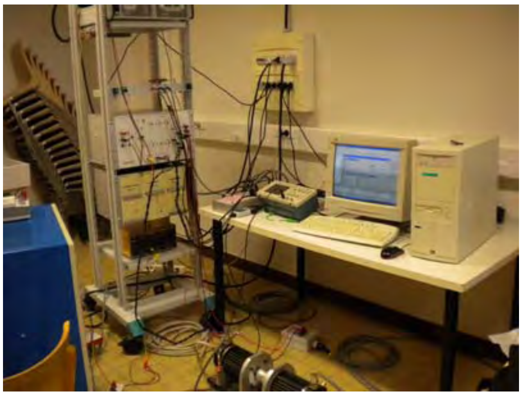


Fig 5 Photo of the test bench

The experimental setup shown in Fig. 5 is constructed to inspect the effectiveness of the proposed controller. Two identical permanent magnet synchronous machines PMSM are used, one as a motor and the other as a load. Their parameters that were used are given in table 1 below

TABLE I
Parameter of the PMSM

Machine power	1KW
Rated current	6.5A
Pole pair number (*p*)	2
d-axis inductance *Ld*	4.5mH
q-axis inductance *Lq*	4mH
Stator resistance *R*	0.56 Ω
Machine inertia *J*	$2.08.10^{-3}$Kg.m^2
Friction coefficient *fc*	$3.9.10^{-3}$Nm.s.rad^{-1}
Magnet flux constant *Φ*	0.064wb

The experimental set-up assembled in the LAII laboratory is made of a Dspace 1104 controller card with its interface, a voltage inverter using the space vector PWM (SVPWM) technique and a permanent magnet synchronous servo-motor (PMSM) equipped with 1024 pulse/revolution incremental encoder. The current measurements are obtained via hall effect sensors. The control algorithm was compiled by the software Matlab / SIMULINK package, and implemented to machine language via software control desk. The digital sampling period was taken equal to 0.1ms.

In order to verify the effectiveness of the designed controller, a series of measurements has been accomplished.

The results are the machine d axis current and the rotor speed in which we observe the performance specifications of the synthesized robust controller to the PMSM. Indeed, the measured speed and d-axis current track well the trajectory of reference one with almost no static error and this over the whole speed range

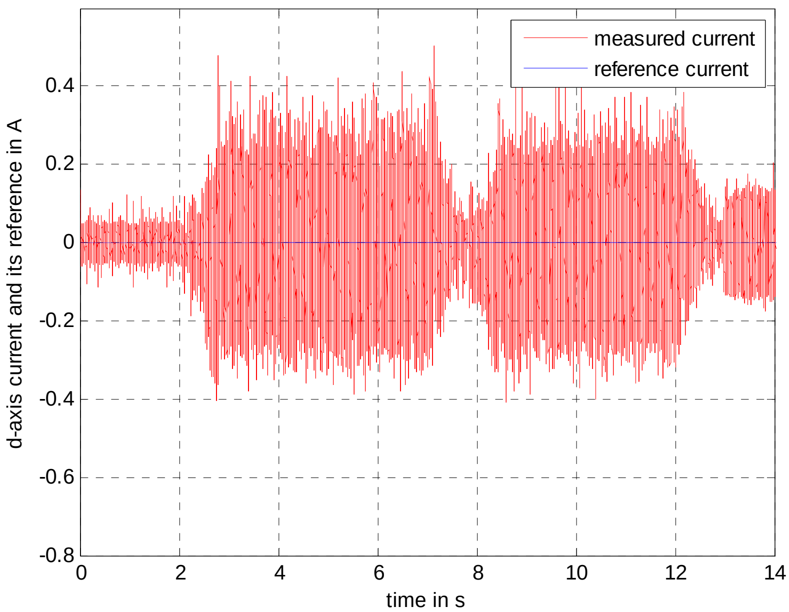


Fig 6 d-axis current and its reference

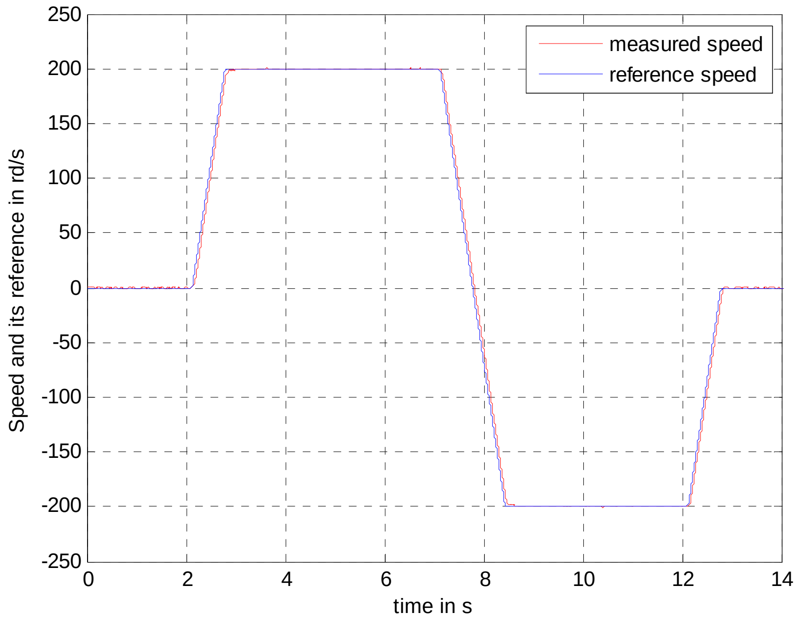


Fig 7 Speed and its reference

V. CONCLUSION

This paper develops a methodology to design a robust monovariable H_∞ controller in order to ensure good performance robustness under uncertainty perturbations.

An efficient algorithm based on iterative technique is proposed to minimize the H_∞ norm of an augmented plant, includes nominal model of the PMSM, the controller K and weighting functions, in a way it is less than a positive constant γ

The controller has been implemented with success on a permanent magnet synchronous machine (PMSM).

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