

# Modeling of Critical Blank Holder Force Based on a Gap Limit and Unbending Strain Energy in Deep Drawing Process

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**Abstract**— This study is aimed to predict the minimum varying blank holder force (VBHF) during the punch stroke, in order to eliminate wrinkle on cup deep drawing product. The slab method was used to develop mathematical modeling of the minimum VBHF base on a gap limit and unbending energy. The mathematical modeling has been validated to FE simulations for the prevention of wrinkling in the same criterion. Steel sheet of SPCD grade with thickness of 0.2 mm is used to generate the cylindrical cup-shaped product with 40 mm diameter. Analytical Results of minimum VBHF have a similar trend result compared to FE simulation. However, the minimum VBHF can be quite effective for preventing the occurrence of excessive wrinkle.

**Keyword**— Modeling blank holder force, Deep drawing, Wrinkling, FE simulation.

## I. INTRODUCTION

Wrinkling is one type of product defects that are often found in the deep drawing process. In order to avoid these defects, the process parameters, i.e. the blank holder force (BHF) must be determined in the proper value [1-20]. The constant blank holder force during the process, it's frequently not capable to prevent wrinkling, effectively. To achieve the proper process performance, the blank holder force should be set to make process deformation with the balance of material flow, tangential stress and radial stress [10-17]. The lowest of BHF magnitude could give consequences for increasing tangential stress, and gap [15-17]. If the gap between the blank holder and die more than 180% of material thickness, it will potentially occur the wrinkling products [10-14]. To prevent this problem, the BHF over the punch stroke should be set in proper value to anticipate the deformation condition dynamically.

R. Venkat Renddy et al. [1] conducted a study of the influence of process parameters, i.e. the constant BHF, punch-die radius and coefficient of friction against the onset of wrinkling in a deep drawing cylindrical cup. This study was also investigating the mechanism of wrinkling initiation and growth in cylindrical cup deep drawing process.

Anupam Agrawal et al. [2] developed the mathematical modeling of minimum constant BHF to avoid wrinkling of cylindrical cups, base on calculating of restraining energy and considering the aspect of thinning material. The constant BHF applied a spring type blank holder. The wrinkling would occur when the buckling strain as tangential strain is excessive.

Xi Wang et al. [3] and qin si-ji et al. [9] have developed the buckling analysis to predict the onset of wrinkling, and determined the BHF magnitude by using the theory of strain energy. This paper develops an analytical approach to calculate the critical wrinkling by using a constant blank holder force, base on the theory of restraining energy and buckling criterion.

M. Kadkhodaya et al. [4], E. Chu et al. [5] Joao Pedro et al. [6], and Mohammad Amin Shafaat et al. [7][8] have developed a model of wrinkle with the analysis of elastic-plastic by using the method of bifurcation function and Tresca criterion. The results of the analysis were predicting the minimum constant BHF without wrinkling, during the punch stroke.

M Gavas et al. [10] [11], Wang Wu-rong et al. [12], Lin Zhong-qin et al. [13] and T. Yagami et al. [14] have researched related to wrinkling prevention by using criteria limits of the maximum gap. The Application of gap criterion can effectively prevent the occurrence of excessive wrinkling, therefore the flow of material can still move into the die cavity completely. The normal gap, which still permitted between 110% to 180% of the initial thickness of the material. An analytical study, continued by S. Candra et al. [15] [16], which aims to control

tangential and radial stress base on limits the maximum gap and the fracture condition, to estimate the maximum varying blank holder force (VBHF).

The current study will develop a different mathematical models to obtain a minimum variable blank holder force each punch stroke, based on a gap limit and strain energy on Cylindrical Drawn Cup Product, and its also compared to simulation to predict the critical limit of wrinkling. By using the a gap limits and strain energy, can be expected simultaneously prevents wrinkling. The analytical calculation of the minimum VBHF, will be validated by the FE simulation of cup deep drawing process.

**II. MATHEMATICAL MODELING OF CRITICAL BLANKHOLDER FORCE**

In the cup deep drawing process, the sheet material will deform the flange to follow a rule of force and energy equilibrium. The force equilibrium diagrams in a small element and ignoring friction on flange, as shown in Fig. 1, the mathematical equation of formation of radial stress can be written as [16][17][21]

$$r d\sigma_r d\alpha s_0 + \sigma_r dr d\alpha s_0 + 2|\sigma_t| s_0 dr \sin\left(\frac{d\alpha}{2}\right) = 0 \tag{1}$$

Where  $\sigma_f$  is the radial stress,  $r$  is the inside radius at the moment,  $d\alpha$  is a small segment angle of the flange,  $s_0$  is the initial thickness of material, and  $\sigma_t$  is the tangential stress in the flange.

An angle of the segment  $d\alpha/2$  is very small, then it can be considered to be  $\sin(d\alpha/2) \approx d\alpha/2$ , so the equation (1) becomes:

$$d\sigma_r = (\sigma_r - \sigma_t) \left(\frac{dr}{r}\right) \tag{2}$$

Where  $R_1$  is the outer flange radius.

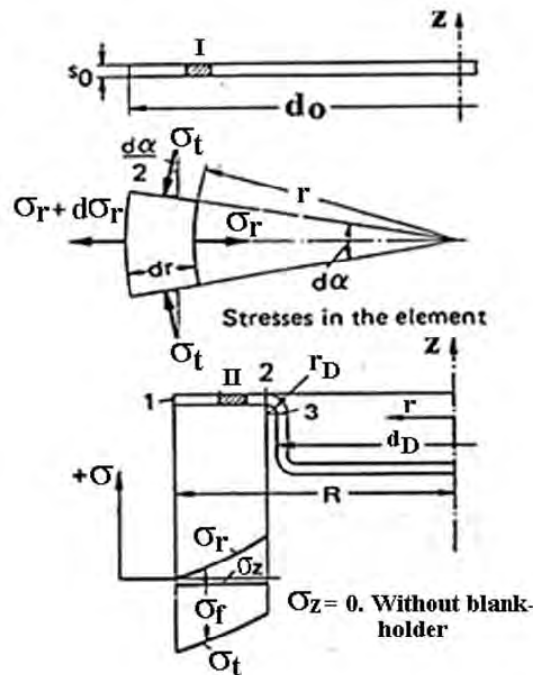


Fig. 1 The Force equilibrium diagrams in a small element on flange, [21]

By using the criteria Tresca and the equivalent stress [14], the relationship between radial and tangential stress under the condition of plain strains can be obtained.

$$(\sigma_r - \sigma_t) = \sqrt{\frac{2(\bar{R}_n+1)}{1+2\bar{R}_n}} \sigma_f \tag{3}$$

Where  $\sigma_f$  is the flow stress on flange from the initial position until another position, and  $\bar{R}_n$  is the mean anisotropy coefficient of the material.

Substituting Eq. (3) into Eq. (2) and integrating with the boundary of  $r = r_m$  ( $d = d_m$ ) until  $r = R_1$ , or ( $d = d_1$ ) the equation of ideal stress ( $\sigma_{r,i}$ ) formation on the flange every punch position obtained as follows,

$$\sigma_{r,i} = \sqrt{\frac{2(\bar{R}_n+1)}{1+2\bar{R}_n}} \sigma_{fml,i} \ln(R_{1,i}/R_m) \tag{4}$$

Where  $\sigma_{fm,i}$  is the mean flow stress on flange from point 1-2 over punch stroke,  $R_{1,i}$  is the outer local radius of flange - function of punch stroke (point 1),  $R_m$  is the average radius of cylindrical cup deep drawing ( $R_m = R_D - s_0$ ) and  $R_D$  is the die radius.

In order to simplify the calculation of mean flow stress on flange, the equivalent strain will be divided into two sections, as at point 1 and point 2. The strain at point 1 ( $\epsilon_{1,i}$ ) and 2 ( $\epsilon_{2,i}$ ) can be calculated respectively by Eq. (5), [9]

$$\epsilon_{1,i} = \sqrt{\frac{2(\bar{R}_n+1)}{1+2\bar{R}_n}} \ln\left(\frac{R_0}{R_{1,i}}\right) \tag{5}$$

And

$$\epsilon_{2,i} = \sqrt{\frac{2(\bar{R}_n+1)}{1+2\bar{R}_n}} \ln\left(\frac{R_{int,i}}{R_D+r_d}\right) \tag{6}$$

Where  $R_0$  is the blank size radius (initial blank sheet),  $r_d$  is the radius of die edge and  $R_{int,i}$  is the unknown inside diameter (point 2),  $R_{int,i} = \sqrt{R_0^2 + (R_D + r_d)^2 - R_{1,i}^2}$

The mean flow stress between point 1 and point 2 can be calculated by Eq. (7),

$$\sigma_{fm,p1-2,i} = \left(\frac{1}{2}\right) K(\epsilon_{1,i}^n + \epsilon_{2,i}^n) = \left(\frac{1}{2}\right) \left(\sqrt{\frac{2(\bar{R}_n+1)}{1+2\bar{R}_n}}\right)^n K \left[ \left(\ln \frac{d_0}{d_{1,i}}\right)^n + \left(\ln \frac{d_{int,i}}{d_D+2r_d}\right)^n \right] \tag{7}$$

Substituting Eq. (7) into Eq. (4) and considering the friction on the flange surface, thus the equation of radial stress will be determined as follows,

$$\sigma_{r,i} = \left\{ \sqrt{\frac{2(\bar{R}_n+1)}{1+2\bar{R}_n}} (\sigma_{fm,i}) \left( \ln \frac{R_{1,i}}{R_m} \right) \right\} + \left\{ \frac{\mu F_{bh,i}}{\pi R_{1,i} s_0} \right\} \tag{8}$$

To determine radial strain and radial stress each step of punch stroke, the changing of diameter should be calculated under the condition a constant volume.

Wrinkling is often approached through buckling phenomena approach, which is caused by too excesses the magnitude of local tangential stress on the flange. Buckling prediction was still an ongoing, conducted by researchers with the wrinkles criteria, e.g. buckling studies based geometric model change [3] [9]. M Gavvas et al [10-11], Lin Zhong et al [13] have conducted research and experiments with the result, that limit gap (safe gap) is under  $(180\% + 1) s_0$ . The maximum gap limit is useful to set the minimum blank holder force for preventing wrinkling. Moreover, wave number and behavior of the buckling are strongly influenced by the material mechanical properties, thickness and the punch-die dimensions. To eliminate local buckling at each position of the punch stroke, VBHF is applied and determined by using a limit gap criterion and a theory of strain energy. And to predict the condition of the flange area, analysis and simulation study, this approach will use the assumption that follows the reference height buckling previous studies [3][4][9][10][11][13]. The schematic model of the ideal buckling and local stress that it will be used to predict the VBHF on this analysis defined as illustrated Fig. 2.

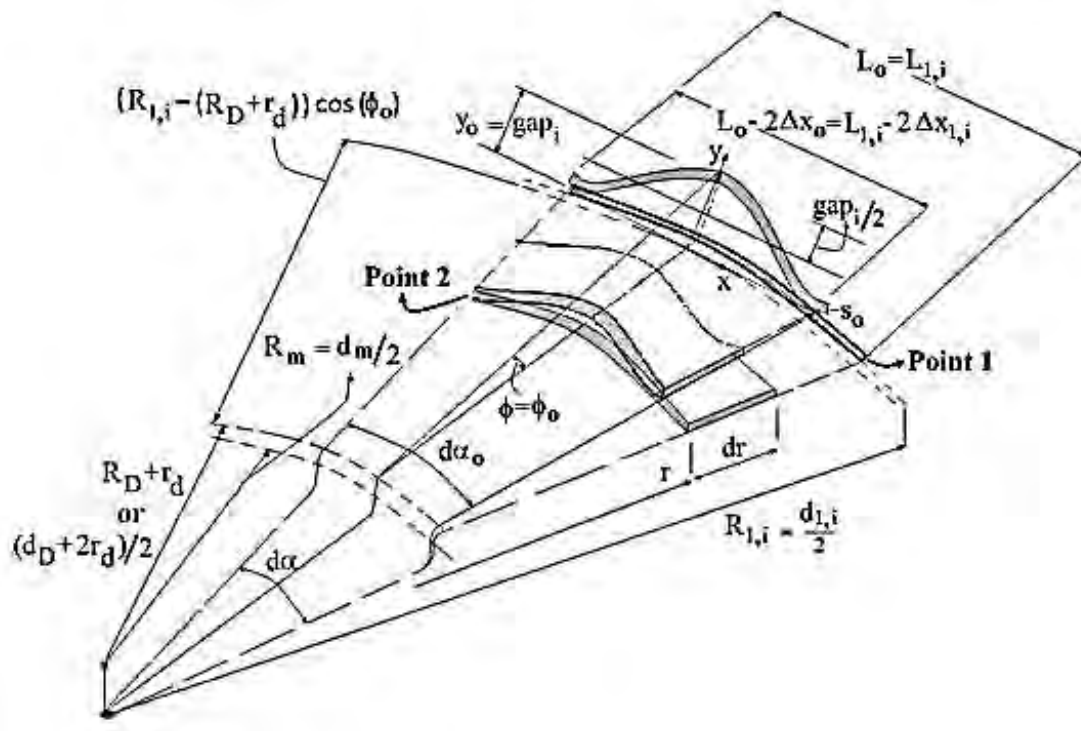
If the waveform is assumed sinusoidal buckling, and the height of the buckling wave angle ( $\phi_0 = \phi$ ) is assumed constant from beginning until the end of the process (see Fig. 2), as well as the angle of wave height can be estimated:

$$\phi_0 = \phi \approx \arcsin\left(\frac{gap_1}{R_{1,i} - R_D - r_d}\right) \tag{9}$$

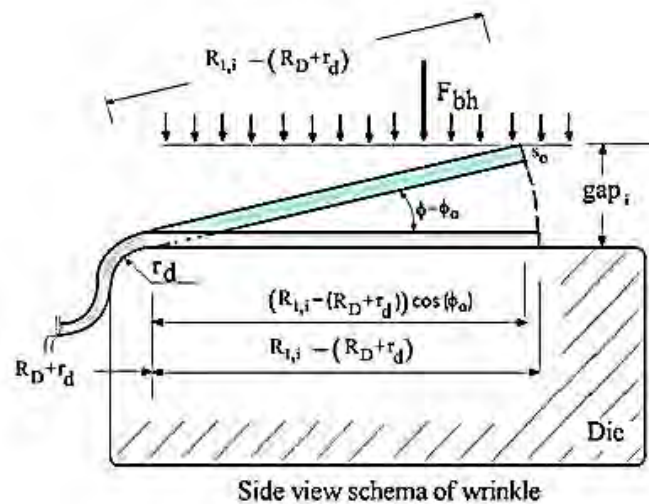
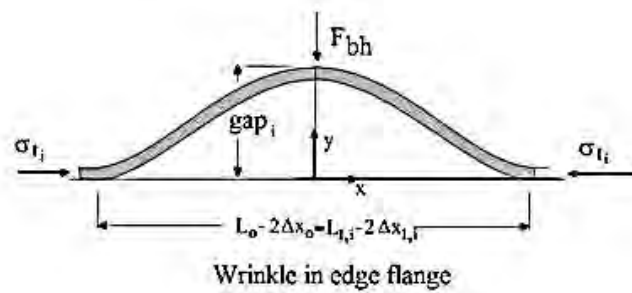
Where  $gap_1$  is the gap at the beginning of the punch stroke, the  $gap_i = gap_1$  on punch stroke (i) is assumed to arise early wrinkling occurs in early stroke punch that  $i = 1$ . And assuming a constant angle  $\phi_0 = \phi$ . Therefore, the gap height prediction punch stroke function approximated by the following equation.

$$gap_i \approx (R_{1,i} - R_D - r_d) \sin(\phi_0) \tag{10}$$

Where  $R_{1,i}$  is the outer local flange radius under the function of the punch stroke (the height of cup or h),  $R_D$  is die radius and  $r_d$  is the edge die radius and  $\phi_0$  is the angle of wave height.



(a)



(b)

Fig. 2. a. Three dimension of the ideal buckling scheme of sheet material on flange, b. Front and side view of the ideal buckling scheme

The changing of flange dimension can be calculated from the material displacement of points 1 and 2 during the punch stroke gradually. Therefore, it can be described as follows.

Under the condition of the product without a wrinkle (see Fig. 2), the circumference of the outer flange and

dimension changing of flange (point 1 position) can be estimated as a function of radius and the wrinkling wave number over punch stroke (i) as the following equation:

$$L_{1,i} = (2\pi R_{1,i})/N_i \tag{11}$$

$$(L_{1,i} - 2\Delta x_{1,i}) = \left(2\pi\left((R_{1,i} - R_D - r_d) \cos(\phi_0)\right) + (R_D + r_d)\right)/N_i \tag{12}$$

$$2\Delta x_{1,i} = (L_{1,i}) - (L_{1,i} - 2\Delta x_{1,i}) \tag{13}$$

$$2\Delta X_{1,i} = (2\pi R_{1,i}) - \left(2\pi\left((R_{1,i} - R_D - r_d) \cos(\phi_0)\right) + (R_D + r_d)\right) \tag{14}$$

Where,  $L_{1,i}$  is the circumference of the outer flange (position of point 1) for an insignificant wrinkling wave,  $R_{1,i}$  is radius of an outer flange (position of point 1),  $2\Delta x_{1,i}$  is reduction in the circumference of the outer ring flange for a wrinkle wave (position point 1), when appearing wrinkling, and  $N_i$  is the wave number of wrinkle each punch stroke.

Based on research conducted by M. Kadkhodayan et al. [4], the number of wrinkle wave can be estimated by using the equation (15).

$$N_i = 1,65 \frac{R_{mean}}{W} = 1,65 \left(\frac{(R_{1,i} + (R_D - r_d))/2}{(R_{1,i} - (R_D - r_d))}\right) \tag{15}$$

Where,  $R_{mean}$  is the mean radius of flange and  $W$  is the flange width.

In the same calculation as above, the length of the circumference and the dimensional change of flange at point 2 for an insignificant wrinkling wave (see Fig. 2) can use the equations as follows:

$$L_{2,i} = \left(2\pi\left((R_{int,i} - R_D - r_d) \cos(\phi_0)\right) + (R_D + r_d)\right)/N_i \tag{16}$$

$$(L_{2,i} - 2\Delta x_{N,i}) = 2\pi(R_D - r_d)/N_i \tag{17}$$

$$2\Delta x_{2,i} = (L_{2,i}) - (L_{2,i} - \Delta x_{2,i}) \tag{18}$$

$$2\Delta X_{2,i} = \left(\left(2\pi\left((R_{int,i} - R_D - r_d) \cos(\phi_0)\right) + (R_D + r_d)\right) - (2\pi(R_D - r_d))\right)/N_i \tag{19}$$

Where,  $L_{2,i}$  is the circumference on the position of point 2 for an insignificant wrinkling wave,  $R_{int,i}$  is unknown inside radius-function of the punch stroke (point 2),  $2\Delta x_{2,i}$  is reduction in the circumference of the flange inside radius for a wrinkle wave (position point 2).

The average length of the circumference ( $\bar{L}_i$ ) to the wave wrinkle and the average displacement direction of the x axis (length reduction circular flange) between point 1 and 2, for a wrinkle wave is:

$$\bar{L}_i = (L_{1,i} + L_{2,i})/2 \tag{20}$$

$$\bar{\Delta x}_i = (\Delta X_{1,i} + \Delta X_{2,i})/2 \tag{21}$$

In case of buckling or wrinkling due to tangential stress over punch stroke, furthermore the strain energy balance on the surface of the flange can follow the equation [3][9]:

$$E_{t,i} = E_{bending(wrinkle),i} + E_{bh,i} \tag{22}$$

Where  $E_{t,i}$  is the strain energy to straighten wrinkle wave,  $E_{bending(wrinkle),i}$  is the bending energy of buckling due to tangential stress,  $E_{bh,i}$  is the strain energy due to the application of blank holder force or energy unbending, and which serves to maintain the tangential stress is not excessive, with reference to the allowable gap.

Strain energy can also be represented by a magnitude formation based on the value of the circumference reduction, which can be approximated as the following equation (23).

$$E_{t,i} = \int_{R_D+r_d}^{R_{1,i}} \sigma_{t,i} s_0 S' dR \tag{23}$$

Where  $S'$  is the additional length of the circumference (the incremental of circumferential length) when wrinkling occurs, i.e.  $S' = \int_0^1 (1 + \bar{\Delta x}_i) dx - \int_0^1 dx$ , and  $S' = \int_0^1 \bar{\Delta x}_i dx$ ,  $R$  is the outer local radius of the flange,  $\sigma_{t,i}$  is the tangential stress and  $s_0$  is the initial thickness of material.

By knowing the magnitude of increase in length of the tour, furthermore the equation (23) would be

$$E_{t,i} = \int_{R_D+r_d}^{R_{1,i}} \int_0^1 (\bar{\Delta x}_i) \sigma_{t,i} s_0 dx dR \tag{24}$$

$$E_{t,i} = \int_{R_D+r_d}^{R_{1,i}} \sigma_{t,i} s_0 \bar{\Delta x}_i dR \tag{25}$$

$$E_{t,i} = \sigma_{t,i} s_0 (R_{1,i} - (R_D + r_d)) \bar{\Delta x}_i \tag{26}$$

Substituting equation (3) and (8) into the equation (26), thus the strain energy ( $E_{t,i}$ ) for one of wrinkle wave would becomes,

$$E_{t,i} = \left( \sqrt{\frac{2(\bar{R}+1)}{1+2\bar{R}}} \sigma_{fml,i} \ln \left( \frac{R_{1,i}}{R_m} \right) + \left\{ \frac{\mu F_{bh}}{\pi R_{1,i} s_0 / N_i} \right\} - \sqrt{\frac{2(\bar{R}+1)}{1+2\bar{R}}} \sigma_{f,i} \right) s_0 \left( R_{1,i} - (R_D + r_d) \right) \bar{\Delta}X_i \quad (27)$$

$$E_{t,i} = \left( 1,1 \sigma_{fml,i} \ln \left( \frac{R_{1,i}}{R_m} \right) + \left\{ \frac{\mu F_{bh}}{\pi R_{1,i} s_0 / N_i} \right\} - 1,1 \sigma_{f,i} \right) s_0 \left( R_{1,i} - (R_D + r_d) \right) \bar{\Delta}X_i \quad (28)$$

Where:

$$\bar{\Delta}X_i = f(R_{1,i} \phi_0)$$

$$\bar{\Delta}X_i = 2\pi \left( (R_{1,i}) - \left( \left( (R_{1,i} - R_D - r_d) \cos (\phi_0) \right) + (R_D + r_d) \right) + \left( \left( (R_{int,i} - R_D - r_d) \cos (\phi_0) \right) + (R_D + r_d) \right) - ((R_D - rd)) \right)$$

$$R_{1,i} = R_{1(i=h)} = f(h_i) = \sqrt{(R_0)^2 + 8R_D (h_i - (0,43r_p - 0,43r_d))}$$

$$R_{int,i} = f(R_{1,i}) = \sqrt{(R_0)^2 + (R_D + r_d)^2 - (R_{1,i})^2}$$

$$\phi_0 = f(N, gap_{0,1})$$

$\sigma_{fml,i}$  is the mean flow stress by considering the alignment wrinkle.

The mean flow stress between point 1 to point 2 ( $\sigma_{fmp1-2,i}$  - function position of punch stroke) can be calculated by:

$$\sigma_{fml,i} = \left( \frac{1}{2} \right) K \left( \left( \varepsilon_{t_{crt-point 1}} \right)^n + \left( \varepsilon_{t_{crt-point 2}} \right)^n \right) \quad (29)$$

Where K is the strength coefficient of material, n is the strain hardening exponential of material,  $\varepsilon_{t_{crt-point 1}}$  is the critical tangential strain at point 1, and  $\varepsilon_{t_{crt-point 2}}$  is the critical tangential strain at point 2.

To estimate the critical tangential strain can be used the follows equation, respectively [21]:

$$\left( \varepsilon_{t_{crt-point 1}} \right) = \sqrt{\frac{2(\bar{R}+1)}{1+2\bar{R}}} \left\{ \ln \left( \frac{L_{1,i} \text{ point 1}}{L_{1,i} \text{ point 1} - \Delta X_{1,i} \text{ point 1}} \right) + \ln \left( \frac{R_0}{R_{1,i}} \right) \right\} \quad (30)$$

$$\left( \varepsilon_{t_{crt-point 2}} \right) = \sqrt{\frac{2(\bar{R}+1)}{1+2\bar{R}}} \left\{ \ln \left( \frac{L_{2,i} \text{ point 2}}{L_{2,i} \text{ point 2} - \Delta X_{2,i} \text{ point 2}} \right) + \ln \left( \frac{R_{int,i}}{R_D + r_d} \right) \right\} \quad (31)$$

Substituting the equations (30) and (31) into the equation (29), therefore the equation of mean flow stress between point 1 to point 2 becomes,

$$\sigma_{fml,i} = \left( \frac{1}{2} \right) K \left[ \left\{ \sqrt{\frac{2(\bar{R}+1)}{1+2\bar{R}}} \left\{ \ln \left( \frac{L_{1,i} \text{ point 1}}{L_{1,i} \text{ point 1} - \Delta X_{1,i} \text{ point 1}} \right) + \ln \left( \frac{R_0}{R_{1,i}} \right) \right\} \right\}^n - \left\{ \sqrt{\frac{2(\bar{R}+1)}{1+2\bar{R}}} \left\{ \ln \left( \frac{L_{1,i} \text{ point 1}}{L_{1,i} \text{ point 1} - \Delta X_{1,i} \text{ point 1}} \right) + \ln \left( \frac{R_0}{R_{1,i}} \right) \right\} \right\}^n \right] \quad (32)$$

By using the phenomena of bending moment, thus the bending energy magnitude ( $E_{bending(wrinkle),i}$ ) can be approximated by the equation:

$$E_{bending(wrinkle),i} = \int_{R_D+r_d}^{R_{1,i}} \left( \frac{\bar{\sigma}_{bending,i} dR s_0 gap_i}{dI_i} \right) s_0 \bar{\Delta}X_i dR \quad (33)$$

Where  $\bar{\sigma}_{bending,i}$  is the average bending stress between points 1 and 2,  $dI_i$  is the moment of inertia of the cross section area of the flange. If the radius of a moment in the flange segment is constant R and the thickness of the material that is  $s_0$ , then change the moment of inertia in every place ( $dI_i$ ) can be shown equation.

$$dI_i = \frac{dR}{12} (s_0)^3 / \left( \frac{s_0}{2} \right) \quad (34)$$

By integrating the equation (33), and combine it into the equation (34), furthermore the equation will be obtained:

$$E_{bending(wrinkle),i} = 6(\bar{\sigma}_{bending,i} gap_i) \bar{\Delta}X_i \left( R_{1,i} - (R_D + r_d) \right) \quad (35)$$

The average of bending stress is a function and a reduction of the circumference length average between points 1 and 2. If the value of  $\varepsilon_{t_{crt-point 1}}$  and  $\varepsilon_{t_{crt-point 2}}$  is obtained, the average of bending stress ( $\bar{\sigma}_{bending,i}$ ) between points 1 and 2 can be approximated by the equation:

$$\bar{\sigma}_{bending,i} = K \left( \left( \sqrt{\frac{2(\bar{R}+1)}{1+2\bar{R}}} \left( \ln \left( \frac{\bar{L}_{i-0}}{\bar{L}_{i-0} - \bar{\Delta}X_{i-0}} \right) + \ln \left( \frac{\bar{L}_i}{\bar{L}_i - \bar{\Delta}X_i} \right) \right) \right)^n \right) \quad (36)$$

While the unbending energy is the energy required to align the sheet material that has been buckling. Unbending energy on flange surfaces approximated by the following equation.

$$E_{bh,i} = F_{bh} \text{ gap}_i \tag{37}$$

Where  $F_{bh}$  is blank holder force (BHF).

The physical meaning of this formula is the energy to straighten a wrinkle, where the magnitude is a multiplication between the blank holder force with a high amount of wrinkle (gap). Then, based on the principle of energy conversion which has considered the normal formation energy and energy buckling, the equation blank holder force to perform unbending one wrinkle wave can be determined as follows.

$$F_{bh} \text{ gap}_i = (E_{t,i} - E_{bending(wrinkle),i}) \quad ; \text{ for } N_i = 1 \tag{38}$$

Then the equations (28) and (37) respectively substituted into the equation (38), taking into account the friction factor, it will obtain the equation of blank holder force of each punch stroke ( $F_{bh} \text{ gap}_i$ ) as follows:

$$F_{bh} \cdot \text{gap}_i = \left\{ \left( \sqrt{\frac{2(\bar{R}+1)}{1+2\bar{R}}} \sigma_{fm,i} \ln \left( \frac{R_{1,i}}{R_m} \right) + \left\{ \frac{\mu F_{bh}}{\pi R_{1,i} s_0 / N_i} \right\} - \sqrt{\frac{2(\bar{R}+1)}{1+2\bar{R}}} \sigma_{f,i} \right) s_0 (R_{1,i} - (R_D + r_d)) \bar{\Delta x}_i \right\} - \left\{ 6(\bar{\sigma}_{bending,i} \text{ gap}_i) \bar{\Delta x}_i [R_{1,i} - (R_D + r_d)] \right\} \tag{39}$$

And

$$F_{bh} \left( \text{gap}_i - \frac{\mu (R_{1,i} - (R_D + r_d)) \bar{\Delta x}_i}{\pi R_{1,i} s_0 / N_i} \right) = \left\{ \left[ \left( \sqrt{\frac{2(\bar{R}+1)}{1+2\bar{R}}} \sigma_{fm,i} \ln \left( \frac{R_{1,i}}{R_m} \right) - \sqrt{\frac{2(\bar{R}+1)}{1+2\bar{R}}} \sigma_{f,i} \right) s_0 (R_{1,i} - (R_D + r_d)) \bar{\Delta x}_i \right] - \left[ 6(\bar{\sigma}_{bending,i} \text{ gap}_i) \bar{\Delta x}_i \{R_{1,i} - (R_D + r_d)\} \right] \right\} \tag{40}$$

Furthermore, the equation (40) can be transformed into the equation (41), which can be used to predict the magnitude of the minimum varying blank holder force (VBHF) without wrinkling during the punch stroke, as follows.

$$F_{bh,i \text{ min.}} = \left( \frac{\left\{ \left( \sqrt{\frac{2(\bar{R}+1)}{1+2\bar{R}}} \sigma_{fm,i} \ln \left( \frac{R_{1,i}}{R_m} \right) - \sqrt{\frac{2(\bar{R}+1)}{1+2\bar{R}}} \sigma_{f,i} \right) s_0 (R_{1,i} - (R_D + r_d)) \bar{\Delta x}_i \right\} - \left\{ 6(\bar{\sigma}_{bending,i} \text{ gap}_i) \bar{\Delta x}_i (R_{1,i} - (R_D + r_d)) \right\}}{\left( \text{gap}_i - \frac{\mu (R_{1,i} - (R_D + r_d)) \bar{\Delta x}_i}{\pi R_{1,i} s_0 / N_i} \right)} \right) \tag{41}$$

The total of minimum VBHF without wrinkles on the entire surface of the flange, can determined by the following equation.

$$F_{bh,i \text{ min. TOTAL}} = \left( \frac{\left\{ \left( \sqrt{\frac{2(\bar{R}+1)}{1+2\bar{R}}} \sigma_{fm,i} \ln \left( \frac{R_{1,i}}{R_m} \right) - \sqrt{\frac{2(\bar{R}+1)}{1+2\bar{R}}} \sigma_{f,i} \right) s_0 (R_{1,i} - (R_D + r_d)) \bar{\Delta x}_i \right\} - \left\{ 6(\bar{\sigma}_{bending,i} \text{ gap}_i) \bar{\Delta x}_i (R_{1,i} - (R_D + r_d)) \right\}}{\left( \text{gap}_i - \frac{\mu (R_{1,i} - (R_D + r_d)) \bar{\Delta x}_i}{\pi R_{1,i} s_0 / N_i} \right)} \right) N_i \tag{42}$$

Where :  $\sigma_{fm,i}$  is the average flow stress in the flange function metrial punch stroke, and  $\sigma_{f,i}$  is the bending stress at the die radius,  $\bar{\sigma}_{bending,i}$  is the unbending stress of wrinkling.

The equation (42) is useful for predicting the minimum VBHF, which can be applied as the lower limit of the distribution profile of blank holder force based on the theory of buckling and unbending energy, where the product is not expected to experience wrinkling.

### III.FE MODELING

In this study, FE modeling provided the essential data, that will be compared to analytical solution. Element formation is carried out using FE of elastic-plastic deformation and ignore the effects of spring back. The curve of flow stress is following the Swift formulation by considering the yield stress and plastic flow stress. Simulation of wrinkling, and tearing (cracking) was done using the implicit AUTOFORM ver3.1. FE simulations had been done for cylindrical cups deep drawing as shown in Fig. 3. Dimensions of product, tools and used mechanical properties can be shown in Table I and table II below.

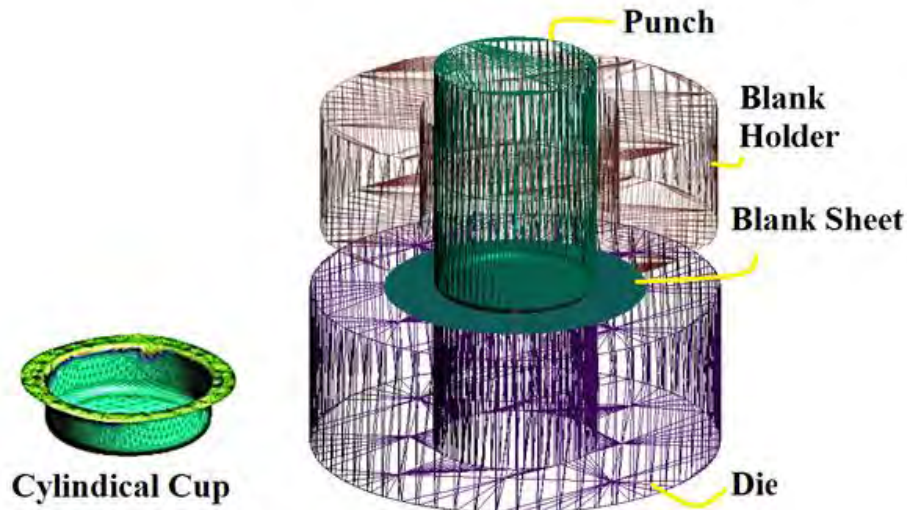


Fig. 3. Finite element model of cylindrical cup, by using AUTOFORM

TABLE I  
Dimension of Tools

Dimension of tools	Remarks
$d_0$ (Blank sheets diameter)	64 mm
$d_D$ (Die diameter)	40 mm
$d_p$ (Punch diameter)	39.58 mm
2 x Clearance	0.43 mm
$s_0$ (Initial thickness of sheet material)	0.2 mm
$r_d$ (Die edge radius)	1 mm
$r_p$ (Edge punch radius)	2 mm
$h$ (height of cylindrical cup)	15-34 mm

TABLE II  
Mechanical Properties and Processes Variable

Steel Sheet grade SPCD equivalent to DIN D03		Processes Variable	
K	559 N/mm <sup>2</sup>	Coefficient of friction	0.4-0.45 (dry) 0.18-0.2 (Palm oil)
n	0.176	Punch velocity	1 mm/Sec.
$\bar{R}_n$	2	BHF	Variable or Varying
UTS	355 N/mm <sup>2</sup>		
Yield Stress	232 N/mm <sup>2</sup>		

The analysis result of the minimum VBHF is incorporated into the process generator functions of FE software for generating an animation and describing the quality of a cylindrical cup drawing during the forming process.

#### IV. RESULT AND DISCUSSION

Based on mathematical approaches and FE modeling, then it obtained the results as shown in Tables III and IV. The FE and analysis results are covering the calculation of the gap, the material displacement, strain energy and minimum VBHF. The magnitude of bending strain and wrinkle straightening energy increase drastically at the beginning of the punch stroke. This caused by the magnitude of tangential stress is very high. Furthermore, the strain energy magnitude will decrease at a middle - stroke, and increase at the end of the punch stroke. The application of minimum VBHF would be used to produce a product without wrinkling, where it was using the wrinkling gap limit around 180 % of the initial material thickness. The Height of gap is always changing in accordance with the strain energy conditions, and its identify the magnitude of tangential stress and wrinkling wave. To anticipate the occurrence of an excessive gap, BHF must be raised properly. The strain energy is highest at the beginning of the punch stroke, furthermore declined in the middle of the punch stroke, and finally increased at the end of the punch stroke, as shown in Table III.

The analytical approach has been validated and confirmed by the FE simulation. Table III has also shown the results of FE simulation related to the minimum VBHF. The FE simulation results of minimum VBHF are in the range of mathematical analysis results based on gap limits around 180% of the initial thickness material.



The minimum VBHF increases from the beginning until the end of the stroke punch. Overall, the minimum VBHF result of mathematical modeling has shown an average increasing value in similar percentage around of 9.3 % (from 360 N up to 890 N) compared to the FE simulation results (from 587 N up to 1432), as shown in table III.

TABLE III  
Strain Energy and Minimum VBHF without Wrinkling – Analytical Result and FE Simulation with Gap 180% of the Initial Thickness

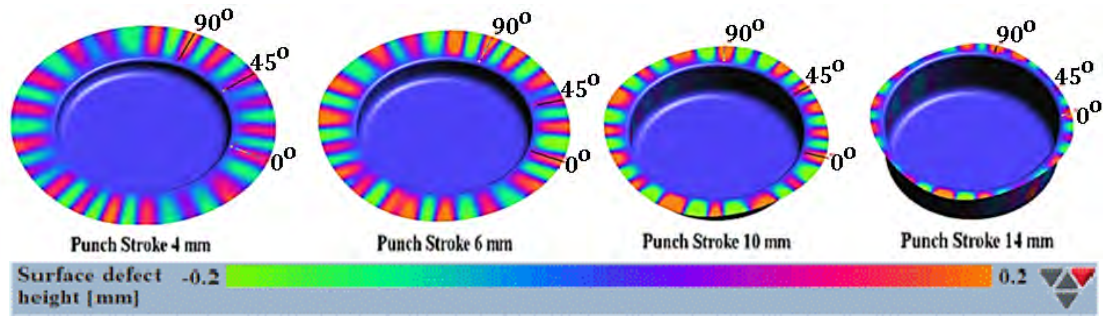
i=h (mm)	Gap <sub>i</sub>	θ (rad)	Δ $\bar{X}$ Wrinkle At Point 1	Δ $\bar{X}$ Wrinkle At Point 2	$\bar{L}_i$	N <sub>i</sub>	E <sub>i</sub> Bending	E <sub>bb,i</sub> Unbending	F <sub>bb,i_Min</sub> Analitical (N)	F <sub>bb,i_Min</sub> FE Simulation (N)
1	0.34	0.0310	0.058796	0.208718	200.65	3.55	43.87	60.04	587	360
2	0.33	0.0310	0.058150	0.211790	198.17	3.64	40.49	63.68	677	252
3	0.31	0.0310	0.057084	0.217206	194.17	3.81	35.38	61.39	707	400
4	0.29	0.0310	0.055961	0.223396	190.08	3.99	30.55	57.51	740	580
5	0.27	0.0310	0.054770	0.230527	185.91	4.21	26.01	53.06	770	605
6	0.25	0.0310	0.053497	0.238814	181.64	4.46	21.77	48.35	802	602
7	0.22	0.0310	0.052122	0.248537	177.26	4.77	17.85	43.50	837	625
8	0.20	0.0310	0.050616	0.260076	172.78	5.14	14.25	38.59	877	646
9	0.18	0.0310	0.048936	0.273938	168.18	5.60	11.00	33.65	925	664
10	0.16	0.0310	0.047018	0.290815	163.44	6.19	8.12	2.72	983	703
11	0.13	0.0310	0.044756	0.311628	158.57	6.96	5.62	2.80	1055	702
12	0.11	0.0310	0.041959	0.337492	153.54	8.04	3.54	1.91	1146	730
13	0.08	0.0310	0.038253	0.369129	148.34	9.65	1.91	1.06	1260	787
14	0.05	0.0310	0.032781	0.403037	142.95	12.29	0.76	9.25	1392	876
15	0.03	0.0310	0.023035	0.402736	137.35	17.48	0.13	4.49	1432	890
16	0.03	0.0310	0.023035	0.402736	131.51	32.35	0.13	4.49	1432	683
Etc.										

Table III shows the gap limits around 180% of initial thickness, is still permitted to use as a reference. This analytical approach could be validated and confirmed by the FE simulation. Therefore, the gap limits and unbending energy will be used to determine the minimum VBHF with no excessive wrinkle.

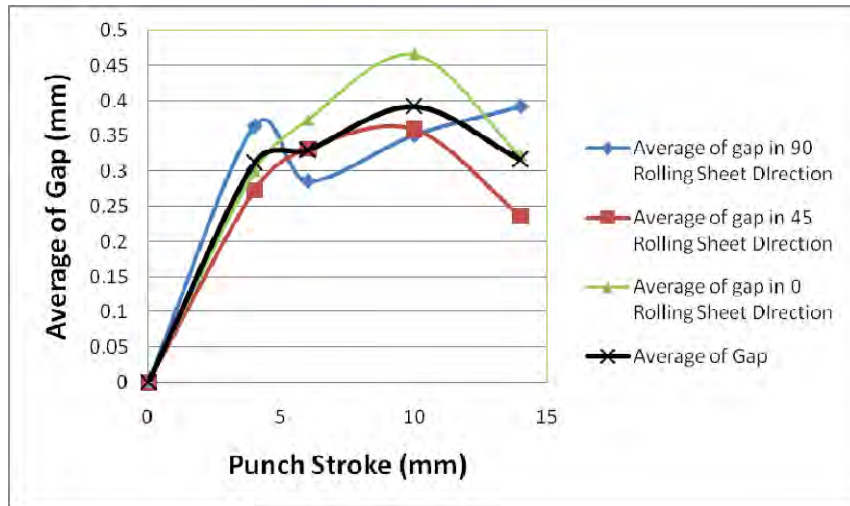
In addition, the application of minimum VBHF has maintained the maximum gap without excessive wrinkle product of 0.395 mm or in around 180% of the initial material thickness, as shown in Table IV and Fig. 4. The results of simulation agree with Gavas’s [23] [24] and Lin’s gap criteria [28], so their method can be considered feasible to prevent excessive wrinkling.

TABLE IV  
Height of Wrinkle and Average Gap in The Several Cross Section Direction (90°, 45° and 0° Rolling Sheet Direction); FE Simulation Result.

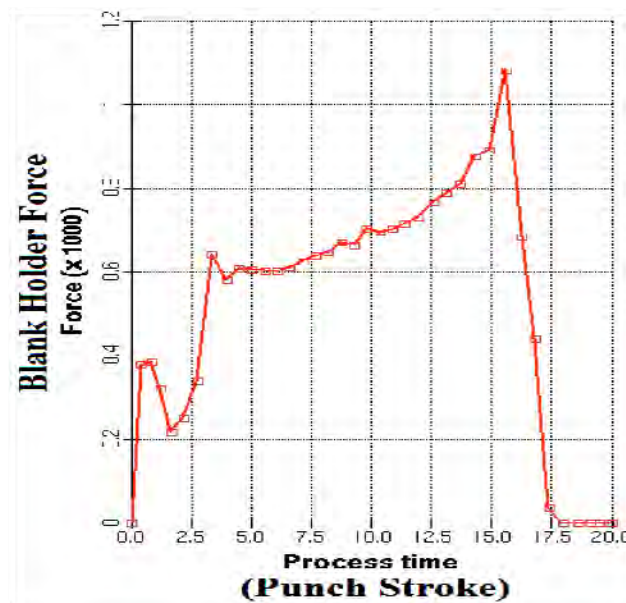
i=h Punch Stroke (mm)	Average Height of Wrinkle and Gap (mm)							Total Average of Wrinkle Height	Total Average of Gap <sub>i</sub>	Maximum Height of Wrinkle (mm)	Maximum of Gap <sub>i</sub> (mm)
	Cross section Direction										
	45°		0°		90°						
	Height of Wrinkle	Gap <sub>i</sub>	Height of Wrinkle	Gap <sub>i</sub>	Height of Wrinkle	Gap <sub>i</sub>					
0	0	0	0	0	0	0	0.000	0.000	0.192	0.392 (around 180% of initial thickness)	
4	0.164	0.364	0.730	0.273	0.100	0.300	0.113	0.313			
6	0.086	0.286	0.132	0.332	0.137	0.373	0.130	0.330			
10	0.152	0.352	0.160	0.360	0.266	0.466	0.192	0.392			
14	0.192	0.392	0.037	0.237	0.122	0.322	0.117	0.317			
Etc.											



(a)



(b)



(c)

Fig. 4. FE Simulation results of cylindrical cup product with gap around 180 % of  $s_0$ ; a. Virtual image of height wrinkle wave every punch stroke in cross section direction of 90°, 45° and 0°, b. Average of Gap during punch stroke, c. The minimum VBHF during punch stroke.

The wrinkle wave height image in the other cross section is shown in Fig 5 and 6. Those figures provide the information, which the application of mathematical modeling of minimum VBHF into AUTOFORM software would maintain the height of gap not more than 0.392 mm. Where, the maximum height of the gap is obtained by summing of the maximum wave height of 0.192 mm with the material initial thickness of 0.2 mm. Those figures also show, that the highest wave height wrinkles are on the outer flange diameter (“a” and “c” position in graphic). Furthermore, the sheet material closer to the die radius (“b” position), wave height of wrinkles

decreased. These results while ensuring mathematical modeling results shown in table III, have a relatively similar trend results.

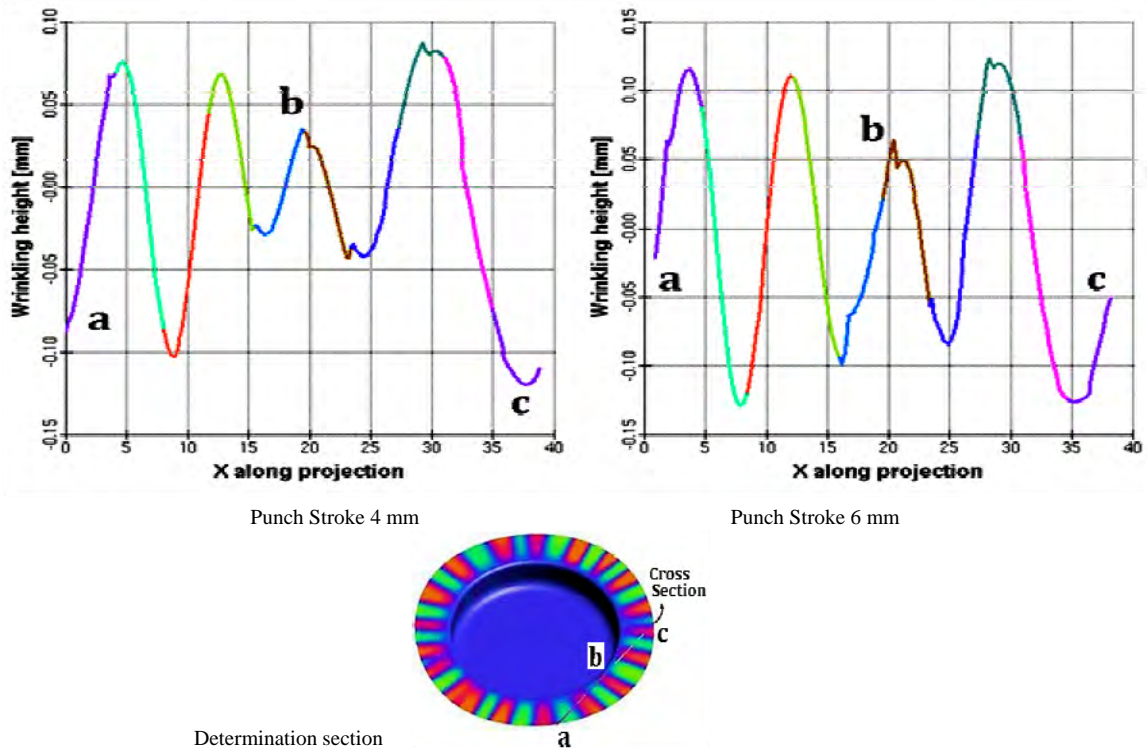


Fig. 5. FE simulation; Wrinkling wave height of cross section of flange at punch stroke of 4 mm and 6 mm.

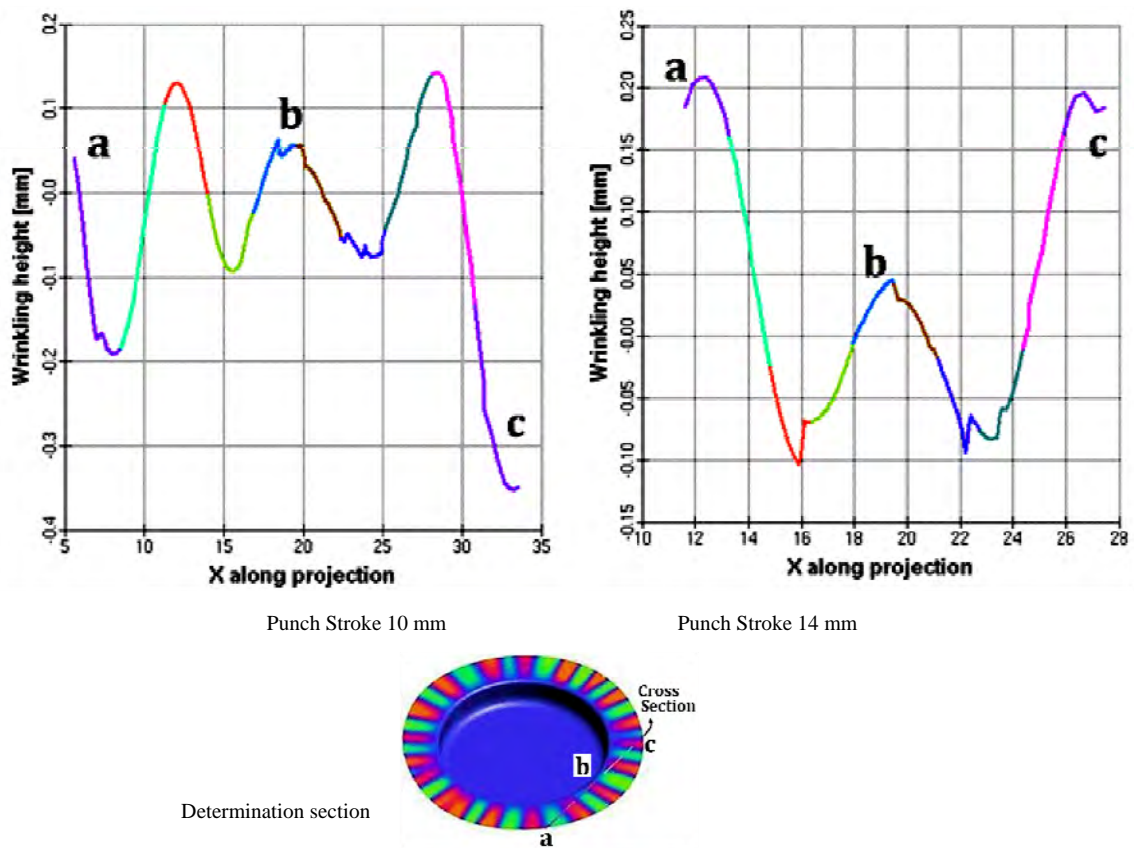


Fig. 6. FE simulation results; Wrinkling wave height of cross section of flange at punch stroke of 10 mm and 14 mm

The minimum VBHF, as a condition of safe limit in order to produce a product without cracking and wrinkling, can be shown in Fig. 7. This figure also shows the similarity value between the results of analytical calculations compared to the FE simulation results, related to the distribution of the minimum VBHF. In practice, the magnitude of VBHF minimum was recommended slightly above that value. With this minimum VBHF will maintain a safe area of the gap and the tangential stress on flange surface.

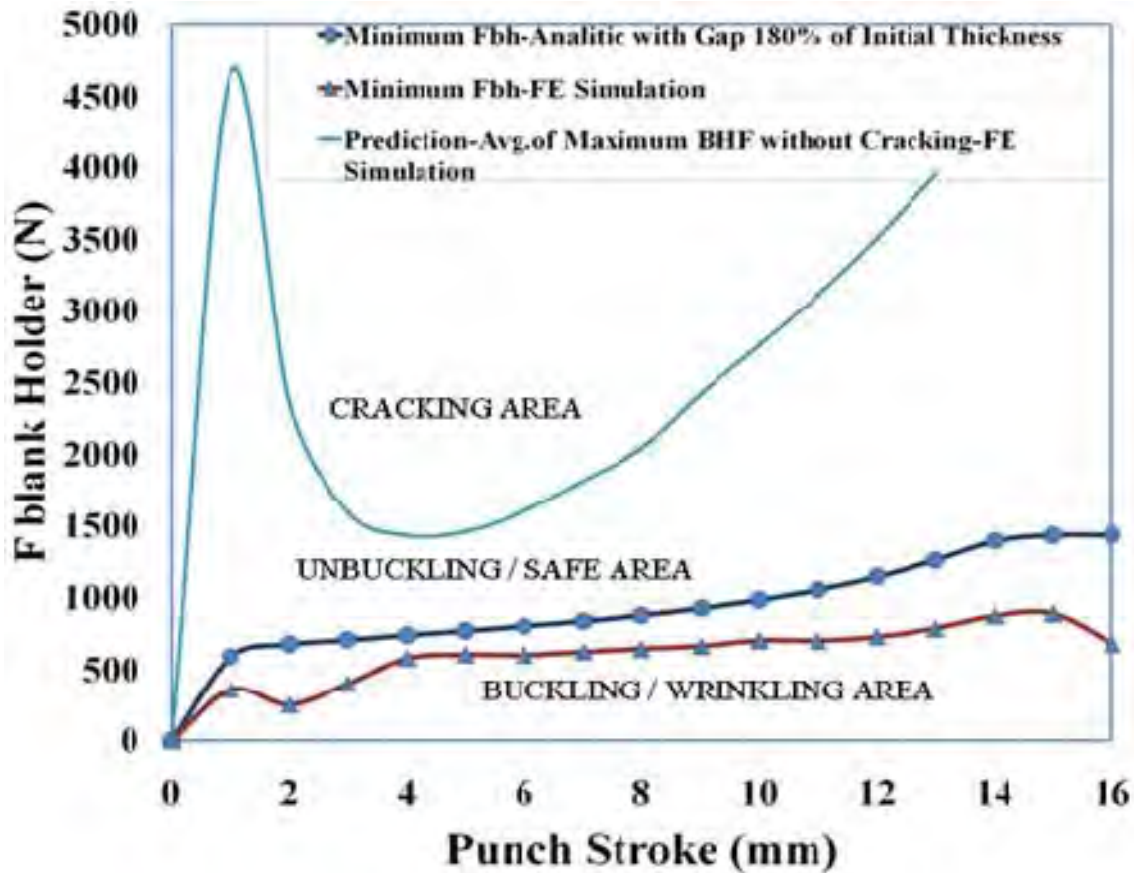


Fig. 7. The minimum VBHF to avoid wrinkling, by using the slab method (analytic calculation) and FE method;  $d_0=64$  mm;  $\mu=0.45$

## V. CONCLUSION

Base on the gap limit and strain energy, the mathematical model of minimum VBHF has been described. It could be used as reference in deep drawing process, to prevent wrinkling. A good correlation between the analytical predicted results and FE simulation in order to determine the magnitude of minimum VBHF without excessive wrinkling was obtained. The application results of the minimum VBHF models have shown similar trends compared to the FE simulation results. The gap around 180% of the initial thickness is still permitted to use as a gap limit with realistic strain energy (not too high). The maximum gap without excessive wrinkle product is 0.395 mm of initial material thickness or around 180% of the thickness. Therefore, this gap limit can be used as a basic criteria to determine minimum VBHF to prevent excessive wrinkling with maintaining the maximum gap around that range. The critical VBHF is recommended slightly higher than the results of this study.

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