

# Fuzzy Gain Scheduled PI Controller for a Two Tank Conical Interacting Level System

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**Abstract—** This paper emphasis the need of a new fuzzy gain scheduled PI controller (FGSPI controller) for a two tank conical interacting level system (TTCILS). A mathematical model is first obtained for the conical interacting process. It is then followed by the development of a fuzzy gain scheduling scheme by PI controller for the process. Fuzzy rules and reasoning are utilized to tune the PI controller parameters. Simulation results demonstrate that the FGSPI controller achieves satisfactory performance in terms of settling time and ISE.

**Keyword-** Mathematical model, fuzzy gain scheduled adaptive controller, two tank conical interacting level system, ISE.

## I. INTRODUCTION

Conical tanks are best suited for food process industries, concrete mixing industries, hydrometallurgical industries and waste water treatment industries. Its shape contributes to better drainage of solid mixtures, slurries and viscous liquids. To achieve a satisfactory performance using conical tanks, its controller design becomes a challenging task because of its non-linearity. This non-linearity arises due to its shape. It is broad at the end and becomes narrow in the lower end. The primary task of a controller is to maintain the process at the desired set point and to achieve optimum performance when facing various types of disturbances [1].

Conventional controllers are widely used in industries since their design is simple and robust. These controllers are best suited for applications where the process parameters do not change. But under situations where the process parameters vary the conventional controllers does not provide satisfactory results. The solution is the controller parameters have to be continuously adjusted [2].

In this paper, a two tank conical interacting system is considered and the controller parameters are adapted based on parameter estimation, which requires certain knowledge of the process. Such controllers are called dynamic or adaptive PID controllers. The applications of knowledge based system in process control is growing especially in the field of fuzzy control. In fuzzy control [5], linguistic descriptions of human expert in controlling a process are represented in fuzzy rules. This knowledge base is used in conjunction with some knowledge of the states of process by an inference mechanism to determine control actions. Theerawut et.al [6] discussed about optimal fuzzy gain scheduling of PID controller of super conducting magnetic energy storage for power system stabilization. Leehter et.al [7] designed a gain scheduled fuzzy PID controller based on genetic algorithm for a second order process. Soft computing based controllers implementation for non linear process is discussed by S.Nithya et.al [8]. A rule based scheme for gain scheduling of PID controllers as discussed by Zhen-Yu Zhao [4] is experienced for a two tank conical interacting system. Based on fuzzy rules, human expertise is utilized with ease for PI controller gain scheduling.

The paper is organized as follows. In section II the two tank conical interacting system considered for simulation study has been discussed. In section III, a fuzzy gain scheduled PI controller has been explained. In section IV simulation results of fuzzy gain scheduled PI controller is discussed. Finally the paper ends with Conclusion in section V.

## II. PROCESS DESCRIPTION

The two tank conical interacting system consists of two identical conical tanks (Tank 1 and Tank 2), two identical pumps that deliver the liquid flows  $F_{in1}$  and  $F_{in2}$  to Tank 1 and Tank 2 through the two control valves  $C_{v1}$  and  $C_{v2}$  respectively as shown in Fig. 1. These two tanks are interconnected at the bottom through a manually controlled valve,  $MV_{12}$  with a valve coefficient  $\beta_{12}$ .  $F_{out1}$  and  $F_{out2}$  are the two output flows from Tank 1 and Tank 2 through manual control valves  $M_{v1}$  and  $M_{v2}$  with valve coefficients  $\beta_1$  and  $\beta_2$  respectively.

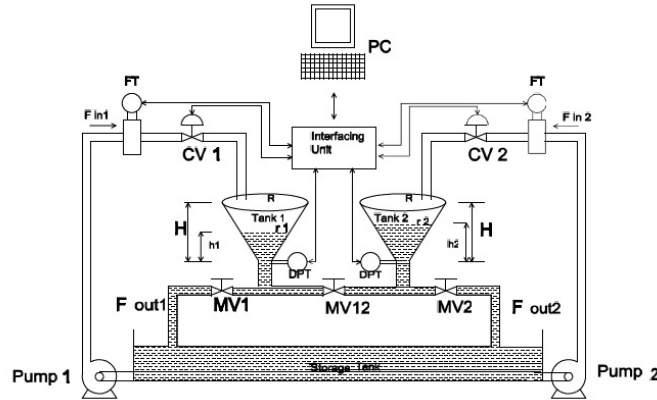


Fig. 1. Schematic of TTCIS

The operating parameters of the interacting conical tank process is shown in Table.I

Table I Operating parameters of TTCIS

Parameter	Description	Nominal Value
R	Top radius of conical tank	19.25cm
H	Maximum height of Tank1&Tank2	73cm
$F_{in1}$ & $F_{in2}$	Maximum inflow to Tank1&Tank2	400 & 100cm <sup>3</sup> /sec
$\beta_1$	Valve coefficient of MV <sub>1</sub>	35 cm <sup>2</sup> /sec
$\beta_{12}$	Valve coefficient of MV <sub>12</sub>	78.28 cm <sup>2</sup> /sec
$\beta_2$	Valve coefficient of MV <sub>2</sub>	19.69 cm <sup>2</sup> /secs

In this work, TTCIS is considered as two inputs two output process in which level  $h_1$  in Tank 1 and level  $h_2$  in Tank 2 are considered as output variables and  $F_{in1}$  and  $F_{in2}$  are considered as manipulated variables. The mathematical model of two tank conical interacting system is given by [3]

$$\frac{dh_1}{dt} = \left[ \frac{F_{in1} - h_1 \frac{dA(h_1)}{dt} - \beta_1 \sqrt{h_1} - \text{sign}(h_1 - h_2) \beta_{12} \sqrt{|h_1 - h_2|}}{\frac{1}{3} \pi R^2 \frac{h_1^2}{H^2}} \right] \quad (1)$$

$$\frac{dh_2}{dt} = \left[ \frac{F_{in2} - \beta_2 \sqrt{h_2} + \text{sign}(h_1 - h_2) \beta_{12} \sqrt{|h_1 - h_2|} - h_2 \frac{dA(h_2)}{dt}}{\frac{1}{3} \pi R^2 \frac{h_2^2}{H^2}} \right] \quad (2)$$

where

$A(h_1)$  = Area of Tank 1 at  $h_1$ (cm<sup>2</sup>)

$A(h_2)$  = Area of Tank 2 at  $h_2$ (cm<sup>2</sup>)

$h_1$  = Liquid level in Tank 1 (cm)

$h_2$  = Liquid level in Tank 2 (cm)

The open loop responses of  $h_1$  and  $h_2$  are shown in Fig. 2.

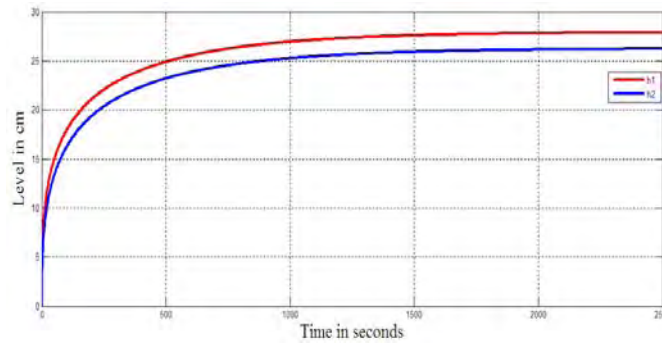


Fig. 2. Open loop response of h<sub>1</sub> and h<sub>2</sub>

### III.FUZZY BASED PI CONTROLLER

Fig. 3. shows the control system with a fuzzy gain scheduling PID controller. The approach taken here is to make use of fuzzy rules and reasoning to generate controller parameters .

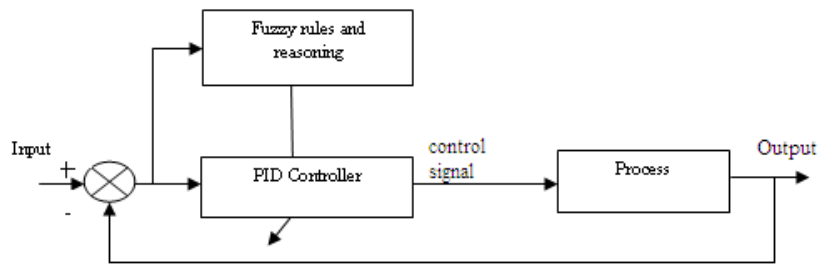


Fig. 3. Control system with rule based PID dynamic controller

It is assumed that  $K_p$ ,  $K_d$  are in prescribed ranges,  $(K_{pmin}, K_{pmax})$  and  $(K_{dmin}, K_{dmax})$  respectively, for convenience,  $K_p$  and  $K_d$  are normalized into the range below zero and one as

$$K_p^l = (K_p - K_{pmin}) / (K_{pmax} - K_{pmin}) \tag{3}$$

$$K_d^l = (K_d - K_{dmin}) / (K_{dmax} - K_{dmin}) \tag{4}$$

In fuzzy gain scheduling scheme, PI parameters are determined based on the current error  $e(k)$  and its first difference  $\Delta e(k)$ . The integral time constant is determined with reference to the derivative time constant (ie)  $T_i = \alpha T_d$ , and the integral gain is thus obtained by

$$K_i = K_p / (\alpha T_d) = K_p^2 / (\alpha K_d) \tag{5}$$

The fuzzy rules of gain scheduling are of the form

If  $e(k)$  is  $A_i$  and  $\Delta e(k)$  is  $B_i$ ,

Then  $K_p^l$  is  $C_i$ ,  $K_d^l$  is  $D_i$  and  $\alpha = \alpha_i$ ;  $i = 1, 2, \dots, m.$

Here  $A_i$ ,  $B_i$ ,  $C_i$  and  $D_i$  are fuzzy sets on the corresponding supporting sets and  $\alpha_i$  is a constant. The membership functions for  $e(k)$  and  $\Delta e(k)$  is shown in Fig. 4. where N represents negative, P- positive, ZO- zero, S- small, M - medium, B - big thus NM stands for negative medium, PB for positive big and so on.

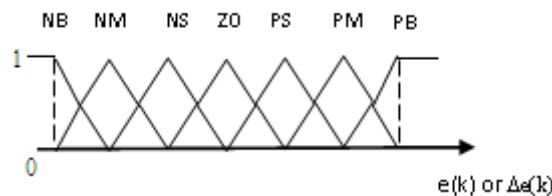


Fig. 4. Membership functions of  $e(k)$  and  $\Delta e(k)$

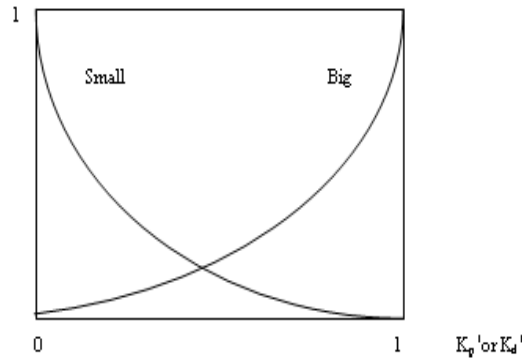


Fig. 5. Membership functions for  $K_p$  and  $K_d$

The fuzzy sets  $C_i$  and  $D_i$  may be either big or small, that are characterized by the membership functions as shown in Fig.5.

The fuzzy rules may be determined heuristically based on the step time response of the process. A Set of rules may be used to adapt the proportional gain ( $K_p^l$ ), derivative gain ( $K_d^l$ ) and  $\alpha$  as shown in Tables II,III and IV.

TABLE II  
Fuzzy tuning rules for  $K_p^l$

$\Delta e(k)$ $e(k)$	NB	NM	NS	ZO	PS	PM	PB
NB	B	B	B	B	B	B	B
NM	S	B	B	B	B	B	S
NS	S	S	B	B	B	S	S
ZO	S	S	S	B	S	S	S
PS	S	S	B	B	B	S	S
PM	S	B	B	B	B	B	S
PB	B	B	B	B	B	B	B

TABLE III  
Fuzzy tuning rules for  $K_d^l$

$\Delta e(k)$ $e(k)$	NB	NM	NS	ZO	PS	PM	PB
NB	S	S	S	S	S	S	S
NM	B	B	S	S	S	B	B
NS	B	B	B	S	B	B	B
ZO	B	B	B	B	B	B	B
PS	B	B	B	S	B	B	B
PM	B	B	S	S	S	B	B
PB	S	S	S	S	S	S	S

TABLE IV  
Fuzzy tuning rules for  $\alpha$

$\Delta e(k)$ $e(k)$	NB	NM	NS	ZO	PS	PM	PB
NB	2	2	2	2	2	2	2
NM	3	3	2	2	2	3	3
NS	4	3	3	2	3	3	4
ZO	5	4	3	3	3	4	5
PS	4	3	3	2	3	3	4
PM	3	3	2	2	2	3	3
PB	2	2	2	2	2	2	2

The controller parameters are obtained as follows

$$K_p = (K_{pmax} - K_{pmin}) K_p^1 + K_{pmin} \tag{6}$$

$$K_d = (K_{dmax} - K_{dmin}) K_d^1 + K_{dmin} \tag{7}$$

$$K_i = K_p^2 / (\alpha K_d) \tag{8}$$

Based on an extensive simulation study on various processes, a rule of thumb for determining the range of  $K_p$  and the range of  $K_d$  is given as

$$K_{pmin} = 0.32K_u \quad K_{pmax} = 0.6K_u \tag{9}$$

$$K_{dmin} = 0.08K_u T_u \quad K_{dmax} = 0.15K_u T_u \tag{10}$$

Where  $K_u$  and  $T_u$  are respectively the ultimate gain and the period of oscillation.

Fig.6 shows the use of fuzzy gain scheduled PI controller for the process with decoupler block.

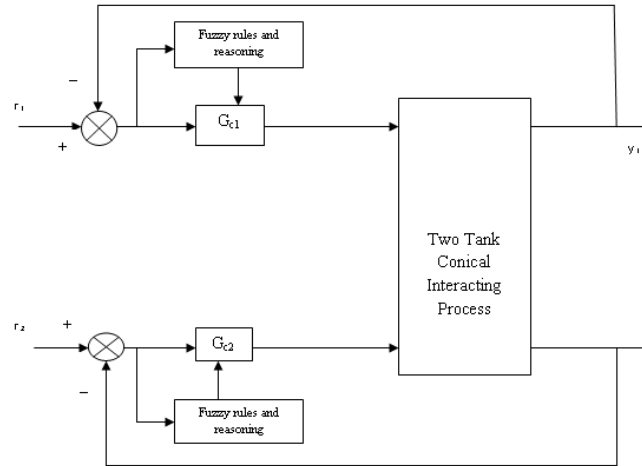


Fig. 6. Fuzzy gain scheduled PI Controller

#### IV. SIMULATION RESULTS

A fuzzy gain scheduled PI controller is designed for TTCILS and the performance is evaluated through MATLAB/SIMULINK software. The simulation is carried out by considering the nominal values of  $h_1$  and  $h_2$ . ( $h_1 = 28\text{cm}$  and  $h_2 = 26\text{cm}$ ). Servo and regulatory responses are taken for tank1 and tank2.

##### A. Servo Performance

The setpoint variations are introduced for understanding the tracking capability of fuzzy gain scheduled PI controller as shown in Fig. 7 and Fig. 8.

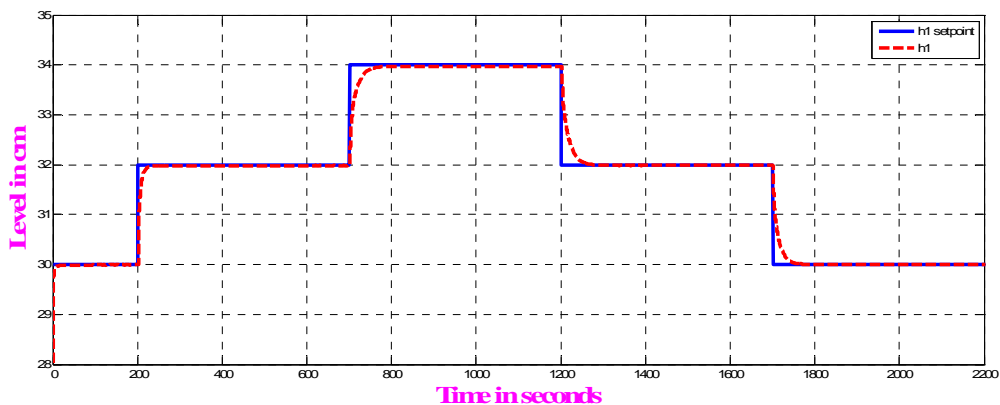
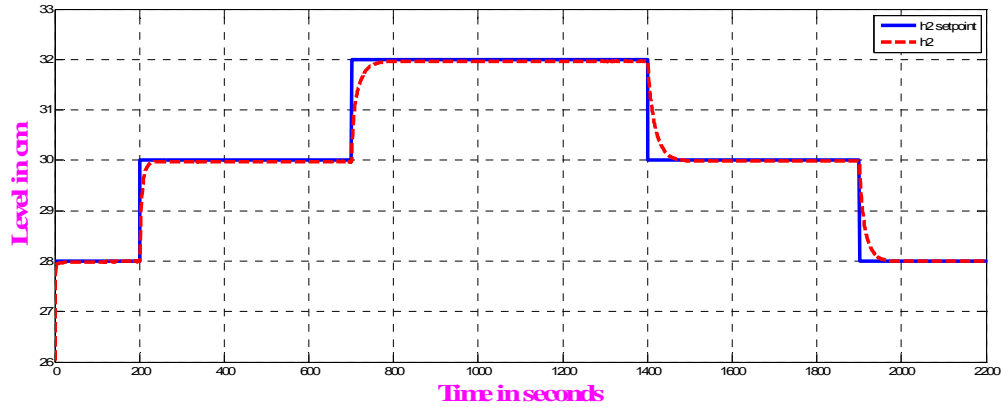


Fig. 7. Servo response of  $h_1$  in TTCILS using FGPI controller

Fig. 8. Servo response of  $h_2$  in TTCILS using FGSPI controller

From the responses, it is inferred that FGSPI controller is able to maintain the tank levels  $h_1$  and  $h_2$  at the respective setpoints with better settling time and integral square error. The performance indices for FGSPI controller is summarized in Table V and VI.

TABLE V  
Servo performance of FGSPI controller with respect to  $h_1$

Operating points of $h_1$ in cm	FGSPI controller	
	Settling time (secs)	ISE
28-30	10	$3.51 \times 10^{-5}$
30-32	50	0.000647
32-34	100	0.002454
34-32	40	0.004707
32-30	100	0.006332

TABLE VI  
Servo performance of FGSPI controller with respect to  $h_2$

Operating points of $h_2$ in cm	FGSPI controller	
	Settling time (secs)	ISE
26-28	10	$4.24 \times 10^{-5}$
28-30	60	0.000777
30-32	100	0.002939
32-30	150	0.005813
30-28	100	0.007918

### B. Regulatory Performance

The simulation results clearly indicate how FGSPI controller effectively rejects the disturbance. The step change in input flow rates  $F_{in1}$  and  $F_{in2}$  which corresponds to 25% change in output level in tank1 and tank2 are introduced as disturbances. Disturbances are introduced at output levels of  $h_1 = 40$  cm and  $h_2 = 36$  cm. Fig. 9 and 10 show regulatory response of TTCILS using FGSPI controller.

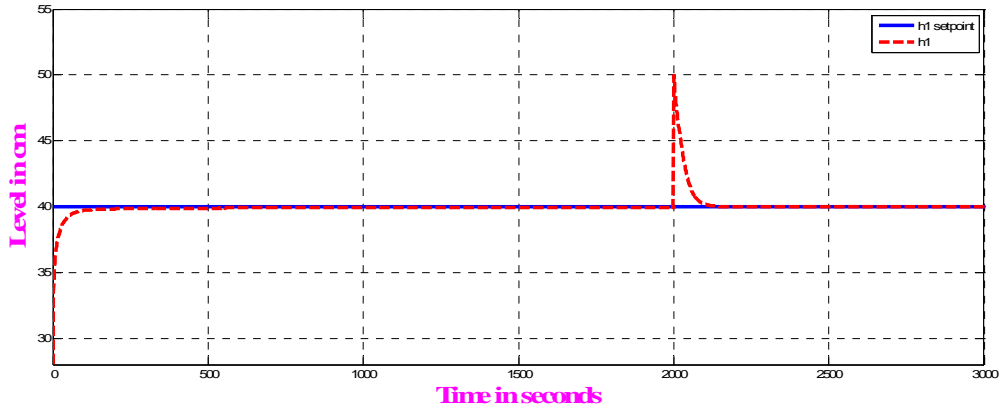


Fig. 9. Regulatory response of  $h_1$  in TTCILS using FGSPI controller

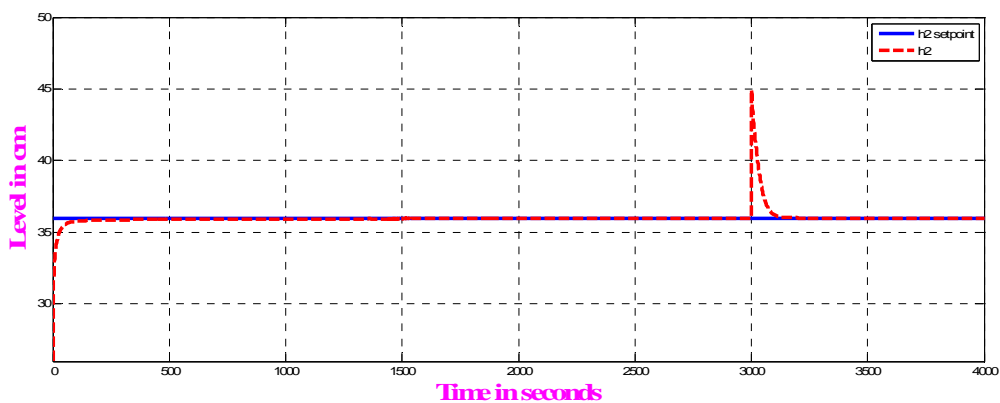


Fig. 10. Regulatory response of  $h_2$  in TTCILS using FGSPI controller

TABLE VII  
Regulatory performance of FGSPI controller

Controller	25% Disturbance in $h_1$		25% Disturbance in $h_2$	
	Settling time(secs)	ISE	Settling time(sec s)	ISE
<b>FGSPI controller</b>	100	0.1824	80	0.1443

**V. CONCLUSION**

The proposed gain scheduling scheme uses fuzzy rules and reasoning to determine the PI controller parameters. The scheme has been tested on two tank conical interacting system and satisfactory results are obtained in simulation. Set point tracking responses and regulatory responses are taken for different set points as shown in section IV. It is clear from the simulation response that fuzzy based PI controller offer minimum integral square error and also settles faster.

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