

Automated Production Flow Line Failure Rate Mathematical Analysis with Probability Theory

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Abstract— Automated lines have been widely used in the industries especially for mass production and to customize product. Productivity of automated line is a crucial indicator to show the output and performance of the production. Failure or breakdown of station or mechanisms is commonly occurs in the automated line in real condition due to the technological and technical problem which is highly affect the productivity. The failure rates of automated line are not express or analyse in terms of mathematic form. This paper presents the mathematic analysis by using probability theory towards the failure condition in automated line. The mathematic express for failure rates can produce and forecast the output of productivity accurately

Keyword- Automated Flow Line, Probability Theory, Failure Rates

I. INTRODUCTION

In virtual, for all modern manufacturing systems, most of the actual numerous processing or assemblies operation is accomplished by machines or tools in production line. The production line can be classified as manually, semi-automated and automated. Automated production lines are typically used for mass production that's required multiple processing operations. It consists of numerous workstations that are automated and linked together by a work handling system to transfer parts from one station to another station.[1] Automated production line has a wide variety of potential advantages. One of the most crucial advantages is the increased ability to respond for changes in demand, which is essential in today's view of short production cycles. Other advantages included the decrease of lead times, reduction in the work-in-process levels and improved machine utilization. The typical designs of industrial automated lines can be presented using linear or rotary arrangement. All these arrangements are presented by three types of structure: serial, parallel action and serial-parallel action. [2]

The serial production line of linear arrangement has been the most common method of production and the 'transfer line' refer to parts that are moved from one station to the next upon completion in a straight line. The stations transfer their parts from one station to another at the same time and series system is productive when all the stations in the line are up and operating. Normally, a serial automated line will be divided into sections with equal reliability and have a different number of serial stations. It is possible to design such lines if level of reliability of station is known. Small variations in the reliability of stations will not have a big influence on the result of the output from a serial automated line [3-4]. By the ways, the model of calculation is same with the rotor-type serial action since the technological and technical attributes are similar and only different in the arrangement. The single rotor-type automatic machine is a multi-station's machine with parallel action. Each single rotary-type automatic machine is equipped with a transport and work machine feeder.

A parallel system of linear arrangement is productive when at least one of the stations is operating. In the parallel arrangement of workstation, stations perform the same operation. The processing rate of the parallel system does not change as the number of operating station change in time. A parallel action of multi-station production line contains identical parallel stations with the same reliability. The system parallelism included designing of independent production lines in parallel with an independent material-handling system servicing its stations. [5,6] Otherwise, the productivity rate equation for rotor-type automatic with parallel station does not have an extreme of the function.

Normally, manufacturers of various industries produce automated lines with a different number of parallel p and serial q stations where the designs present peculiarities of technological processes and restrictions in machine designs. Two types of machine with complex structures represent the design of automated lines with serial-parallel action. The first type has a rotor-type arrangement where application is found in various industries like pressing, coining, filling of bottles and cans and others. Parallel-serial structure is basically designed with two, and very rarely designed with three parallel flows. This structure can combine different numbers of serial stations and such an automated line is the most complex in design. [7,8] The second type of automated production line of serial-parallel actions has a linear arrangement, which is applied for processes that

are having long machining times like machining of housing parts, shafts of complex design and others. A multi-station's automated line with serial-parallel actions and linear design has several parts feeders equal to the number of parallel stations. The serial automated line is divided into several sections with equal reliability, which may have a different number of serial stations. [9]

Productivity is considered as an important indicator to show the performance in an industry [10]. In the perspective of economic, productivity can be defined as the ratio of outputs (goods and services) divided by the inputs (resources such as labour and capital) [11]. Failure or breakdown of station or mechanisms is commonly occurs in the automated line in real condition due to the technological and technical problem which is highly affected the productivity. The productivity rate of automated lines included technological parameters (machine time), technical parameters (auxiliary time, idle time and capacity of buffers) and structural parameters (number of serial and parallel stations and number of sections of automated lines). The current equations of productivity only included the technological and technical aspects of the automated lines. Aspects of management or maintenance of automated lines at prescribed planned overhaul repair time are not included in the equations. The concept of management considers the quality of the production process while the concept of maintenance considers the planned repair and service of machines that are stopped for overhaul processes. The aggrandizement of manufacturing system such as complex design of automated line has increased the probability of failure problem. The failure of any single station can lead to stoppage of the entire automated lines. The random stoppages in complex design of automated lines consisting of independently operated station are calculated by using probability theory. [12]

The mathematical form of productivity with average reliability for station and mechanism is still not very accurate to express the actual productivity since the different failure rates during automated line are not considered in the mathematical form. Therefore, different types of failure have to be investigated before to enhance the mathematical form of productivity with average reliability to a more robust and accurate forecast for productivity in automated line. This paper presents a mathematical approach to express the failure rate and failure types of the automated line with probability theory to present the failure rate accurately and clearly.

II. METHODOLOGY

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There are several publications that presented the usage of probability theory for further analysis of failure or frequency in different type of area such as biology and chemical[13-16]. Application of probability theory to the failure rate of automated are required to differentia and express the failure rate of working automated in term of mathematic. In the probability theory, there are few basic axioms that required to match before using the theory and the basic axioms are presented as below[17]:

Probability Basic Axioms

1. $P(A) \geq 0$ for all events A
2. $P(\Omega) = 1$
3. $P(A \cup B) = P(A) + P(B)$ for disjoint events A and B
4. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ for joint events A and B
5. $P(A \cap B) = P(A) \times P(B)$ for joint or disjoint events A and B

First axiom present the probability of event A must equal or more than zero. The basic theory of probability for any events stated that no negative probability exists. For condition of an automated line, there must be an occurrence of failure or breakdown during the automated line working time. Since there are certain types of automated line failure (f_i) which is not zero, the probability for certain type failure of automated line, $P(f_i)$ is more than zero and no negative figures. Therefore, the failure for automated line is fulfil the basic requirement of probability theory.

The second axiom presents the total sample space (Ω) which is the total coverage of events in the consideration boundary must be obtained the total probability is equal to one or $P(\Omega) = 1$. Total of the failure rate or breakdown is consider as total sample space which present the total of probability for failure rate in automated line is equal to one or $P(f) = 1$. With the combination of first and second axioms, probability of total event for all type of failure in automated line is equal to the probability of total sample space or total of failure in automated.

Following axioms which are third, fourth and fifth axioms that stated above are discussed about the mathematical express between both or more events relationship. If the condition of events in the sample size can occurs independently, this mean that the probability is in 'or' condition and the probability mathematical express is $P(A \cup B)$. For 'or' condition, there are two condition might be happened which are disjoint event or joint event. The Fig 1(a) and 1(b) below will present the both condition of 'or' in term of schematic diagrams.

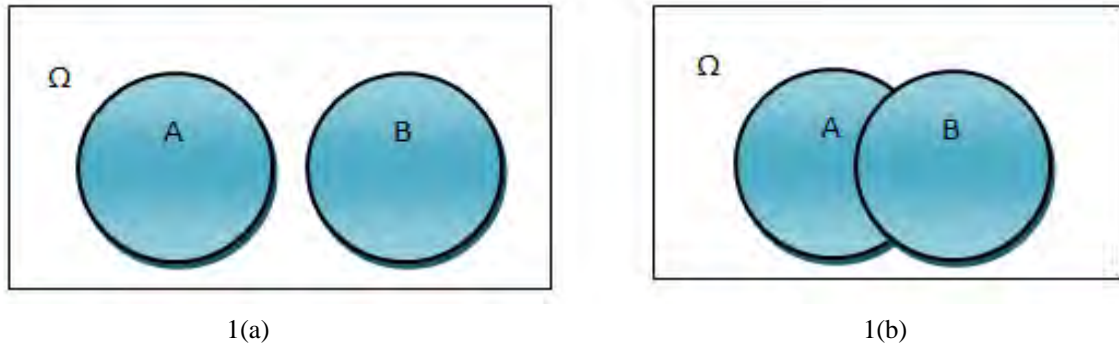


Fig 1: (a) $P(A \cup B)$ occurs independently with disjoint events. (b) $P(A \cup B)$ occurs independently with joint events.

For the $P(A \cup B)$ occurs independently or ‘or’ condition with disjoint events is expressed to $P(A) + P(B)$ since the probability is considered either A or B events to occur. $P(A \cup B)$ occurs independently with joint events is expressed as $P(A) + P(B) - P(A \cap B)$ which present probability is either A or B events occurred but required to minus the subset part between A and B. For dependently condition, this is considered as ‘and’ condition which presents the probability is considered when A and B events occur together. The probability for dependent or ‘and’ condition is expressed as $P(A \cap B) = P(A) \times P(B)$ which is using probability event A multiply with probability event B and figure 2 below is showing the dependent or ‘and’ condition.

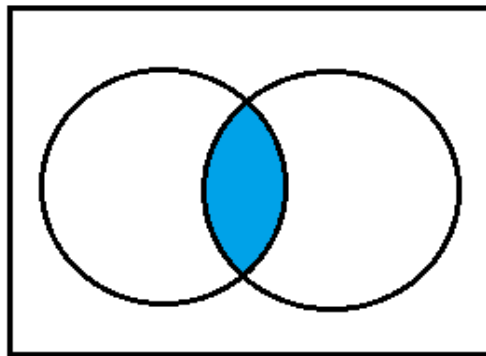


Fig 2. $P(A \cap B)$ occur dependently between both A and B event with joint events.

Third axiom which presents the relationship of different of probability’s between two or more events. The finding of matching with the case of failure rate of automated line is fulfills the first and second axioms. Then, the failure of automated line is happened independently to cause the failure or breakdown of the whole line. The independent failure such as failure of first station, transport system or control system are occurred itself without cause by other. Once there is any failure happened in the automated, it will cause whole line to stop. This means that any failure will cause the increasing of the total probability of failure. Therefore, the condition of third axiom is matched with the condition of failure of automated line which occurs independently with disjoint events. The equation 1 and 2 below will show the relationship of failure of automated line with the third axiom.

$$P(\Omega) = P(A \cup B) = P(A) + P(B) \tag{1}$$

Where

$P(\Omega)$ = Total Probability of Sample Space

$P(A)$ = Probability of A event

$P(B)$ = Probability of B event

$$P(f) = P(f_1 \cup f_2) = P(f_1) + P(f_2) \tag{2}$$

Where

$P(f)$ = Total Probability of failure in automated line

$P(f_1)$ = Probability of first event failure,

$P(f_2)$ = Probability of second event failure,

The third axioms which is A and B or f_1 and f_2 are occurs independently in the condition of Union (\cup) that show in equation 1 and 2 present the meaning of the result of the condition Union is the sum of the independent

events. In other words, the result Union's condition is consider the when either event A or event B occurs. In the term of failure rate (λ) which present the failure of any stations or mechanism in automated production line with respect to time is also occurs independently due to any station or any mechanism which will stop the automated production line. Failure rate is an important parameter in reliability theory for calculation and explanation purpose [18-19]. Since that all or any of the failure rate is contribute to the failure of production line, so that this condition is match to the Union condition or 'or' condition in probability theory which present the sum result in mathematical express. The mathematical expression of failure rate that based on the probability theory with Union or 'or' condition is present in equation 3 below.

$$\lambda = (\lambda_1 \cup \lambda_2) = \lambda_1 + \lambda_2 \tag{3}$$

λ = Total failure rate in automated production line

λ_1 = Failure rate for first event

λ_2 = Failure rate for second event

III.RESULT AND DISCUSSION

In order to express the probability theory towards the automated line failure rates, the recognition of the types of automated line in industry is very important since there are different of the probability mathematical failure expression due to different working properties of automated line. There are two types of arrangement of automated line in current industry which are linear type and rotor type and each type consists of three different working actions which are serial, parallel, and serial parallel action[9,20].

The first type of arrangement is linear with serial working action which is shown in Fig. 3 below. Since each of the workstation consists of different of operations and type of processes, the failure probability of each station is different. However, the failure of transport system and control system is same due to the same transport line and main control. The equation 4 below will present the mathematical expression of total probability of failure and failure rate expression in equation 5 for linear type with serial action automated line.

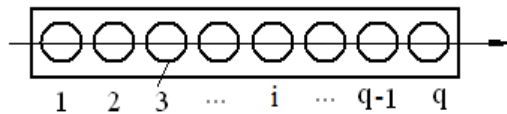


Fig. 3. Linear Type Automated Line with q Station of Serial Workstation

$$P(f) = P(f_{s,1}) + P(f_{s,2}) + P(f_{s,3}) + \dots + P(f_{s,q}) + P(f_{tr}) + P(f_c) \tag{4}$$

$$\lambda = \lambda_{s,1} + \lambda_{s,2} + \lambda_{s,3} + \dots + \lambda_{s,q} + \lambda_{tr} + \lambda_c \tag{5}$$

Where

$P(f)$ = Probability for total failure rate

$P(f_{s,1})$ = Probability for failure of workstation 1

$P(f_{s,2})$ = Probability for failure of workstation 2

$P(f_{s,3})$ = Probability for failure of workstation 3

$P(f_{s,q})$ = Probability for failure of workstation q

$P(f_{tr})$ = Probability for failure of transport system

$P(f_c)$ = Probability for failure of automated line control system

λ = Total of failure rate of automated line

$\lambda_{s,1}$ = Failure rate of workstation 1

$\lambda_{s,2}$ = Failure rate of workstation 2

$\lambda_{s,3}$ = Failure rate of workstation 3

$\lambda_{s,q}$ = Failure rate of workstation q

- λ_{tr} = Failure rate of transport system
- λ_c = Failure rate of automated line control system

For linear type with parallel and serial-parallel action that shows in the Fig. 4(a) and 4(b) below consist of different mathematical expression due to different working properties.

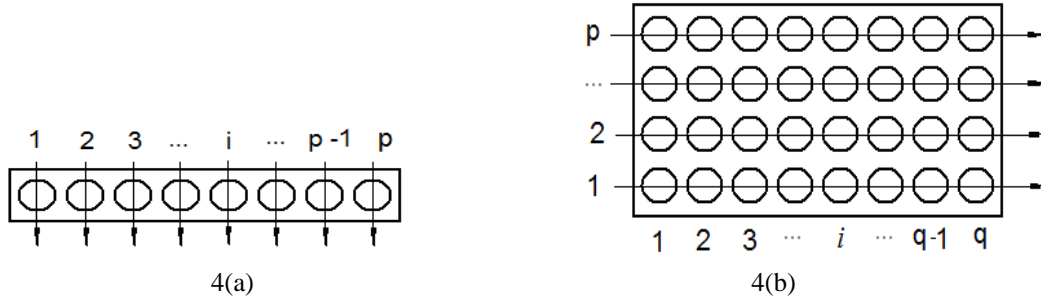


Fig 4. (a) Parallel Action of Linear Arrangement Automated Line with P Stations, (b) Linear-Type of Serial Parallel Action Automated Line with q Serial Stations and p Parallel Stations

For parallel action with linear arrangement automated line, all the machines are the same in all aspect so that this automated is expressed in one of probability times p station with the failure of transportation system due to feeder or transport system in this type of line. The probability of this type of line is expressed in equation 6 while equation 7 presents the failure rate expression and p is the number of parallel station in automated line.

$$P(f) = p \times [P(f_s) + P(f_{tr})] + P(f_c) \tag{6}$$

$$\lambda = p \times [\lambda_s + \lambda_{tr}] + \lambda_c \tag{7}$$

For the serial parallel action with linear arrangement consists of different failure rate in the q serial workstation but same failure rate in p parallel workstation. The mathematical expression for serial parallel action is shows in equation 8 including the transportation system and control system. Failure rate of serial parallel for linear automated line is shows in equation 9.

$$P(\lambda) = p \times [P(\lambda_{s,1}) + P(\lambda_{s,2}) + P(\lambda_{s,3}) + \dots + P(\lambda_{s,q})] + P(\lambda_{tr}) + P(\lambda_c) \tag{8}$$

$$\lambda = p \times [\lambda_{s,1} + \lambda_{s,2} + \lambda_{s,3} + \dots + \lambda_{s,q}] + \lambda_{tr} + \lambda_c \tag{9}$$

For rotor-type serial action, the model of calculation is same with the linear-type serial action since the technological and technical attributes are similar and only different in arrangement. The probability expression of rotor-type serial action automated line is same with the linear type serial action. The second working action for rotor type is parallel action where all the workstation in the automated line is same and the failure rate is multiple with p station. Rotor-type parallel action automated line is shown in Fig. 5 below and the probability expression is presented in equation 10 and failure rate express in equation 11 below.

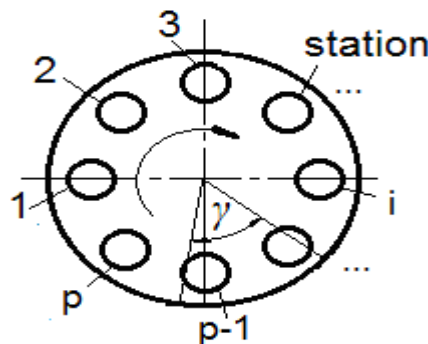


Fig 5. Rotor-Type in Single Pure Parallel Action Automated Lines

$$P(\lambda) = p \times P(\lambda_s) + P(\lambda_{tr}) + P(\lambda_c) \tag{10}$$

$$\lambda = p \times \lambda_s + \lambda_{tr} + \lambda_c \tag{11}$$

Where

P = Number of parallel workstation in a rotor

$P(\lambda)$ = Probability for total failure

$P(\lambda_s)$ = Probability for one workstation failure in rotor

$P(\lambda_{tr})$ = Probability for transport failure in rotor

$P(\lambda_c)$ = Probability for automated line control system failure in rotor

λ = Total of failure rate of automated line

λ_s = Failure rate of one workstation in rotor

λ_{tr} = Failure rate of transport system

λ_c = Failure rate of automated line control system

Since the pure parallel action automated lines are seldom exist in single line, the third rotor-type which is serial-parallel action is widely use in the industrial production system for mass and fast production. The serial-parallel action of rotor-type automated line is shown in Fig. 6 below.

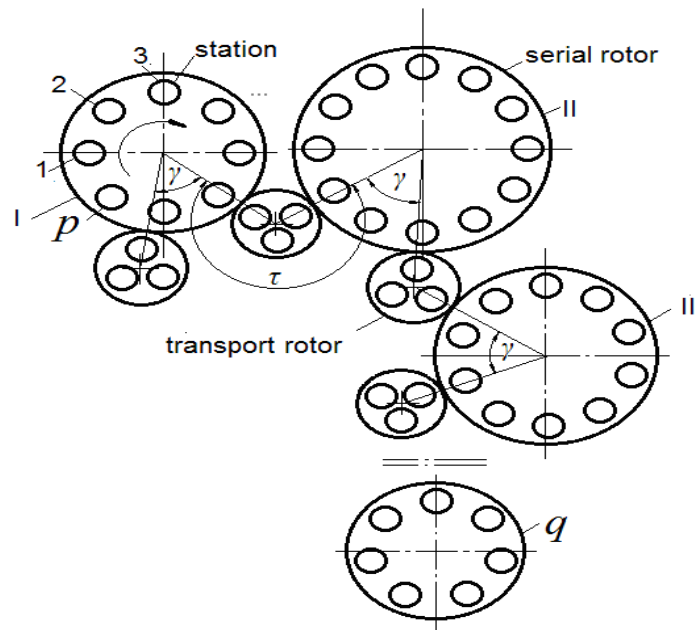


Fig. 6. Serial-parallel Action of Rotor-type Automated Production Line

This rotor-type with serial parallel action is considered as complicated or complex design of automated line in industry. This line is the combination of serial action and parallel rotor to increase the productivity. The number of q working rotors and transport rotors in serial arrangement consist of different failure rate due to the different workstation and working properties. However, the p number of workstation in each of rotor is considered as parallel action which is working for all similar workstation with the same working process. The failure rate for q working rotors and transport rotors are required to multiply by the number of parallel station, p. The failure of control system for this type is considered in one group of probability due to the same control system in the whole automated line and the equation of probability failure for serial parallel of rotor type is express in equation 12 while equation 13 shows the failure rates equation below.

$$P(\lambda) = p \times [P(\lambda_{s,1}) + P(\lambda_{s,2}) + \dots + P(\lambda_{s,q}) + P(\lambda_{tr,1}) + P(\lambda_{tr,2}) + \dots + P(\lambda_{tr,q})] + P(\lambda_c) \quad (12)$$

$$\lambda = p \times [\lambda_{s,1} + \lambda_{s,2} + \lambda_{s,3} + \dots + \lambda_{s,q} + \lambda_{tr,1} + \lambda_{tr,2} + \lambda_{tr,3} + \dots + \lambda_{tr,q}] + \lambda_c \quad (13)$$

Where:

$P(\lambda_{s,1})$ = Probability for serial rotor 1 failure

$P(\lambda_{s,2})$ = Probability for serial rotor 2 failure

$P(\lambda_{s,3})$ = Probability for serial rotor 3 failure

$P(\lambda_{s,q})$ = Probability for serial rotor q failure

$P(\lambda_{tr,1})$ = Probability for transport rotor 1 failure

$P(\lambda_{tr,2})$ = Probability for transport rotor 2 failure

$P(\lambda_{tr,3})$ = Probability for transport rotor 3 failure

$P(\lambda_{tr,q})$ = Probability for transport rotor q failure

$\lambda_{s,1}$ = Failure rate of serial rotor 1

$\lambda_{s,2}$ = Failure rate of serial rotor 2

$\lambda_{s,3}$ = Failure rate of serial rotor 3

$\lambda_{s,q}$ = Failure rate serial rotor q

$\lambda_{tr,1}$ = Failure rate of transport rotor 1

$\lambda_{tr,2}$ = Failure rate of transport rotor 2

$\lambda_{tr,3}$ = Failure rate of transport rotor 3

$\lambda_{tr,q}$ = Failure rate of transport rotor q

Since the theory of probability is applied in the aspect of reliability, it can show clearly the different levels of reliability regarding to the workstation, transportation and control system in rotor-type automated line. The analysis of the failure rate for all common types of automated line in industry is useful for further research about productivity. The reliability part of mathematical model which is availability is applied by this probability methodology to improve the current mathematical model with average level of reliability.

IV. CONCLUSION

Analysis of the failure rate or breakdown in automated line by using the theory of probability has been done to express the probability of failure and failure rate of the automated line into probability mathematical equation. The probability equation for each type of automated line is clearly stated and expressed regarding to the working action and type of arrangement of workstation. The expression of equation in terms of mathematically is useful for engineer or researcher to calculate or forecast the failure rate of the automated line and for improvement of productivity mathematical model to obtain a more robust model due to the consideration of failure rate in distinguishes categories. Simulation and validation are required in future to be applied to the probability failure rate model expression. Improvement of the probability in terms of failure rate in automated line is important as future work.

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