Development of a method of ICP algorithm accuracy improvement during shaped profiles and surfaces control

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Abstract. In this paper we propose a method of improvement of operating accuracy of iterative closest point algorithm used for metrology problems solving when determining a location deviation. Compressor blade profiles of a gas turbine engine (GTE) were used as an object for application of the method of deviation determining. It is proposed to formulate the problem of the best alignment in the developed method as a multi-objective problem including criteria of minimum of squared distances, normal vectors differences and depth of camber differences at corresponding points of aligned profiles. Variants of resolving the task using an integral criterion including the above-mentioned were considered. Optimization problems were solved using a quasi-Newton method of sequential quadratic programming. The proposed new method of improvement of the registration algorithm based on geometric features showed greater accuracy in comparison with the discussed methods of optimization of a distance between fitting points, especially if a small quantity of measurement points on the profiles was used.

Keywords: ICP-algorithm; profiles alignment ; least squares method; NURBS; nonlinear optimization, profile curvature.

1. INTRODUCTION

Today the aerospace and automotive industries manufacture products with a shaped form of surface and high production accuracy requirements. Requirements for the form of such components are dictated by numerous functional requirements as well as by aesthetic considerations. Examples of details with a shaped form of surface are airfoils in the aerospace industry (compressor and turbine blades), body parts in the automotive industry, dies, molds.

When controlling such items production problems with adequacy of their shape assessment arise. At measurements a part is compared to the CAD-model of the part. The main tool for simulation of shaped surfaces in the CAD-system is NURBS (not rational B-spline). As a result of coordinate metrology a cloud of points, which are compared with the characteristic points of the CAD-model, is loaded into CMM system. Moreover the shape of the measured part can be evaluated only after evaluation of the location deviation (alignment of the measured part coordinate system with the CAD model).

In most software programs the best alignment of parts with shaped form of surface a complicated surface shape in software for CMM is performed using iterative closest point algorithm (ICP) based on the least squares method [1].

In the practice of compressor blades of gas turbine engines production the blade airfoil geometry is critical for engine performance. In order to evaluate the geometrical parameters characterizing the shape of blades accurately it is necessary to align a measured part with its CAD-model accurately and adequately.

In the most common terms, the algorithm is an iterative algorithm that solves a problem (task) of optimal rotation and displacement for alignment of the point cloud to the CAD-model nominal points. The problem consists in finding the minimum of the following function:

$$f(R,t) = \frac{1}{n}\sum_{i=1}^{n} ||Rp_i + t - q_i||^2 \quad , \qquad (1)$$

where n - number of fitting points (measured points);

p_i - coordinates of the i^{th} measured point;

R - point rotation matrix;

t - displacement vector;

q_i – coordinates of a point on the CAD-model.

The key point of fitting is to find t and R of a measurement profile relative to the nominal profile and to convert coordinates of the measurement profile, eliminating the location error.

Coordinates of a point of a real profile fitted to the points of the nominal profile can be found by the matrix product of coordinates of real profile points of a transformation matrix:

$$P_{np} = P \cdot M, \tag{2}$$

where M - fitting matrix, which has the following form:

$$M = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ t_x & t_y & 1 \end{bmatrix}, \tag{3}$$

where α - angle of profile rotation, t_x, t_y - elements of the matrix t.

Form deviation is calculated as the deviation of points of the nominal profile and "fitted" measurement profile.

There is a lot of literature concerning methods of improvement of the traditional ICP method for the best fit of measured and nominal parts [2].

For the purposes of elimination of the parts location deviation, there should be corresponding points on the measured part and the CAD-model. Existing search methods are mainly based on selection of the nearest points on the measured and nominal model. This imposes a limit in the sense that localization can operate correctly when the condition of closeness of the measured component coordinate system of the model coordinate system is fulfilled. Ainsworth in the work [3] applied manual adjustment to find a number of relevant points of the measurement surface and the model surface to make a rough alignment in order to ensure good conversion accuracy in cases where the model and the measured item were roughly offset initially. Then ICP was used for subsequent alignment.

Another method is to divide points into groups according to their location. Search for the nearest neighboring points is accelerated because of elimination of a set of points in the calculation and reduction of the amount of computation of distances between points. One way to do perform this search is to use a method of k-dimensional decision trees, considered in relation to the problem of objects aligning by Bentley in the works [4,5].

In the work [6] Balasubramanian et al. proposed to optimize the ICP algorithm using self-organizing neural network. The neural network was trained on transformations data in a homogeneous form, and then it determined relative transformation parameters. According to the authors, the use of the neural network allowed to reduce computation time and to improve obtained data stability.

The problem of accuracy of location deviation parameters calculation is first of all associated with a discrepancy of measured points to points on the CAD-model with a location deviation. In this paper we consider algorithms of alignment allowing to overcome a discrepancy of points that part in the ICP algorithm. The computational of the basic ICP method, discussed in the first part of Chapter 3, provided similar results with the data of fitting of similar profiles in the software PC-DMIS.

2. MATERIALS AND METHODS

2.1 Formulation of an optimization problem of appropriate points search for the best alignment

This section describes the method that allows to overcome the problem of non-compliance of points on the nominal profile with the measured points. In mathematical terms the problem is to select the location of control points on the nominal profile curve so that to minimize objective functions characterizing the accuracy of profiles alignment.

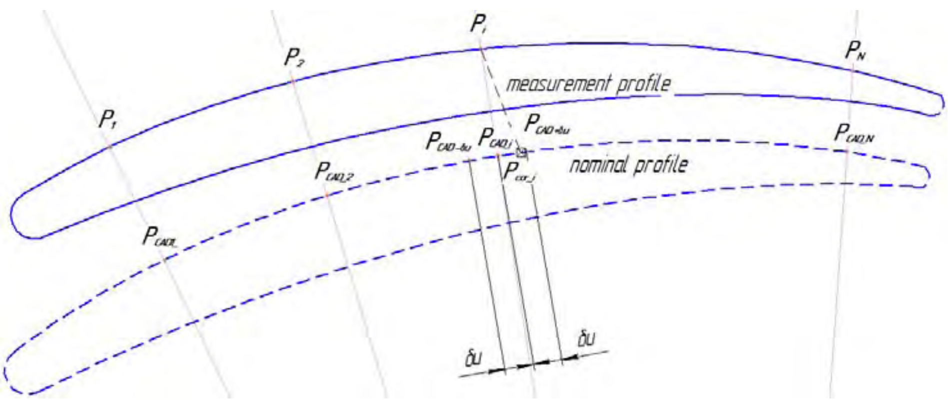

Fig. (1). Model of optimization in case of appropriate points of the nominal profile search by the measurement profile.

Profiles (nominal and measurement) curves are presented in the form of NURBS-curves. This is the standard of curves and surfaces representation in CAD-systems [7]. NURBS-curves are piecewise splines, which mathematically can be represented as:

$$P(u) = \frac{\sum_{i=0}^{n} h_i P_i N_{i,k}(u)}{\sum_{i=0}^{n} h_i N_{i,k}(u)} \quad , \tag{4}$$

where P_i - vector (x_i, y_i, z_i), combining the coordinates of the i-th defining point in the three-dimensional space, as well as for non-rational B-splines. Parameter value range is [t_{k-1}, t_{n+1}]. h_i - points weight coefficients.

By adjusting the parameters of the points u_i by the values of Δu_i new coordinates of the points on the curve can be calculated. The problem of finding of corresponding points is the optimization task. Multiparameteric optimization problem consists in finding of a minimum of a nonlinear function with constraints. The paper proposes to use three criteria as objective functions in the above optimization problem. The first criterion is the sum of distances between points after alignment according to the basic ICP algorithm. The second and third criteria are respectively the sum of angles between the normal vectors and depth of camber differences at corresponding points of aligned profiles.

2.2 Mathematical description of the proposed optimization methods

The objective function, which is the sum of distances between the points after alignment according to the basic ICP algorithm, has the following form:

$$f(\Delta u) = \sum_{i=1}^{N} \left\| P_{\text{н}i}(u_i + \Delta u_i) - P_{bfi} \right\| \to min, \tag{5}$$

where $P_{\text{н}i}(u_i + \Delta u_i)$ - coordinates of points on the nominal profile, defined by the parameter $u_i + \Delta u_i$;

P_{bfi} - points on the fitted profile according to the ICP method.

The objective variable is Δu_i. The set of constrains for the object can be represented as follows:

$$\begin{cases} lb_1 \le \Delta u_1 \le ub_1, \\ \qquad \dots \\ lb_i \le \Delta u_i \le ub_i. \end{cases} \tag{6}$$

where, lb, ub - upper and lower limits for the constructed parameter u.

Now let’s consider optimization criteria, based on the sum of absolute differences of tilt angles of normals and curvature values at points of the CAD-model and the fitted profile according to the ICP algorithm.

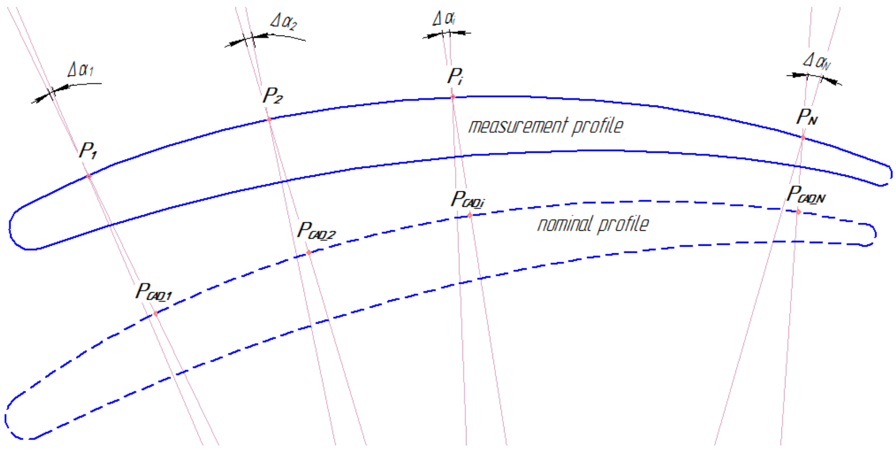

Fig. (2). Angular difference between the normals to the CAD-model and the measurement profile at fitting points

Until the profile is not aligned with the nominal in the best way, angles between the normals in the corresponding points are large. After the best alignment, provided that there is a low shape deviation, the difference of normals angles will tend to 0.

The same is true for curvature of appropriate points of the CAD-model and the measurement profile. The curvature of their surfaces and profiles of parts with a shaped surface form is an important characteristic. The higher the curvature is the more difficult it is to treat the surface and respect all necessary tolerances. Details of the type of gas turbine engine blades, molds, dies, a number of body parts have a significant curvature. In case of the best selection of nominal point corresponding to appropriate measured points their curvature will be close. If an item has a deviation of the shape, the curvature at corresponding points of the measurement and the nominal profile will be different, as well as the angle of normals tilt. But the sum of absolute values of curvature deviations at corresponding points will be minimal.

Thus, using the curvature and tilt angles of normals at points as criteria, it is possible to form two optimization functions:

$$\begin{cases} f_\alpha(\Delta u) = \sum_{i=1}^{N} \Delta\alpha_i(u_i + \Delta u_i) \to min \\ f_k(\Delta u) = \sum_{i=1}^{N} \Delta k_i(u_i + \Delta u_i) \to min' \end{cases} \quad (7)$$

where, $\Delta\alpha_i(u_i + \Delta u_i)$ – absolute difference of normals tilt angles at profiles points;

$\Delta k_i(u_i + \Delta u_i)$ – absolute difference of curvature corresponding profiles points. The formula for calculation of curvature of the curve *f(u)* given parametrically has the following form [8]:

$$k = \frac{|f_x'(u)\cdot f_y''(u) - f_x''(u)\cdot f_y'(u)|}{((f_x'(u))^2 + (f_y'(u))^2)^{\frac{3}{2}}}. \quad (8)$$

We can move from multi-objective optimization problem to the problem of single-objective optimization combining the objective functions into a single function, which has the form

$$f(\Delta u) = \sum_{i=1}^{N} \Delta\alpha_i(u_i + \Delta u_i) \cdot \Delta k_i(u_i + \Delta u_i) \to min. \quad (9)$$

2.3 Methods for optimization problems solving used in the work

Thus, we have the conditional nonlinear optimization problem consisting of a nonlinear objective function and linear constraints.

Let's denote the constraints (6) as a system of inequalities $g(\Delta u) \le 0$.. If the vector Δu^* under the imposed conditions is the solution of the problem, then there is a positive vector of Lagrange multipliers $\lambda \in R^m$ such that for the Lagrange function [9]:

$$L(\Delta u) = f(\Delta u) + \sum_{i=1}^{m} \lambda_i \cdot g_i(\Delta u_i) \to min\ , \quad (10)$$

the following conditions are fulfilled:

$$\begin{cases} \min\,(L(\Delta u)) = L(\Delta u^*), \\ \lambda_i \cdot g_i(\Delta u^*) = 0, i = 1 \dots m, \\ \lambda_i > 0, i = 1 \dots m. \end{cases} \quad (11)$$

The optimization problem was solved by a method of sequential quadratic programming (SQP). The method allows to accurately simulate Newton's method for constrained optimization [10]. Newton's method is based on finding of a zero gradient of the objective function $\nabla f(\Delta u)$ and reduces to the solution of the following equations [11]:

$$\nabla f(\Delta u_j) + H(\Delta u_j) \cdot (\Delta u_{j+1} - \Delta u_j) = 0, j = 1,2, \dots, n, \quad (12)$$

where $\boldsymbol{H}(\Delta \boldsymbol{u_j})$ – Hessian of the function $\boldsymbol{f}(\Delta \boldsymbol{u})$, $\boldsymbol{H}(\Delta \boldsymbol{u_j}) = \left[\frac{\partial^2 \boldsymbol{f}(\Delta \boldsymbol{u})}{\partial \Delta \boldsymbol{u}^2}\right]$.

In the SQP method at each major iteration Hessian approximation takes place for the Lagrangian using a modified quasi-Newton method.

Alignment accuracy can be estimated using the relative values of its parameters comparison [12]. In this paper, the accuracy of alignment parameters search is characterized by the relative indicators: deviation of the profile rotation matrix and deviation of transposition matrix.

Deviation of the rotation matrix is calculated as:

$$e_R = \frac{|R - R'|}{|R|} \cdot 100\%. \quad (10)$$

where R - planted profile rotation matrix;

R '- calculated profile rotation matrix as a result of the algorithm.

Deviation of the transposition matrix is calculated as follows:

$$e_T = \frac{|T - T'|}{|T|} \cdot 100\%. \quad (11)$$

where T – planed profile transposition matrix;

T '- calculated profile transposition matrix as a result of the algorithm

3. RESULTS

GTE compressor blade profile was used as the simulation object. Different values of profile rotation and displacement were considered, coordinates of profile points with a location deviation (formula 2) and with a form deviation were calculated from coordinates of point of the nominal profile. A cloud of points of the nominal and measurement profiles was approximated by NURBS-splines of the 3 degrees. X offset varied from 0.1 to 0.3 mm, y offset varied in the range of 0.1-0.5 mm, the angle of turn was 0.1 - 0.7 degrees. Overlay deviation of the profile shape has a limit of 38.5 microns.

Table 1 shows the results of errors of the rotation and transposition matrix of the profile e_t and e_r, which are obtained using the ICP method, implementing the best alignment according to arrays of measured and nominal points.

Table 1. Relative errors of alignment parameters of a profile

Ranges of variation of location deviation along y axis and turn angle	Form deviation value, mm	X offset 0,1mm e_T ,%	X offset 0,1mm e_R ,%	X offset 0,3 mm e_T ,%	X offset 0,3 mm e_R ,%	Ranges of variation of location deviation along x and y axes	Turn angle 0,1 degrees e_T ,%
y=0,1..0,5 mm	0	9,05	3,03	4,37	3,08	y=0,1..0,5 mm	7,76
α=0..0,7 degrees	0,0039	9,42	3,17	4,53	3,21	x=0,1..0,5 mm	8,13
	0,0108	10,09	3,41	4,83	3,46		8,80
	0,0177	10,76	3,41	5,12	3,71		9,45
	0,0246	11,43	3,64	5,41	3,95		10,12
	0,0316	12,10	3,86	5,71	4,20		10,79
	0,0385	12,77	4,08	6,00	4,12		11,45

Relative errors parameters resulting from optimization of the ICP algorithm using the target function (5) are shown in Table 2.

Table 2: Relative errors of alignment parameters of a profile using the method of optimization of location of points of the nominal profile according to the first criterion

Ranges of variation of location deviation along y axis and turn angle	Form deviation value, mm	X offset 0,1mm e_T ,%	X offset 0,1mm e_R ,%	X offset 0,3 mm e_T ,%	X offset 0,3 mm e_R ,%	Ranges of variation of location deviation along x and y axes	Turn angle 0,1 degrees e_T ,%
y=0,1..0,5 mm	0	1,20	0,06	1,21	0,34	y=0,1..0,5 mm	0,20
α=0..0,7 degrees	0,0039	1,08	0,15	1,21	0,11	x=0,1..0,5 mm	0,49
	0,0108	1,41	0,44	1,35	0,44		1,41
	0,0177	2,59	0,80	1,52	0,71		2,59
	0,0246	3,39	1,03	1,84	1,04		3,39
	0,0316	4,36	1,32	2,22	1,36		4,36
	0,0385	5,49	1,66	2,61	1,60		5,49

At a relatively low deviation of the form the algorithm gives values close to the values of the base method. For worse fabricated profiles the algorithm gives more accurate solutions regarding the transposition matrix - the error is reduced approximately by a factor of 3. However, the use of the first criterion in the optimization function did not solve the problem of increase of profiles alignment errors with increase of shape deviation, although it gave more accurate data compared to the results of the basic ICP method.

Relative parameters obtained as a result of ICP algorithm optimization using the objective function (9) are shown in Table 3.

Table 3. Relative errors of alignment parameters of a profile using the method of optimization of location of points of the nominal profile at the objective function of normals curvature-deviation

Ranges of variation of location deviation along y axis and turn angle	Form deviation value, mm	X offset 0,1mm		X offset 0,3 mm		Ranges of variation of location deviation along x and y axes	Turn angle 0,1 degrees
		e_T ,%	e_R ,%	e_T ,%	e_R ,%		e_T ,%
y=0,1..0,5 mm	0	1,19	0,01	1,21	0,02	y=0,1..0,5 mm	0,17
α=0..0,7 degrees	0,0039	1,19	0,34	1,21	0,02	x=0,1..0,5 mm	0,49
	0,0108	1,21	0,22	1,21	0,44		0,23
	0,0177	1,47	0,43	1,21	0,42		1,47
	0,0246	1,44	1,16	1,21	0,42		1,44
	0,0316	1,42	0,31	1,40	0,87		0,16
	0,0385	1,09	0,06	2,22	0,19		0,17

Errors obtained when solving the non-linear function of normals angles difference and curvature in respective points of the profile do not increase with increase of form deviation. We can say that in case of optimization with the objective function (12) the solution result does not change significantly.

4. DISCUSSION

The proposed variants of implementation of the method allow to improve the accuracy of determining of the profile rotation and transposition matrixes by a factor of 2.5-3 at maximum simulated form error of 38.5 microns. At minimum form deviation the error e_R was reduced by a factor of 30-300.

The influence of shape deviations on the accuracy of calculation of location deviation parameters was minimized by minimization of deviations of normales tilt angles and curvature at points of the aligned profile and the CAD-model (Table 4). The proposed new method can significantly (up to 5 times) improve alignment accuracy of the measurement profile with its model at form deviation of 38.5 microns. It is the method of optimization with respect to geometric features which is the most applicable in practice as during parts manufacturing form deviations are inevitable.

5. CONCLUSIONS

This paper proposes and confirms the method of ICP algorithm accuracy improvement based on the solution of a multi-objective optimization problem used for identification of location deviation parameters for profiles and surfaces with shaped form of surface.

The main reason of an error of location deviation parameters determination is in discrepancy of points of the nominal and measurement profiles, which are used for the best alignment using the method of least-squares. Profiles shape deviation, depending on its degree deteriorates alignment by 5-40%.

In the proposed method, we considered the use of two criteria. The first criterion uses distance between aligned points of the measured profile and points of the nominal profile. The second criterion used geometric features of a measuring object, such as its curvature and tilt angle of normals at points. The studies revealed that in case of high form deviations the use of the second criterion is more accurate, but it is more time consuming.

6. ACKNOWLEDGEMENT

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