Channel Estimation in MIMO Ad Hoc Network with Binomial Node Distribution and Transmit Antenna Selection

K.Rajeswari #1, M.Anitha #2, S.J.Thiruvengadam #*3
# Department of ECE, Thiagarajar College of Engineering, Madurai, 625015, India
1 rajeswari@tce.edu
2 anithatcedgl@gmail.com
* TIFAC CORE in Wireless Technologies, Thiagarajar College of Engineering, Madurai, 625015, India
3 sjtece@tce.edu

Abstract— Reliable channel estimation is an essential task in the development of receivers for multiple input multiple output (MIMO) finite ad hoc networks. The major problem in the estimation of channel coefficients in a MIMO ad hoc network is the interference from the other nodes. In this paper, channel estimation is performed for a finite MIMO ad hoc network where transmitter employs antenna selection. The spatial node distribution in a finite network, containing finite number of nodes in a finite region is characterized using the binomial point process. Linear Minimum Mean Square Error (LMMSE) algorithm is developed for estimating channel coefficients using training sequences. The algorithm uses the spatial correlation of receive antennas. The MSE performance of proposed LMMSE channel estimator for MIMO ad hoc network in the interference limited environment is analysed through simulations.

Keywords— Binomial Point Process, Guard Zone, Linear Minimum Mean Square Error, Multiple Input Multiple Output (MIMO), Transmit Antenna Selection.

I. INTRODUCTION

Wireless ad hoc networks are considered as an important part of next generation wireless communication systems. Since there is no centric controller in the network, the performance of the ad hoc network is constrained by the interference from other users [1]. Recently, finite area networks with node positions modeled using a Poisson Point Process (PPP) have been studied for applications such as personal or local area networks [2-4]. However, a practical network is often finite in terms of both the number of nodes and the service area, which is referred as a finite network. In such a finite ad hoc network, a Binomial Point Process (BPP), rather than a PPP, models the node distribution [5-7].

In a homogeneous PPP, there is finite number of nodes in a finite region. The number of nodes in any subset of this region follows a binomial distribution. In PPP, realizations of the process may have more number of nodes than the number of nodes deployed. Hence, when number of nodes is known, PPP is not a good model. Further, if all the nodes are presented in certain part of network area, the remaining area is empty; PPP does not capture this fact. There is a need to study and characterize finite uniformly random network and model the network using the more realistic case of BPP. Binomial network finds application in mobile ad hoc networks, sensor networks and wireless networks with infrastructure, such as cellular telephony networks.

It is proved that multiple input multiple output (MIMO) systems are capable of managing interference and improving link reliability through spatial multiplexing and diversity techniques [8]. To harness the advantage of MIMO technologies, ad hoc networks with each node equipped with multiple antennas, are proposed and investigated [9-13]. However, to exploit the benefits of MIMO systems, accurate Channel State Information (CSI) is needed at the transmitter and/or receiver [14]. For example, the performance of transmit beamforming depends on the accuracy of the CSI at the transmitter. The performance of space time decoders is also entirely determined by the availability of an accurate CSI at the receiver. Hence, accurate channel estimation plays a key role in MIMO communications [15-17]. One of the most popular and widely used approaches for MIMO channel estimation is to employ training sequences for estimation.

There are several methods of channel estimation for flat and frequency selective fading MIMO channels. The Maximum Likelihood (ML) method using orthogonal pilot signals is applied for estimation of MIMO channels [17]. Further study of this estimator is reported in [18]. Least Squares (LS) channel estimation technique is developed for systems with multiple transmit antennas [19]. The Linear Minimum Mean Square Error (LMMSE) channel estimation technique improves the Mean Square Error (MSE) performance of the conventional LS method [20]. Optimal training signals are developed for channel estimation schemes, including the LS and
LMMSE for systems with multiple transmit antennas and single receive antenna [21, 22]. Orthogonal training sequence is employed for LMMSE estimator in [23, 24].

Since LMMSE is an optimal technique for channel estimation in wireless systems and channel estimation for ad hoc network characterized using BPP is not found in the literature, this paper aims to develop an optimal channel estimator for MIMO ad hoc network with transmit antenna selection. Transmit antenna selection is performed based on the channel gain between transmit and receive antennas. The antenna selection information is transmitted through a control channel to the receiver [27]. The channel state information between the selected transmit antenna and the receiver is estimated using the antenna selection information.

The paper is organized as follows. In Section II, the system model is described. In Section III, proposed LMMSE channel estimator is derived for finite ad hoc network. Simulation results are presented in section IV to analyse the MSE performance of proposed LMMSE estimator. Section V concludes the paper.

II. SYSTEM MODEL

Consider a finite ad hoc network with finite number of nodes $N_n$ in finite area $\mathcal{R}$ represented by 2-D Euclidean space as shown in Figure 1. The location of transmitting nodes is modeled using Binomial Point Process (BPP). The set $\Phi$ represents the independent and uniformly distributed nodes in a finite area $\mathcal{R}$. The subset constructed from the finite area $\mathcal{R}$ is known as Borel subset $\tilde{\mathcal{B}}$. The number of nodes of $\Phi$ falling in $\tilde{\mathcal{B}}$ is represented as $\Phi(\tilde{\mathcal{B}})$. The probability that $\Phi(\tilde{\mathcal{B}}) = n$, follows a binomial distribution. It is given by

$$\Pr(\Phi(\tilde{\mathcal{B}}) = n) = \binom{n}{N_{\infty}} \tilde{p}^n (1-\tilde{p})^{N_{\infty}-n}, n = 0,..,N_{\infty}-1$$

where $\tilde{p} = |\tilde{\mathcal{B}}| / |\mathcal{R}|$ denotes the ratio of the area of $\tilde{\mathcal{B}}$ to that of $\mathcal{R}$.

A test transceiver is deployed in $\mathcal{R}$, which itself is not a part of the BPP. At the origin $O$ of a circular area of radius $R$ within $\mathcal{R}$ consist of test receiver. In order to avoid the power of interfering signals from outside this circular area, the radius $\mathcal{R}$ is chosen to be sufficiently large. The guard zone is represented within concentric circular area with radius $\epsilon$, where interfering transmissions are neglected. Consequently, the interferer within the corresponding annular area denoted as $\tilde{\mathcal{B}}$ cause effect on the test receiver. The set of interferers within $\tilde{\mathcal{B}}$ is referred as $\Phi \cap \tilde{\mathcal{B}}$.

The distance between test transceiver is considered as $r_0$. The distance between the $i^{th}$ interferer and the test receiver is denoted as $r_i$, $\forall i \in \Phi \cap \tilde{\mathcal{B}}$ and $r_i$’s are independent identically distributed (i.i.d) random variables with a common cumulative distribution function [9]

$$\Pr(r_i \leq r) = \frac{r^2 - \epsilon^2}{R^2 - \epsilon^2}, \ \epsilon \leq r \leq R$$
Each node in the network is assumed to be equipped with $n_t$ transmit antennas and $n_r$ receive antennas. Inter node channels are assumed to be flat Rayleigh fading channel with i.i.d. circularly symmetric complex Gaussian random variables of zero mean and unit variance.

The spatial correlations of MIMO channels are modeled by a Kronecker structure with separable transmit and receive antenna correlations. The channel matrix $G$ between transmitter and receiver arrays is given by

$$G = R_s^{1/2} G_w (R_r^{1/2})^T$$  \hspace{1cm} (3)$$

where $R_s$ is a $n_r \times n_r$ receive correlation matrix, $R_t$ is a $n_t \times n_t$ transmit correlation matrix and $G_w$ is a $n_r \times n_t$ matrix with i.i.d circular symmetric complex Gaussian with zero mean and unit variance. The channel vector is given by

$$g = vec(G)$$  \hspace{1cm} (4)$$

One of the $n_t$ transmit antennas is selected for transmission. The criterion for selecting $j^{th}$ transmit antenna is given by

$$j = \arg \max_{j \in [1, \ldots, n_t]} \left\{ \sum_{m=1}^{n_r} \| g_m \|^2 \right\}$$  \hspace{1cm} (5)$$

Let $x_0$ and $x_i$ denote the $T_t \times 1$ signal vectors transmitted from the test transmitter and $i^{th}$ interferer respectively over the time $T_t$, $\forall i \in \Phi n \in \tilde{B}$. The $T_t n_r \times 1$ received signal vector at the test receiver is given by

$$y = r_0^{-\alpha/2} x_0 g_s + \sum_{i \in \Phi n \in \tilde{B}} r_i^{-\alpha/2} x_i g_s + w$$  \hspace{1cm} (6)$$

where $X_0$ is given by $X_o = x_o \otimes I_{n_r}$ and $X_i = x_i \otimes I_{n_r}$. The $T_t n_r \times 1$ vector $w$ is additive white Gaussian noise with mean zero and covariance matrix $\sigma^2 \alpha I_{n_r}$. As a part of the channel modeling, path loss is included where path loss exponent is given by $\alpha (\alpha \geq 2)$. The $n_r \times 1$ vector $g_s$ is selected channel vector with the criterion given in (5), given by

$$g_s = S g$$  \hspace{1cm} (7)$$

where $S = I_{n_r} \otimes s$ and $s$ is a $1 \times n_t$ vector with its $j^{th}$ element as 1 if $j^{th}$ antenna is selected for transmission. It is assumed that transmit antenna selection is employed in all the interfering nodes. The $n_r \times 1$ vectors $g_s$ are selected channel vectors from the interfering nodes.

### III. Proposed Channel Estimation Algorithm

The LMMSE estimation of the channel $h$ is given by

$$\hat{g_s} = Qy$$  \hspace{1cm} (8)$$

where $Q$ is the LMMSE transformation matrix and $Q$ is derived by minimizing the mean square error between desired and estimated channel coefficients. It is defined as

$$g_{MSE} = Tr \left( E \left[ \| g_s - \hat{g_s} \|^2 \right] \right)$$  \hspace{1cm} (9)$$

The term $E \left[ \| g_s - \hat{g_s} \|^2 \right]$ is expanded as

$$E \left[ \| g_s - \hat{g_s} \|^2 \right] = E \left[ (g_s - \hat{g_s})^H (g_s - \hat{g_s}) \right]$$  \hspace{1cm} (10)$$

Equation (10) is rewritten as

$$E \left[ \| g_s - \hat{g_s} \|^2 \right] = E \left[ g_s g_s^H \right] - E \left[ g_s \hat{g_s}^H \right] - E \left[ g_s^H \hat{g_s} \right] + E \left[ \hat{g_s} \hat{g_s}^H \right]$$  \hspace{1cm} (11)$$

The first term in (11) is

$$E \left[ g_s g_s^H \right] = R_s$$  \hspace{1cm} (12)$$

The Kronecker model for spatial MIMO channel, models the correlation matrix $R_s$ of the channel as a Kronecker product of receive and transmit correlation matrices as [28]
The receive and transmit correlation matrices are given by

\[ R_{rr} = J_0 \left( \frac{2\pi}{\lambda} d_{r(i,j)} \right) \]

where \( J_0(x) \) is the Bessel function of the first kind of the zeroth order, in which \( \Delta \) is the angle spread, \( \lambda \) is the wavelength and \( d_{i,j} \) are the distance from the antenna element \( i \) to antenna element \( j \) in the transmitter and receiver respectively. As only one antenna is selected for transmission, \( R_{sr} = R_r \). The second and third terms are simplified as

\[ E \left[ g_i^H g_i^H \right] = r_i^{-\alpha/2} Q^H R_{rr} X_i^H \]
\[ E \left[ g_i^H g_i^H \right] = r_i^{-\alpha/2} X_i R_{sr} Q \]

respectively. Substituting (8) in the fourth term of (11) results in

\[ E \left[ g_i^H g_i^H \right] = r_i^{-\alpha/2} Q^H X_i R_{rr} X_i^H + E \left[ Q^H \left( \sum_{\substack{i \neq j \in \Phi}} r_{ij}^{-\alpha} X_i g_i g_j X_j^H \right) \right] + \sigma_q^2 Q^H Q \]

The middle term in (16) can be rewritten as

\[ E \left[ Q^H \left( \sum_{\substack{i \neq j \in \Phi}} r_{ij}^{-\alpha} X_i g_i g_j X_j^H \right) \right] = Q^H \left( \sum_{\substack{i \neq j \in \Phi}} E \left[ r_{ij}^{-\alpha} X_i g_i g_j X_j^H \right] \right) Q \]

As the distances \( r_i \) are independent to \( X_i \) and \( g_{s,i} \),

\[ E \left[ r_{ij}^{-\alpha} X_i g_i g_j X_j^H \right] = E \left[ r_{ij}^{-\alpha} \right] E \left[ X_i g_i g_j X_j^H \right] \]

Let \( r_{i}^{-\alpha} = v_i \). From (2), the probability density function of \( v_i \) is given by [9]

\[ f_v(v) = \frac{2}{b \alpha (R^2 - v^2)^{\alpha/2}} \]

where \( b = \frac{2}{\alpha (R^2 - e^2)} \) for \( v \in [R^{-\alpha}, e^{-\alpha}] \). \( E[v] \) is derived as

\[ E[v] = \int_{v=e^{-\alpha}}^{v=R^{-\alpha}} v f_v(v) dv \]

Evaluating the integral in (20) gives

\[ E[v] = \frac{b \alpha}{\alpha - 2} (e^{2\alpha} - R^{2\alpha}) \]

The second term \( E \left[ X_i g_i g_j X_j^H \right] \) in (18) is computed from the nature of \( X_i g_i \). The vector \( X_i g_i \) is written as

\[ X_i g_i = [x_i g_{i,1}, x_i g_{i,2}, \ldots, x_i g_{i,n}, \ldots, x_i g_{i,1}, x_i g_{i,2}, \ldots, x_i g_{i,n}, \ldots, x_i g_{i,1}, x_i g_{i,2}, \ldots, x_i g_{i,n}]^T \]

With \( E \left[ x_i^T g_{i,n} (x_i^T g_{i,n})^T \right] = E \left[ x_i^T (x_i^T)^T g_{i,n} (g_{i,n})^T \right] \) and as the data and channel are independent of each other,

\[ E \left[ x_i^T (x_i^T)^T g_{i,n} (g_{i,n})^T \right] \]

can be written as

\[ E \left[ x_i^T (x_i^T)^T g_{i,n} (g_{i,n})^T \right] = E \left[ x_i^T (x_i^T)^T \right] E \left[ g_{i,n} (g_{i,n})^T \right] \]

where \( E \left[ x_i^T (x_i^T)^T \right] = \begin{cases} \frac{P_{s,avg}}{t_i = t_2} & \text{if } t_i = t_2 \\ 0 & \text{if } t_i \neq t_2 \end{cases} \) and \( P_{s,avg} \) is the average single interferer power. The term \( E \left[ g_{i,n} (g_{i,n})^T \right] \) is given by

\[ E \left[ g_{i,n} (g_{i,n})^T \right] = R_i (r_i - r_i) \]

\[ E \left[ g_{i,n} (g_{i,n})^T \right] = R_i (r_i - r_i) \]
Using (23) and (24), \( E\left[ X g_i g_i^H X_i^H \right] \) can be rewritten as
\[ E\left[ X g_i g_i^H X_i^H \right] = P_{\text{avg}} \left( I_{r_i} \otimes R_i \right) \] (25)

Using (18), (21) and (25), equation (17) given by
\[ E \left[ Q^H \left( \sum_{\alpha=0}^{\infty} r_{v}^{-\alpha} X g_i g_i^H X_i^H \right) Q \right] = Q^H \left( N_i \frac{b\alpha}{\alpha-2} \left( \epsilon^{2-\alpha} - R^{2-\alpha} \right) P_{\text{avg}} \left( I_{r_i} \otimes R_i \right) \right) Q \] (26)

where \( N_i \) is the number of interfering nodes. Substituting (12), (14), (15) and (16) which is calculated using (26), in (11) gives
\[ E \left[ g_i - \hat{g}_i \right] = R_{\alpha} - r_{v}^{-\alpha/2} Q^H R_{\alpha} X_i^H - r_{v}^{-\alpha/2} X_i R_{\alpha} Q + \sigma_v^2 Q^H Q \] (27)

Differentiating \( \text{Tr} \left( E \left[ g_i - \hat{g}_i \right] \right) \) with respect to \( Q \) and equating to zero gives the LMMSE filter matrix \( Q \) as
\[ Q = r_{v}^{-\alpha/2} R_{\alpha} X_i^H \left[ N_i \frac{b\alpha}{\alpha-2} \left( \epsilon^{2-\alpha} - R^{2-\alpha} \right) P_{\text{avg}} \left( I_{r_i} \otimes R_i \right) \right] + \sigma_v^2 I_{r_i}^{-1} \] (28)

IV. SIMULATION RESULTS

In this section, MSE performance of proposed LMMSE channel estimator in MIMO ad hoc network with transmit antenna selection is analysed. The following parameters are used for simulation as mentioned in Table I.

<table>
<thead>
<tr>
<th>Sl.No.</th>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Number of transmit antennas, ( n_t )</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>Number of receive antennas, ( n_r )</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>Distance between test transmitter and test receiver, ( r_o )</td>
<td>5 m</td>
</tr>
<tr>
<td>4</td>
<td>Radius of Guard zone, ( \varepsilon )</td>
<td>8 m</td>
</tr>
<tr>
<td>5</td>
<td>Radius of the area B, ( R )</td>
<td>100 m</td>
</tr>
<tr>
<td>6</td>
<td>Distance between successive antenna elements in the transmitter, ( d_t )</td>
<td>0.2( \lambda )</td>
</tr>
<tr>
<td>7</td>
<td>Distance between successive antenna elements in the receiver, ( d_r )</td>
<td>0.15( \lambda )</td>
</tr>
<tr>
<td>8</td>
<td>Path loss exponent, ( \alpha )</td>
<td>2.3</td>
</tr>
</tbody>
</table>

Figure 2 shows the MSE performance of the LMMSE channel estimator in MIMO ad hoc network modeled using BPP and employing transmit antenna selection, with 20, 50 and 80 interfering nodes. The single interferer power is 20dB. It can be observed that the proposed LMMSE estimator works well in the presence of strong interference also. MSE of \( 10^{-3} \) is achieved at the SNR of 18 dB with 20 interfering nodes. With 80 interfering nodes, same MSE performance is reached only at 22 dB.
Figure 3 shows the MSE performance of the LMMSE channel estimator in ad hoc network modeled using BPP for varying single interferer power. The number of nodes is fixed to be 80. MSE of $10^{-2}$ is achieved with the single interferer power of 17 dB and SNR of 20 dB. With single interferer power of 30 dB, MSE of 0.1194 is achieved when SNR is 20 dB. With SNR of 10 dB, MSE of $10^{-1}$ is obtained for single interferer power of 13.5 dB. Same MSE performance is obtained at 21.5 dB and 29.5 dB of single interferer power, when SNR is 15 dB and 20 dB respectively.

V. CONCLUSION

In this paper, a finite multi antenna ad hoc network with nodes distributed as a BPP is considered. Each node of this network employs transmit antenna selection based on the channel gain. The LMMSE channel estimation algorithm is developed to estimate the channel coefficients of finite ad hoc network with interference limited environment. The proposed algorithm includes derivation of average distance for the interferers. Simulations are carried out to analyze the MSE performance of proposed LMMSE channel estimator for varying interferer power and varying number of interferer nodes.
REFERENCES


