Abstract— In this paper we present a broadband matching technique for the design of low noise amplifiers. This technique is based on the use of coupled lines filters and quarter wave transformers for the adaptation and stabilization of these amplifiers, presenting the theory and the design process of these circuits. The type of transistors used for modeling this amplifier is the HEMT of Alpha Industries®. The results we found show that this amplifier is unconditionally stable with a satisfactory gain of about 20 dB and good impedance matching across the band of interest [10-12] GHz. The amplifier modeled in this work can be integrated in satellite receiving systems and radar systems.

Keyword- Microstrip, Coupled lines, Filter, Amplifier, Quarter wave transformer, Impedance matching

I. INTRODUCTION

The circuits which can eliminate or select signals or waves in a given frequency band are called filters. They play an important role in many RF / microwave applications. The electromagnetic spectrum is limited and must be shared, the filters are used to select or confine the RF / microwaves signals within the assigned limits. In new applications such as wireless communications, the quality of RF filters is measured in terms of high performance, a reduced size, a lighter weight and lower cost. According to the requirements and specifications the RF / microwaves filters can be designed with lumped or distributed elements. They can be realized in various structures of transmission lines such as waveguides, coaxial lines and microstrip lines [1-4].

The response of a microwave filter should check the constraints fixed by a template. A good design gives a filter with a template approaches the ideal template. This response can be assimilated to different approximation functions satisfying the template, such as functions of Chebyshev type, Butterworth or Elliptic. The method of study of a microwave filter consists in determining at first, the prototype low-pass filter equivalent to the desired synthesis. Once this prototype is established, the band pass filtering function is obtained by a transformation using a change of variables [1].

The main objective of this study is to model a bandpass coupled lines filter for adaptation of a broadband amplifier, giving satisfactory results in terms of gain and input and output reflection parameters compared with the results of other works presenting the intermediate steps leading to the modeling of our broadband amplifier.

II. MICROSTRIP LINE

The microstrip line is a conductor of width W and of thickness t, remote a ground plane by a distance h. The material separating the two conductors, the conductor band and the ground plane, has a dielectric constant \( \varepsilon _R \). The propagation is carried out simultaneously in the material and air, in this case, it is assumed that the propagation mode is the quasi-TEM. This is not entirely accurate, but this approximation is sufficient in most cases. The Fig. 1 shows a microstrip line [5-7].

A. Characteristic Impedance of the Microstrip Line

The parameters characterizing the microstrip are:

- For the substrate, its thickness h and its relative dielectric constant \( \varepsilon _R \);
- For the microstrip, its width W and its thickness t.

In general, \( 0.1 \leq \frac{W}{h} \leq 1 \) and \( \frac{t}{h} \ll 10 \).

The difficulty in studying the propagation in a microstrip is that the propagation takes place simultaneously in the substrate and in air [7-13].

The substrate and the air have by definition, different relative dielectric constant \( \varepsilon _R \). The ratio \( \frac{\varepsilon _R \text{substrate}}{\varepsilon _R \text{air}} \) is generally between 2 and 10 [7].

There is therefore a large number of approximate formulas resulting from the work of all the researchers who contributed to the modeling of microstrip. The main results were obtained in the late 1970s. The main objective
of these works was to give approximate formulas as accurate as possible to the characteristic impedance \( Z_0 \) and the propagation velocity \( v_0 \) [7-11].

Fig. 1. Microstrip placed above a ground plane and separated of the latter by a material of relative permittivity \( \varepsilon_R \)

The first approximation is to evaluate a new equivalent dielectric constant \( \varepsilon_{RE} \) which takes into account the simultaneous propagation in the two environments [7].

- \( Z_0 \) in function of \( \frac{W}{h} \)

An approximate formula gives:

\[
\varepsilon_{RE} = \frac{\varepsilon_R + 1}{2} + \frac{\varepsilon_R - 1}{2} \frac{1}{1 + \frac{1}{1 + 12 \frac{W}{h}}} \tag{1}
\]

To evaluate the characteristic impedance \( Z_0 \), we dissociate the two cases according to the ratio \( \frac{W}{h} \).

If \( \frac{W}{h} \leq 1 \)

\[
Z_0 = \frac{\eta}{2 \pi \sqrt{\varepsilon_{RE}}} \ln \left( \frac{\varepsilon_R + 1}{W} + \frac{W}{\varepsilon_R} \right) \tag{2}
\]

If \( \frac{W}{h} \geq 1 \)

\[
Z_0 = \frac{\eta}{\sqrt{\varepsilon_{RE}}} \left[ \frac{W}{h} + 1.393 + 0.667 \ln \left( \frac{W}{h} + 1.444 \right) \right]^{-1} \tag{3}
\]

In these relations, we have:

\[
\eta = \frac{\mu_0}{\varepsilon_0} \tag{4}
\]

\( \mu_0 \): Magnetic permeability of vacuum = \( 4 \pi \cdot 10^{-7} \) H/m

\( \varepsilon_0 \): The vacuum permittivity = \( 10^{-7} / 4 \pi c^2 \)

\( C \): Velocity of electromagnetic waves in vacuum = \( 2,997 \, 925 \cdot 10^8 \) m/s

\( C \approx 3 \cdot 10^8 \) m/s

\[
\eta = \sqrt{\frac{4 \pi \cdot 10^{-7}}{10^7 - 4 \pi c^2}} = 4 \pi \cdot 10^{-7} c
\]

B. Discontinuity in the Lines

The discontinuities in the microstrip lines are such that they allow the realizing filters, resonators or impedance transformers [7].

- Discontinuity in the width

Two lines of width \( W_1 \) and \( W_2 \) and length \( \lambda_0/4 \) shown in Fig. 2 can be used to adopt two stages having input and output impedances \( R_S \) and \( R_L \).

The line connected to the quadruple having an impedance \( R_S \) will have a characteristic impedance \( Z'_0 \), the line connected to a quadruple having an impedance \( R_L \) will have a characteristic impedance \( Z''_0 \) [7, 10].

\[
Z'_0 = \sqrt{R_S^2 R_L} \tag{5}
\]

\[
Z''_0 = \sqrt{R_L^2 R_S} \tag{6}
\]

A second solution consists in adopting a single section of length \( \lambda_0/4 \) and characteristic impedance \( Z_0 \). In this case the configuration is simply that shown in (Fig. 2b).
The characteristic impedance $Z_0$ is related to $R_S$ and $R_L$ by the relation:

$$Z_0 = \sqrt{R_S R_L} \quad (7)$$

In the case of a single line having characteristic impedance $Z_0$, case of Fig. 2b, the adaptation is carried in a narrow band. The configuration of Fig. 2a increases the bandwidth upon which the adaptation is effected. The bandwidth can be increased by increasing the number of sections of different widths [7].

### III. COUPLED LINES BANDPASS FILTERS

The filter design is based on the constraints imposed by specifications established according to the desired application. There are two families of bandpass filters, broadband filters characterized by relative bandwidths between 20% and 80%, and the narrow band filters characterized by a bandwidth less than 20%. The microwaves filters are often used for applications in civilian and military space telecommunication. These applications are often located in a frequency band between 8 and 14 GHz [1-4], this is why, as application, our choice was made on the modeling of a bandpass filter having 11 GHz as a center frequency and a bandwidth of 2 GHz, in order to use it as an impedance matching circuit for a broadband amplifier.

The structure of a filter with lumped elements is unsuitable in high frequency mainly because of the physical dimensions of inductance and capacitance, which are no longer negligible compared with the wavelength. Therefore we choose to make this filter on distributed elements constituted of segments of transmission lines with different characteristic impedances ($Z_i$) [1-4].

#### A. Coupled Lines

Coupled microstrip lines are widely used for implementing microstrip filters. Fig. 3 illustrates the general structure of a pair of coupled microstrip lines considered in this work, where the two microstrip lines of width $W$ are in parallel coupled configuration with a separation $s$ [2].

**Fig. 3. General structure of the coupled microstrip lines**

This coupled line structure supports two quasi-TEM modes, that is, the even and the odd mode, as shown in (Fig. 4). For an even-mode excitation, both microstrip lines have the same voltage potentials or carry the same sign charges, say, the positive ones, resulting in a magnetic wall at the symmetry plane, as shown in Fig. 4a. In the case when an odd mode is excited, both microstrip lines have the opposite voltage potentials or carry the opposite sign charges, so that the symmetric plane is an electric wall, as indicated in Fig. 4b. In general, these two modes will be excited at the same time. However, they propagate with different phase velocities because they are not pure TEM modes, which mean that they experience different permittivities. Therefore, the coupled microstrip lines are characterized by the characteristic impedances, as well as the effective dielectric constants for the two modes [2-3].

1) **Even- and Odd-Mode Capacitances**: In a static approach similar to the single microstrip, the even- and odd-mode characteristic impedances and effective dielectric constants of the coupled microstrip lines may be
obtained in terms of the even- and odd-mode capacitances, denoted by $C_e$ and $C_o$. As shown in Fig. 4, the even- and odd-mode capacitances $C_e$ and $C_o$ may be expressed as [2, 14-16]:

$$C_e = C_p + C_f + C_f'$$

$$C_o = C_p + C_f + C_{gd} + C_{ga}$$

In these expressions, $C_p$ denotes the parallel-plate capacitance between the strip and the ground plane and, hence, is simply given by:

$$C_p = \varepsilon_0 \varepsilon_r W/h$$

$C_f$ is the fringe capacitance as if for an uncoupled single microstrip line and is evaluated by:

$$2C_f = \sqrt{E_{ref}(fZ_j)} - C_p$$

The term $C_f'$ accounts for the modification of fringe capacitance $C_f$ of a single line because of the presence of another line. An empirical expression for $C_f'$ is given below:

$$C_f' = \frac{A}{1 + A(k_b/s) \tanh(2s/h)}$$

Where:

$$A = \exp[-0.1 \exp(2.33-2.53 W/h)]$$

For the odd-mode $C_{ga}$ and $C_{gd}$ represent, respectively, the fringe capacitances for the air and dielectric regions across the coupling gap. The capacitance $C_{gd}$ may be found from the corresponding coupled stripline geometry, with the spacing between the ground planes given by $2h$. A closed-form expression for $C_{gd}$ is:

$$C_{gd} = \frac{\varepsilon_0 \varepsilon_r}{x} \ln \left( \coth \left( \frac{x s}{4h} \right) \right) + 0.65C_f \left( 0.02/\varepsilon_r + 1 - \frac{1}{\varepsilon_r} \right)$$

The capacitance $C_{ga}$ can be modified from the capacitance of the corresponding coplanar strips and expressed in terms of a ratio of two elliptic functions

$$C_{ga} = \varepsilon_0 \frac{k(k)}{k'(k)}$$

Where:

$$k = \frac{s/h}{s/h + 2W/h} \quad \text{and} \quad k' = \sqrt{1 - k^2}$$

and the ratio of the elliptic functions is given by:

$$\frac{k(k)}{k'(k)} = \begin{cases} \frac{1}{\pi} \ln \left( 2 \frac{1+k}{1-k} \right) & \text{for} \quad 0 \leq k^2 \leq 0.5 \\ \frac{\pi}{\ln \left( 2 \frac{1+k}{1-k} \right)} & \text{for} \quad 0.5 \leq k^2 \leq 1 \end{cases}$$

The capacitances obtained by using above design equations are found to be accurate to within 3% over the ranges $0.2 \leq W/h \leq 2$, $0.05 \leq s/h \leq 2$ and $\varepsilon_r \geq 1$ [2].

2) Even -and Odd- Mode Characteristic Impedances and Effective Dielectric Constants: The even- and odd-mode characteristic impedances $Z_{ce}$ and $Z_{co}$ can be obtained from the capacitances. This yields

$$Z_{ce} = \left( c \sqrt{C_e^2 C_o} \right)^{-1}$$

$$Z_{co} = \left( c \sqrt{C_o^2 C_e} \right)^{-1}$$
where $C_e$ and $C_o$ are even- and odd-mode capacitances for the coupled microstrip line configuration with air as a dielectric [14-16].

Effective dielectric constants $\varepsilon_{re}^e$ and $\varepsilon_{re}^o$ for even and odd modes, respectively, can be obtained from $C_e$ and $C_o$ by using the relations:

$$\varepsilon_{re}^e = C_e / C_e^0$$  \hspace{1cm} and  \hspace{1cm} $$\varepsilon_{re}^o = C_o / C_o^0$$  \hspace{1cm} (18)

B. Parallel-Coupled Half-Wavelength Resonator Filters

Fig. 5 illustrates a general structure of parallel-coupled microstrip bandpass filters, which use half-wavelength line resonators. They are positioned so that adjacent resonators are parallel to each other along one-half of their length. This parallel arrangement gives relatively large coupling for a given spacing between resonators and, thus, this filter structure is particularly convenient for constructing filters having a wider bandwidth as compared to the structure for the end-coupled microstrip filters described in the last section. The design equations for this type of filter are given by [2, 14-16]:

$$\frac{L_j}{L_0} = \frac{\pi FBW}{2 g_0 g_1}$$  \hspace{1cm} (19)

$$\frac{L_j}{L_0} = \frac{\pi FBW}{2 g_0 g_{j+1}} \quad j = 1 \text{ to } n - 1$$  \hspace{1cm} (20)

$$\frac{L_n}{L_0} = \frac{\pi FBW}{2 g_0 g_{n+1}}$$  \hspace{1cm} (21)

Where $g_0$, $g_5$, ..., $g_n$ are the element of a lowpass prototype with a normalized cutoff $\Omega_c=1$, and $FBW$ is the fractional bandwidth of bandpass filter. $g_j$ are the characteristic admittances of $J$ inverters and $Y_0$ is the characteristic admittance of the terminating lines. To realize the $J$ inverters obtained above, the even- and odd-mode characteristic impedances of the coupled microstrip line resonators are determined by [2, 14-16]:

$$(Z_{0e})_{j+1} = \frac{1}{Y_0} \left[ 1 + \frac{L_{j+1}}{Y_0} + \left( \frac{L_{j+1}}{Y_0} \right)^2 \right] \quad \text{for} \quad j = 0 \text{ to } n$$  \hspace{1cm} (22)

$$(Z_{0o})_{j+1} = \frac{1}{Y_0} \left[ 1 - \frac{L_{j+1}}{Y_0} + \left( \frac{L_{j+1}}{Y_0} \right)^2 \right] \quad \text{for} \quad j = 0 \text{ to } n$$  \hspace{1cm} (23)

For modeling the bandpass coupled lines filter that will be used to adapt our microwave amplifier, we have chosen the following specifications:

- The center frequency is $f_0 = 11 \text{ GHz}$
- The impedances of the source and the load are $Z_S = Z_L = Z_0 = 50 \text{ } \Omega$
- The filter response is Chebyshev type, with a ripple of $0.1 \text{ dB}$
- The order of the filter is $N = 3$.
- $FBW = \frac{f_2-f_1}{f_0}$ with $f_1 = 10 \text{ GHz}$ and $f_2 = 12 \text{ GHz}$. 
The determination of the coefficients of Chebyshev polynomial corresponding to the desired network. These are determined by using the table 4.05-2 (a) on page 100 of [1] given below.

<table>
<thead>
<tr>
<th>N</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>1</td>
<td>1.0315</td>
<td>1.1474</td>
<td>1.0315</td>
<td>1</td>
</tr>
</tbody>
</table>

From these specifications and previously established equations we found the following values for the impedances of the even and odd modes for the considered filter [Tables II and III].

<table>
<thead>
<tr>
<th>(Z_{0e})_{0,1}</th>
<th>(Z_{0e})_{1,2}</th>
<th>(Z_{0e})_{2,3}</th>
<th>(Z_{0e})_{3,4}</th>
</tr>
</thead>
<tbody>
<tr>
<td>90.1535 Ω</td>
<td>66.5720 Ω</td>
<td>66.5720 Ω</td>
<td>90.1535 Ω</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(Z_{0o})_{0,1}</th>
<th>(Z_{0o})_{1,2}</th>
<th>(Z_{0o})_{2,3}</th>
<th>(Z_{0o})_{3,4}</th>
</tr>
</thead>
<tbody>
<tr>
<td>37.5343 Ω</td>
<td>40.3198 Ω</td>
<td>40.3198 Ω</td>
<td>37.5343 Ω</td>
</tr>
</tbody>
</table>

The general scheme of the modeled filter and the representation of curves S_{11} and S_{21} are illustrated in the following figures (Fig. 6 and 7):

Fig. 6. General scheme of modeled coupled lines filter with N= 3

The obtained template of the filter approaches the ideal template of a bandpass filter (Fig. 7), which means that a good modeling of the filter was done. We have obtained three resonances frequencies corresponding to 3 minimums having lower values or equal to -50 dB (Curve dB(S(1,1))). The curve dB(S(2,1)) shows that there is a maximum power transfer from the input to the output of this filter. On the other side, the curve dB(S(1,1)) indicates that there are only small reflections of power at the input port of the filter.
IV. MODELING OF THE LOW NOISE AMPLIFIER

For the modeling of the amplifier we have used the HEMT transistors type of Alpha Industries\textsuperscript{®} characterized by the S parameters [17]. This type of transistors has a high gain and low noise figure compared with other types of technologies; namely FETs and BJTs. The general structure of an amplifier is composed of an amplification stage comprising one or more transistors characterized by the S parameters and surrounded on either and others by impedance matching networks [18-22].

A. Procedure of Impedance Matching

Firstly we have done the representation of the input and output impedances of the amplifier as shown in the charts of (Fig. 8).

We notice that these impedances are far from the center of the Smith chart (Rel\!(\!Z\!)\! = \!50 \Omega and \!(\!Im\!(\!Z\!)\! = \!0\!) at the center frequency which in our case is \( F = 11 \) GHz. Then we have resonated the impedance to adapt at 11 GHz, by inserting next to the transistor a line of electrical length equal to 21.68° and having a characteristic impedance \( Z_0 \) equal to 50 \( \Omega \) to eliminate imaginary part of the impedance to adapt and a quarter wave transformer of characteristic impedance \( Z = 35.53 \Omega \) to arrive to the center of the Smith chart. The same thing was done for the output (Fig. 9).
The new representation of impedances to adapt is illustrated in the following charts (Fig. 10). We observe that the impedances become resonants at the center frequency $F = 11$ GHz.

![Fig. 9. Impedance transformation at the center frequency](image)

![Fig. 10. New representation of impedances to adapt, (a) for the input, (b) for the output](image)

The second step is to add adaptation coupled lines filters previously modeled to the input and to the output of the amplifier of the scheme of (Fig. 6). The final structure of this amplifier is as follows (Fig. 11). It is composed of an amplification stage in the form of 2 transistors in cascade as shown in (Fig. 9), put into box (Transistors Block), a block of impedance transformation to go to the center of the Smith chart (Input and Output_Impedance_Transformation) and a matching block in the form of coupled lines filter at the input and at the output (Input and Output_Coupled_Lines_Filter).

![Fig. 11. Final structure of the modeled amplifier](image)
We have simulated the performance of the circuit using optimization and adjustments techniques of ADS software of Agilent Technologies® [20] to fix the best values of the circuit components ensuring a good adaptation and unconditional stability of the amplifier. We have obtained the following results (Fig. 12 and 13):

The reflection parameters $S_{11}$ and $S_{22}$ are respectively strictly inferior to $-19.35$ dB and $-32.93$ dB (Fig. 12). For directe transmission $S_{21}$ is greater than 20 dB and reverse transmission $S_{12}$ is less than $-37.63$ dB (Fig. 13).

For the stability of our amplifier [23-26], it is unconditionally stable because all coefficients of stability are greater than 1 in the whole the band of interest (Fig. 14).

In the table IV we have compared the performance of our amplifier with the results of other works. We find that the modeled amplifier in this work presents satisfactory results compared with articles [27-31].
This work  X  <-19.35  <-32.93  >20.02  <-37.63  2  HEMT  2  
[27]  X  <-12  <-10.2  > 7.5  <-15  0.5  HEMT  1  
[28]  X  <-4  <-2  > 22  *  1  mHEMT  3  
[29]  X  <-15  <-0.4  > 7  *  2.4  HBT  3  
[30]  X  <-8.5  <-9.2  *  *  1.5  pHEMT  4  
[31]  X  <-25.75  <-25.28  >14.10  <-33.43  2.5  FET  2  

V. CONCLUSION

Low noise amplifiers and RF filters are among the essential blocks in any chain of receipt for amplification levels of the received signals and selection of useful information for defined applications. The LNAs and RF coupled lines filters modeled in this work are widely used in applications such as wireless devices, base stations, satellite receivers and military applications. We have established a comprehensive study to model a coupled lines filter presenting the used technology for its manufacture and the various steps to make its dimensioning, in order to use it as an impedance matching circuit for broadband amplifier. We have also presented a simple and complete technique of adaptation for the design of broadband amplifiers.

The modeled LNA in this article based on HEMT transistors present satisfactory results compared with other design techniques, with gain greater than 20 dB, unconditional stability and good adaptation.

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