Outage Analysis of PLNC Based Bidirectional Relay Network in the presence of I/Q Imbalance

V.N.Senthil Kumaran[#1], S.Rajkumar[#2], M.N.Suresh[#3], S.J.Thiruvengadam[#4]

[#]Department of Electronics and Communication Engineering, TIFAC CORE in Wireless Technologies
Thiagarajar College of Engineering, Madurai – 625 015, India
[1]vnsenthilkumaran@tce.edu
[2]sraj.tce@gmail.com
[3]mnsece@tce.edu
[4]sjtece@tce.edu

Abstract— Bidirectional relay network consists of two sources and a relay, where each node has a single antenna and operates in half duplex mode. The PLNC based OFDM system transmits numerous high data streams through narrow band flat fading subchannels to achieve high spectral efficiency over wide band channels. However in practice, I/Q imbalance is introduced at the radio frequency sections of the nodes affects the orthogonality among the subcarriers. In this paper, the bidirectional relay network is modelled with I/Q imbalance and the outage performance of Orthogonal Frequency Division Multiplexing (OFDM) based bidirectional relay network which employs physical layer network coding (PLNC) is analyzed in the presence of In-phase and Quadrature (I/Q) imbalance at both the time slots.

Index Terms- Bidirectional relay network, Physical layer network coding (PLNC), Orthogonal Frequency Division Multiplexing (OFDM), I/Q Imbalance, outage probability.

I. INTRODUCTION

Wireless relay assisted networks have received renewed significant attention recently after the pioneering work of [1]**.** In one way relay network (OWRN), the data transmission is unidirectional in which a source node sends the information to the relay and then the relay sends it to the destination [2]. Unlike OWRN, in two way relay network (TWRN), two source nodes exchange their information through a relay node and thereby doubling the data rate. The bidirectional transmission model, first explored by Shannon, is one of the popular types of modern communications where two source nodes simultaneously send information to each other [3]. Many of the TWRNs are analyzed only in the flat fading environments [4]. But, most practical wireless communication systems experience the scattered and delayed propagation paths which are modeled as frequency selective fading channels. One of the methods to overcome the system with frequency selective fading is the Orthogonal Frequency Division Multiplexing (OFDM). It is capable of converting the frequency selective fading channel into orthogonal flat–fading sub channels [5]. However, OFDM system performance is affected in the presence of I/Q Imbalance. Several papers have addressed the issue of I/Q imbalance in OFDM systems [6-8]. The effect of I/Q Imbalance on point to point OFDM systems and the resulting performance degradation are investigated in [9]. The impact of I/Q Imbalance on OFDM receivers is studied and system level algorithms to compensate for the distortions are proposed in [10].

Network coding as proposed firstly in [11] is a potential and powerful tool in designing communication networks. Network coding has the capability of combining and extracting information’s through simple Galois field additions and multiplications. However, it cannot resolve the interference of simultaneously arriving electromagnetic signals at the receiver. Hence, in Physical Layer Network Coding (PLNC) the interfering signals are mapped to Galois field additions of two digital bit streams, so that the interference becomes the part of arithmetic operations in network coding. The optimal mapping functions at the relay nodes that could minimize the transmission Bit Error Rate (BER) were proposed in [12]. In this paper, the effect of I/Q Imbalance on the OFDM based bidirectional relay network with PLNC is analyzed in terms of outage probability.

The rest of the paper is organized as follows. The system model of OFDM based bidirectional relay network in the presence of I/Q Imbalance is presented in Section 2. The outage performance of the network is analyzed in section 3. The simulation results are discussed in Section 4. Finally the concluding remarks are presented in Section 5.

II. SYSTEM MODEL

Consider a generalized bidirectional relay network consisting of two nodes S_1, S_2 and a relay node R, shown in Figure 1. Each node in the network is assumed to have a single antenna and operates in a half duplex mode. During time slot I, both the nodes S_1 and S_2 transmit OFDM signals with N information symbols $\tilde{x}_m(k)$, $0 \le k \le N-1$, $m = 1,2$, $\{\tilde{x}_m(k) \in \pm 1\}$, where N is the number of subcarriers in one OFDM symbol. Let $x_m(n) = x_m^I(n) + jx_m^Q(n)$, $0 \le n \le N-1$, $m = 1,2$ be the time domain samples at the output of Inverse Discrete Fourier Transform (IDFT) block of OFDM transmitter. $x_m^I(n)$ and $x_m^Q(n)$ represent the in-phase and quadrature parts of $x_m(n)$. Let $\mathbf{g}_1$ be the $L_1 \times 1$ vector of the quasi static frequency selective fading channel between S_1 and R, $\mathbf{g}_2$ be the $L_2 \times 1$ vector of the quasi static frequency selective fading channel between S_2 and R respectively. A cyclic prefix (CP) of length L_{CP} is appended to the time domain transmitted samples to avoid inter-symbol interference (ISI). The CP length L_{CP} satisfies the condition that $L_{CP} \ge \max\{L_1 - 1, L_2 - 1\}$.

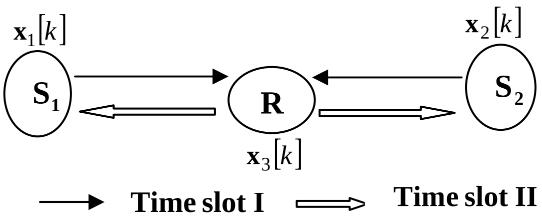

Fig 1. System Model

2.1 I/Q Imbalance at Time slot – I

2.1.1 Signal Transmission from Source Nodes:

A detailed block diagram with I/Q imbalance at source nodes in time slot-I is shown in Figure 2. The OFDM samples are passed through a Digital-to-Analog Converter (DAC) and a pulse-shaping filter. Let $h_m(t)$ be the impulse response of the pulse shaping filter at m^{th} source node. The continuous time outputs of the pulse shaping filters at in-phase and quadrature branches are represented as,

$$\begin{aligned} p_m^I(t) &= \sum_{n=-\infty}^{\infty} x_m^I(n) h_m^I(t - nT) \qquad m = 1,2 \\ p_m^Q(t) &= \sum_{n=-\infty}^{\infty} x_m^Q(n) h_m^Q(t - nT) \qquad m = 1,2 \end{aligned} \tag{1}$$

where T is the symbol duration. The imbalance filter shown in Figure 2 represents the mismatch between I/Q branches. Generally there exists two types of I and Q mismatches, namely frequency dependent (FD) and frequency independent (FI) I/Q imbalances. The frequency dependent I/Q imbalance exists prior to the upconversion due to analog components and D/A or A/D converters. Frequency independent I/Q imbalance is due to the amplitude and phase mismatches in quadrature upconversion.

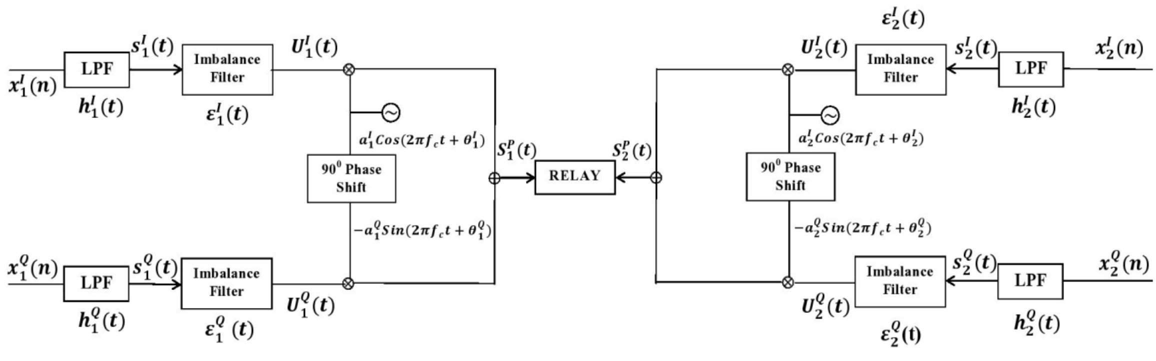

Fig 2. Signal Transmission from the Source nodes

Let $\varepsilon_m^I(t)$ and $\varepsilon_m^Q(t)$ denote the impulse response of the FD imbalance filters at I and Q branches of m^{th} source node. The imbalance filter outputs at respective branches are represented as

$$u_m^I(t) = p_m^I(t) * \varepsilon_m^I(t) \quad m = 1,2 \tag{2}$$

$$u_m^Q(t) = p_m^Q(t) * \varepsilon_m^Q(t) \quad m = 1,2 \tag{3}$$

where, $(*)$ represents convolution operation. The signal components are then upconverted using quadrature tones in the mixer. Ideally, the quadrature tones have equal amplitude and phase differences of 90^0. However, in practice, there exists amplitude and phase difference. It causes FI I/Q imbalance. Let a_m^I, a_m^Q and θ_m^I, θ_m^Q be the amplitude and phase mismatches at the m^{th} source node. In the presence of I/Q imbalance, the pass band signal at the output of the m^{th} source node is given by

$$s_m^p(t) = u_m^I(t) a_m^I \cos\left(2\pi f_c t + \theta_m^I\right) - u_m^Q(t) a_m^Q \sin\left(2\pi f_c t + \theta_m^Q\right) \tag{4}$$

Rearranging the terms, Equation (2) is expressed as

$$s_m^p(t) = s_m^{p_I}(t)\cos(2\pi f_c t) - s_m^{p_Q}(t)\sin(2\pi f_c t) \tag{5}$$

where $s_m^{p_I}(t) = \left(u_m^I(t) a_m^I \cos\theta_m^I - u_m^Q(t) a_m^Q \sin\theta_m^Q\right)$ and

$s_m^{p_Q}(t) = \left(u_m^I(t) a_m^I \sin\theta_m^I + u_m^Q(t) a_m^Q \cos\theta_m^Q\right)$. The baseband equivalent of $s_m^p(t)$ is given by

$$\begin{aligned} s_m^b(t) &= s_m^{p_I}(t) + j s_m^{p_Q}(t) \\ &= \left(u_m^I(t) a_m^I \cos\theta_m^I - u_m^Q(t) a_m^Q \sin\theta_m^Q\right) \\ &\quad + j\left(u_m^I(t) a_m^I \sin\theta_m^I + u_m^Q(t) a_m^Q \cos\theta_m^Q\right) \end{aligned} \tag{6}$$

The I and Q components of $u_m(t)$ can be written as $u_m^I(t) = \frac{u_m(t) + u_m'(t)}{2}$, $u_m^Q(t) = \frac{u_m(t) - u_m'(t)}{2j}$. where, $(')$ represents complex conjugation. Using these definitions, Equation (6) is simplified as,

$$s_m^b(t) = \mu_m u_m(t) + v_m u_m'(t) \tag{7}$$

where $\mu_m = \dfrac{a_m^I e^{j\theta_m^I} + a_m^Q e^{j\theta_m^Q}}{2}$ and $v_m = \dfrac{a_m^I e^{j\theta_m^I} - a_m^Q e^{j\theta_m^Q}}{2}$

Using Equations (2) and (3), Equation (7) is rewritten as,

$$s_m^b(t) = \lambda_m(t) * p_m(t) + \phi_m(t) * p_m'(t) \tag{8}$$

where,

$$\begin{aligned} \lambda_m(t) &= \frac{1}{2}\left((\mu_m + v_m)\varepsilon_m^I(t) + (\mu_m - v_m)\varepsilon_m^Q(t)\right) \\ \phi_m(t) &= \frac{1}{2}\left((\mu_m + v_m)\varepsilon_m^I(t) - (\mu_m - v_m)\varepsilon_m^Q(t)\right) \end{aligned} \tag{9}$$

2.1.2 Signal reception at Relay Node

The relay node introduces a I/Q imbalance. The received signal at the relay node with I/Q imbalance during time slot-I is shown in Figure 3. Let $g_1^p(t)$ and $g_2^p(t)$ be the channel impulse responses of S_1 to R and S_2 to R respectively. The received signal at the relay is given by,

$$y_r^p(t) = \left\{g_1^p(t) * s_1^p(t)\right\} + \left\{g_2^p(t) * s_2^p(t)\right\} + w_r^p(t) \tag{10}$$

where $w_r^p(t)$ is the AWGN with double sided power spectral density σ^2. The baseband equivalent of $y_r^p(t)$ is given by,

$$y_r(t) = \left\{g_1(t) * s_1^b(t)\right\} + \left\{g_2(t) * s_2^b(t)\right\} + w_r(t) \tag{11}$$

where $g_1(t)$ and $g_2(t)$ are the baseband equivalent of $g_1^p(t)$ and $g_2^p(t)$. Let a_r^I, a_r^Q and θ_r^I, θ_r^Q be the amplitude and phase mismatches at the relay node. Further, let

$$\mu_{r_r} = \frac{1}{2}\left(a_{r_r}^I e^{-j\theta_{r_r}^I} + a_{r_r}^Q e^{-j\theta_{r_r}^Q}\right), v_{r_r} = \frac{1}{2}\left(a_{r_r}^I e^{j\theta_{r_r}^I} - a_{r_r}^Q e^{j\theta_{r_r}^Q}\right).$$

Let $\varepsilon_{r_r}^I(t)$ and $\varepsilon_{r_r}^Q(t)$ be the FD I/Q Imbalance at the relay receiver.

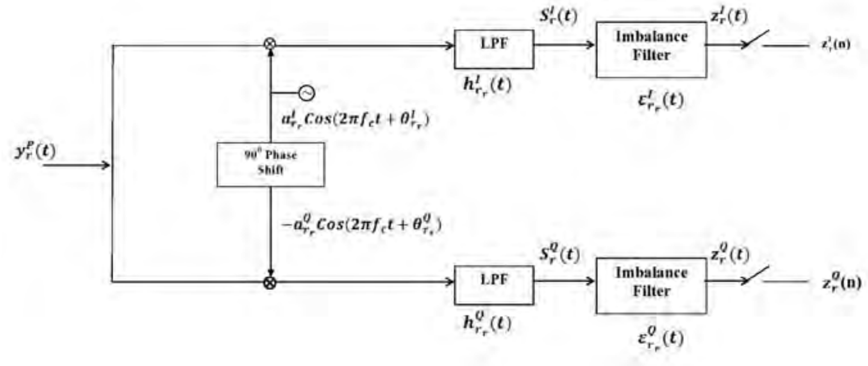

Figure 3. Receive signal at relay node with I/Q imbalance

The baseband equivalent output at the imbalance filter is given by,

$$\begin{aligned} z_r(t) &= z_r^I(t) + jz_r^Q(t) \\ &= \frac{1}{2}\left[\left(\mu_{r_r} + v'_{r_r}\right)\varepsilon_{r_r}^I(t) + \left(\mu_{r_r} - v'_{r_r}\right)\varepsilon_{r_r}^Q(t)\right] * h_{r_r}(t) * y_r(t) \\ &+ \frac{1}{2}\left[\left(\mu'_{r_r} + v_{r_r}\right)\varepsilon_{r_r}^I(t) + \left(v_{r_r} - \mu'_{r_r}\right)\varepsilon_{r_r}^Q(t)\right] * h_{r_r}(t) * y'_r(t) \end{aligned} \tag{12}$$

Let $\lambda_{r_r}(t) = \frac{1}{2}\left[\left(\mu_{r_r} + v'_{r_r}\right)\varepsilon_{r_r}^I(t) + \left(\mu_{r_r} - v'_{r_r}\right)\varepsilon_{r_r}^Q(t)\right]$ and $\phi_{r_r}(t) = \frac{1}{2}\left[\left(\mu'_{r_r} + v_{r_r}\right)\varepsilon_{r_r}^I(t) + \left(v_{r_r} - \mu'_{r_r}\right)\varepsilon_{r_r}^Q(t)\right]$.

Then, (12) can be simplified as

$$z_r(t) = \lambda_{r_r}(t) * h_{r_r}(t) * y_r(t) + \phi_{r_r}(t) * h_{r_r}(t) * y'_r(t) \tag{13}$$

Using eqn. (6) and (9), eqn. (13) is written as

$$\begin{aligned} z_r(t) &= \sum_{m=1}^{2} g_m(t) * \left\{\lambda_m(t) * p_m(t) + \phi_m(t) * p'_m(t)\right\} * h_{r_r}(t) * \lambda_{r_r}(t) \\ &+ \sum_{m=1}^{2} g'_m(t) * \left\{\lambda_m(t) * p_m(t) + \phi_m(t) * p'_m(t)\right\}' * h_{r_r}(t) * \phi_{r_r}(t) + \left\{\left(\lambda_{r_r}(t) * w_r(t)\right) + \left(\phi_{r_r}(t) * w'_r(t)\right)\right\} * h_{r_r}(t) \end{aligned} \tag{14}$$

Upon rearranging (14), it can be written as

$$\begin{aligned} z_r(t) &= \left(\sum_{m=1}^{2}\left[\lambda_{r_r}(t) * g_m(t) * \lambda_m(t) + \phi_{r_r}(t) * g'_m(t) * \phi'_m(t)\right] * p_m(t) * h_{r_r}(t)\right) \\ &+ \left(\sum_{m=1}^{2}\left[\lambda_{r_r}(t) * g_m(t) * \phi_m(t) + \phi_{r_r}(t) * g'_m(t) * \lambda'_m(t)\right] * p'_m(t) * h_{r_r}(t)\right) \\ &+ \left\{\left(\lambda_{r_r}(t) * w_r(t)\right) + \left(\phi_{r_r}(t) * w'_r(t)\right)\right\} * h_{r_r}(t) \quad ; \quad 0 \le t \le T + T_{CP} \end{aligned} \tag{15}$$

where T represents OFDM symbol duration, T_{CP} is the duration of the cyclic prefix. The output of the imbalance filter $z_r(t)$ is sampled and the cyclic prefix is removed. Assuming $\{h_m(t) * h_r(t)\}, \quad m = 1,2$ satisfies Nyquist criterion for pulse shaping, the discrete time samples after removing the cyclic prefix can be represented as

$$\begin{aligned} z_r(n) &= \sum_{m=1}^{2}\left[\lambda_{r_r}(n) \circ g_m(n) \circ \lambda_m(n) + \phi_{r_r}(n) \circ g'_m(n) \circ \phi'_m(n)\right] \circ p_m(n) \\ &+ \sum_{m=1}^{2}\left[\lambda_{r_r}(n) \circ g_m(n) \circ \phi_m(n) + \phi_{r_r}(n) \circ g'_m(n) \circ \lambda'_m(n)\right] \circ p'_m(n) + \left(\lambda_{r_r}(n) \circ w_r(n)\right) + \left(\phi_{r_r}(n) \circ w'_r(n)\right); \quad 0 \le n \le N-1 \end{aligned} \tag{16}$$

Let the sampled version of $\lambda_m(t)$ be $\boldsymbol{\lambda}_{\mathbf{m}} = \left[\lambda_m(0),.....\lambda_m\left(L_{\lambda_m}-1\right),\mathbf{0}^T_{1\times\left(N-L_{\lambda_m}\right)}\right]^T$ $m=1,2$ be a $N\times1$ vector, where, L_{λ_m} is the number of non zero elements in $\boldsymbol{\lambda}_{\mathbf{m}}$. Similarly, the vectors $\boldsymbol{\lambda}_{\mathbf{r}},\boldsymbol{\varphi}_{\mathbf{1}},\boldsymbol{\varphi}_{\mathbf{2}},\boldsymbol{\varphi}_{\mathbf{r}},\mathbf{g}_{\mathbf{1}},\mathbf{g}_{\mathbf{2}},\mathbf{p}_{\mathbf{1}}$ and $\mathbf{p}_{\mathbf{2}}$ are $N\times1$ vector representation of samples from $\lambda_r(t),\phi_1(t),\phi_2(t),\phi_r(t),g_1(t),g_2(t),p_1(t)$ and $p_2(t)$ respectively. Moreover, $\boldsymbol{\Lambda}_1$, $\boldsymbol{\Lambda}_2$, $\boldsymbol{\Lambda}_r$, $\boldsymbol{\Phi}_1$, $\boldsymbol{\Phi}_2$, $\boldsymbol{\Phi}_{\mathbf{r}}$, $\mathbf{G}_1$ and $\mathbf{G}_2$ be the $N\times N$ circulant matrices with first columns as $\boldsymbol{\lambda}_{\mathbf{1}},\boldsymbol{\lambda}_{\mathbf{2}},\boldsymbol{\lambda}_{\mathbf{r}},\boldsymbol{\varphi}_{\mathbf{1}},\boldsymbol{\varphi}_{\mathbf{2}},\boldsymbol{\varphi}_{\mathbf{r}},\mathbf{g}_{\mathbf{1}}$ and $\mathbf{g}_{\mathbf{2}}$ respectively. The matrix representation of (16) after removing the cyclic prefix is represented as

$$\mathbf{z}_{\mathbf{r}} = \sum_{m=1}^{2}\left(\boldsymbol{\Lambda}_{\mathbf{r}}\mathbf{G}_{\mathbf{m}}\boldsymbol{\Lambda}_{\mathbf{m}} + \boldsymbol{\Phi}_{\mathbf{r}}\mathbf{G}^{\mathbf{H}}_{\mathbf{m}}\boldsymbol{\Phi}^{\mathbf{H}}_{\mathbf{m}}\right)\mathbf{p}_{\mathbf{m}} + \sum_{m=1}^{2}\left(\boldsymbol{\Lambda}_{\mathbf{r}}\mathbf{G}_{\mathbf{m}}\boldsymbol{\Phi}_{\mathbf{m}} + \boldsymbol{\Phi}_{\mathbf{r}}\mathbf{G}^{\mathbf{H}}_{\mathbf{m}}\boldsymbol{\Lambda}^{\mathbf{H}}_{\mathbf{m}}\right)\mathbf{p}'_m + \boldsymbol{\Lambda}''_r\mathbf{w}_{\mathbf{r}} + \boldsymbol{\Phi}''_{\mathbf{r}}\mathbf{w}'_{\mathbf{r}} \tag{17}$$

where, $\boldsymbol{\Lambda}''_r$ is a $[N\times(N+L_{CP})]$ matrix and $\mathbf{w}_{\mathbf{r}}$ is a $(N+L_{CP})\times1$ noise vector.

After taking DFT, the frequency domain representation of (17) is given by,

$$\tilde{\mathbf{z}}_{\mathbf{r}} = \sum_{m=1}^{2}\left(\tilde{\boldsymbol{\Lambda}}_{\mathbf{r}}\tilde{\mathbf{G}}_{\mathbf{m}}\tilde{\boldsymbol{\Lambda}}_{\mathbf{m}} + \tilde{\boldsymbol{\Phi}}_{\mathbf{r}}\tilde{\mathbf{G}}^{\#}_{\mathbf{m}}\tilde{\boldsymbol{\Phi}}^{\#}_{\mathbf{m}}\right)\tilde{\mathbf{p}}_{\mathbf{m}} + \sum_{m=1}^{2}\left(\tilde{\boldsymbol{\Lambda}}_{\mathbf{r}}\tilde{\mathbf{G}}_{\mathbf{m}}\tilde{\boldsymbol{\Phi}}_{\mathbf{m}} + \tilde{\boldsymbol{\Phi}}_{\mathbf{r}}\tilde{\mathbf{G}}^{\#}_{\mathbf{m}}\tilde{\boldsymbol{\Lambda}}^{\#}_{\mathbf{m}}\right)\tilde{\mathbf{p}}^{\#}_{\mathbf{m}} + \tilde{\boldsymbol{\Lambda}}''_{\mathbf{r}}\tilde{\mathbf{w}}_{\mathbf{r}} + \tilde{\boldsymbol{\Phi}}''_{\mathbf{r}}\tilde{\mathbf{w}}^{\#}_{\mathbf{r}} \tag{18}$$

where, the frequency domain parameters, $\tilde{\boldsymbol{\Lambda}}_1,\tilde{\boldsymbol{\Lambda}}_2,\tilde{\boldsymbol{\Lambda}}_r,\tilde{\mathbf{G}}_1$, $\tilde{\mathbf{G}}_2,\tilde{\boldsymbol{\Phi}}_1,\tilde{\boldsymbol{\Phi}}_2,\tilde{\boldsymbol{\Phi}}_r$ are the diagonal matrices defined as $\tilde{\boldsymbol{\Lambda}}_1 = \mathbf{F}\boldsymbol{\Lambda}_1\mathbf{F}^{\mathbf{H}}$, $\tilde{\boldsymbol{\Lambda}}_2 = \mathbf{F}\boldsymbol{\Lambda}_2\mathbf{F}^{\mathbf{H}}$, $\tilde{\boldsymbol{\Lambda}}_{\mathbf{r}} = \mathbf{F}\boldsymbol{\Lambda}_{\mathbf{r}}\mathbf{F}^{\mathbf{H}}$, $\tilde{\mathbf{G}}_{\mathbf{1}} = \mathbf{F}\mathbf{G}_{\mathbf{1}}\mathbf{F}^{\mathbf{H}}$, $\tilde{\mathbf{G}}_{\mathbf{2}} = \mathbf{F}\mathbf{G}_{\mathbf{2}}\mathbf{F}^{\mathbf{H}}$, $\tilde{\boldsymbol{\Phi}}_1 = \mathbf{F}\boldsymbol{\Phi}_1\mathbf{F}^{\mathbf{H}}$, $\tilde{\boldsymbol{\Phi}}_2 = \mathbf{F}\boldsymbol{\Phi}_2\mathbf{F}^{\mathbf{H}}$, $\tilde{\boldsymbol{\Phi}}_r = \mathbf{F}\boldsymbol{\Phi}_r\mathbf{F}^{\mathbf{H}}$ with $\mathbf{F}$ as a $N\times N$ DFT matrix. Moreover, $\tilde{\mathbf{z}} = \mathbf{F}\mathbf{z}$, $\tilde{\mathbf{p}}_{\mathbf{m}} = \mathbf{F}\mathbf{p}_{\mathbf{m}}$, $\tilde{\boldsymbol{\Lambda}}^{\#}_1 = \mathbf{F}\boldsymbol{\Lambda}^{\#}_1\mathbf{F}^{\mathbf{H}}$, $\tilde{\boldsymbol{\Lambda}}^{\#}_2 = \mathbf{F}\boldsymbol{\Lambda}^{\#}_2\mathbf{F}^{\mathbf{H}}$, $\tilde{\boldsymbol{\Lambda}}^{\#}_r = \mathbf{F}\boldsymbol{\Lambda}^{\#}_r\mathbf{F}^{\mathbf{H}}$, $\tilde{\mathbf{G}}^{\#}_1 = \mathbf{F}\mathbf{G}^{\#}_1\mathbf{F}^{\mathbf{H}}$, $\tilde{\mathbf{G}}^{\#}_2 = \mathbf{F}\mathbf{G}^{\#}_2\mathbf{F}^{\mathbf{H}}$, $\tilde{\boldsymbol{\Phi}}^{\#}_1 = \mathbf{F}\boldsymbol{\Phi}^{\#}_1\mathbf{F}^{\mathbf{H}}$, $\tilde{\boldsymbol{\Phi}}^{\#}_2 = \mathbf{F}\boldsymbol{\Phi}^{\#}_2\mathbf{F}^{\mathbf{H}}$, $\tilde{\mathbf{w}}_{\mathbf{r}} = \mathbf{F}\mathbf{w}_{\mathbf{r}}$ and $\tilde{\mathbf{w}}^{\#}_{\mathbf{r}} = \mathbf{F}\mathbf{w}'_{\mathbf{r}}$ respectively. In (18), the noise term $\tilde{\boldsymbol{\Lambda}}''_{\mathbf{r}}\tilde{\mathbf{w}}_{\mathbf{r}} + \tilde{\boldsymbol{\Phi}}''\tilde{\mathbf{w}}^{\#}_{\mathbf{r}} = \tilde{\mathbf{n}}_{\mathbf{r}}$ is a complex Gaussian coloured noise with covariance matrix given by

$$\mathbf{R}_{\tilde{\mathbf{n}}_{\mathbf{r}}\tilde{\mathbf{n}}_{\mathbf{r}}} = E\left\{\tilde{\mathbf{n}}_{\mathbf{r}}\tilde{\mathbf{n}}^{\mathbf{H}}_{\mathbf{r}}\right\} = \sigma^2\left(\mathbf{F}\boldsymbol{\Lambda}''_r\boldsymbol{\Lambda}''^H_r\mathbf{F}^H + \mathbf{F}\boldsymbol{\Phi}''_r\boldsymbol{\Phi}''^H_r\mathbf{F}^H\right) \tag{19}$$

where $\tilde{\boldsymbol{\Lambda}}''_{\mathbf{r}}$ and $\tilde{\boldsymbol{\Phi}}''_{\mathbf{r}}$ are the matrices of size $N\times(N+L_{CP})$.

Let $\boldsymbol{\psi}_{\mathbf{11}} = \tilde{\boldsymbol{\Lambda}}_{\mathbf{r}}\tilde{\mathbf{G}}_1\tilde{\boldsymbol{\Lambda}}_1 + \tilde{\boldsymbol{\Phi}}_{\mathbf{r}}\tilde{\mathbf{G}}^{\#}_1\tilde{\boldsymbol{\Phi}}^{\#}_1$, $\boldsymbol{\psi}_{12} = \tilde{\boldsymbol{\Lambda}}_{\mathbf{r}}\tilde{\mathbf{G}}_1\tilde{\boldsymbol{\Phi}}_1 + \tilde{\boldsymbol{\Phi}}_{\mathbf{r}}\tilde{\mathbf{G}}^{\#}_1\tilde{\boldsymbol{\Lambda}}^{\#}_1$, $\boldsymbol{\psi}_{21} = \tilde{\boldsymbol{\Lambda}}_{\mathbf{r}}\tilde{\mathbf{G}}_2\tilde{\boldsymbol{\Lambda}}_2 + \tilde{\boldsymbol{\Phi}}_{\mathbf{r}}\tilde{\mathbf{G}}^{\#}_2\tilde{\boldsymbol{\Phi}}^{\#}_2$, $\boldsymbol{\psi}_{22} = \tilde{\boldsymbol{\Lambda}}_{\mathbf{r}}\tilde{\mathbf{G}}_2\tilde{\boldsymbol{\Phi}}_2 + \tilde{\boldsymbol{\Phi}}_{\mathbf{r}}\tilde{\mathbf{G}}^{\#}_2\tilde{\boldsymbol{\Lambda}}^{\#}_2$. Then eqn. (18) can be written simply as

$$\tilde{\mathbf{z}}_{\mathbf{r}} = \boldsymbol{\psi}_{11}\tilde{\mathbf{p}}_{\mathbf{1}} + \boldsymbol{\psi}_{12}\tilde{\mathbf{p}}^{\#}_{\mathbf{1}} + \boldsymbol{\psi}_{21}\tilde{\mathbf{p}}_2 + \boldsymbol{\psi}_{22}\tilde{\mathbf{p}}^{\#}_2 + \tilde{\mathbf{n}}_{\mathbf{r}} \tag{20}$$

$\tilde{\mathbf{z}}_{\mathbf{r}}$ is a $N\times1$ vector containing superimposed symbols transmitted from source nodes S_1 and S_2. Moreover, for the given values of $a_1,a_2,\theta_1,\theta_2,\varepsilon_1,\varepsilon_2,a_r,\theta_r,\varepsilon_r$, the $\tilde{\mathbf{z}}_{\mathbf{r}}$ is a N - dimensional multivariate Gaussian with a zero mean and covariance matrix, $\mathbf{R}_{\tilde{\mathbf{n}}_{\mathbf{r}}\tilde{\mathbf{n}}_{\mathbf{r}}}$. A N - dimensional ML search is performed at the relay node to detect the superposed symbols, $\tilde{x}_3(k)$, $k=0,1,...,N-1$. Assuming that the channel state information is known the ML estimate of $N\times1$ vector, $\tilde{\mathbf{x}}_{\mathbf{3}}$ is given by

$$\hat{\tilde{\mathbf{x}}}_{\mathbf{3}} = \arg\min_{(A)^N}\left(\left[\tilde{\mathbf{z}}-\boldsymbol{\psi}_{11}-\boldsymbol{\psi}_{12}-\boldsymbol{\psi}_{21}-\boldsymbol{\psi}_{22}\right]^H\mathbf{R}^{-1}_{\tilde{\mathbf{w}}_{\mathbf{r}}\tilde{\mathbf{w}}_{\mathbf{r}}}\left[\tilde{\mathbf{z}}-\boldsymbol{\psi}_{11}-\boldsymbol{\psi}_{12}-\boldsymbol{\psi}_{21}-\boldsymbol{\psi}_{22}\right]\right) \tag{21}$$

After the detection, PLNC mapping is carried out at each subcarrier such that $\{-2,+2\}$ is mapped to bit 0 and $\{0\}$ is mapped to bit 1.

2.2 Time slot II

2.2.1 Signal Transmission from Relay Node

In time slot II, the relay node, R broadcasts the OFDM signal to the destination nodes S_1 and S_2. Let $\mathbf{s}_{\mathbf{3}}$ be the $N\times1$ vector to be broadcasted. The OFDM signal corresponding to the signal $\mathbf{s}_3[k]$, $k=0,1,...N-1$ is generated by taking IDFT and appending CP of length L_{CP}. These broadcasting messages are further affected by the frequency dependent (FD) and frequency independent (FI) imbalances of the transmitter at the relay node. Here the I/Q imbalance at the transmitter of relay is considered.

The OFDM samples are passed through a Digital-to-Analog Converter (DAC) and a pulse-shaping filter. Let $h_{r_t}(t)$ be the impulse response of the pulse shaping filter. The continuous time outputs of the pulse shaping filters at in-phase and quadrature branches are represented as,

$$\begin{aligned} p_3^I(t) &= \sum_{n=-\infty}^{\infty} x_{3,t}^I(n) h_{r_t}^I(t-nT) \\ p_3^Q(t) &= \sum_{n=-\infty}^{\infty} x_3^Q(n) h_{r_t}^Q(t-nT) \end{aligned} \tag{22}$$

Let $\varepsilon_{r_t}^I(t)$ and $\varepsilon_{r_t}^Q(t)$ denote the impulse response of the FD imbalance filters at I and Q branches. The imbalance filter outputs at respective branches are represented as

$$u_{3,t}^I(t) = p_3^I(t) * \varepsilon_{r_t}^I(t) \tag{23}$$

$$u_{3,t}^Q(t) = p_3^Q(t) * \varepsilon_{r_t}^Q(t) \tag{24}$$

Let $a_{3,t}^I, a_{3,t}^Q$ and $\theta_{3,t}^I, \theta_{3,t}^Q$ be the amplitude and phase mismatches due to FI I/Q imbalance at the transmitter section of the relay node. In the presence of I/Q imbalance, the pass band signal at the output of the relay node is given by

$$s_3^p(t) = s_3^{p_I}(t)\cos(2\pi f_c t) - s_3^{p_Q}(t)\sin(2\pi f_c t) \tag{25}$$

where $s_3^{p_I}(t) = \left(u_{3,t}^I(t) a_{3,t}^I \cos\theta_{3,t}^I - u_{3,t}^Q(t) a_{3,t}^Q \sin\theta_{3,t}^Q\right)$ and $s_3^{p_Q}(t) = \left(u_{3,t}^I(t) a_{3,t}^I \sin\theta_{3,t}^I + u_{3,t}^Q(t) a_{3,t}^Q \cos\theta_{3,t}^Q\right)$.

The baseband equivalent of $s_3^p(t)$ is given by

$$s_3^b(t) = \mu_{r_t} u_{3,t}(t) + v_{r_t} u_{3,t}'(t) \tag{26}$$

where $\mu_{r_t} = \dfrac{a_{3,t}^I e^{j\theta_{3,t}^I} + a_{3,t}^Q e^{j\theta_{3,t}^Q}}{2}$, $v_{r_t} = \dfrac{a_{3,t}^I e^{j\theta_{3,t}^I} - a_{3,t}^Q e^{j\theta_{3,t}^Q}}{2}$, $u_{3,t}^I(t) = \dfrac{u_{3,t}(t) + u_{3,t}'(t)}{2}$ and $u_{3,t}^Q(t) = \dfrac{u_{3,t}(t) - u_{3,t}'(t)}{2j}$

Using Equations (23) and (24), Equation (26) is rewritten as,

$$s_3^b(t) = \lambda_{r_t}(t) * p_3(t) + \phi_{r_t}(t) * p_3'(t) \tag{27}$$

where,

$$\begin{aligned} \lambda_{r_t}(t) &= \frac{1}{2}\left(\left(\mu_{r_t} + v_{r_t}\right)\varepsilon_{r_t}^I(t) + \left(\mu_{r_t} - v_{r_t}\right)\varepsilon_{r_t}^Q(t)\right) \\ \phi_{r_t}(t) &= \frac{1}{2}\left(\left(\mu_{r_t} + v_{r_t}\right)\varepsilon_{r_t}^I(t) - \left(\mu_{r_t} - v_{r_t}\right)\varepsilon_{r_t}^Q(t)\right) \end{aligned} \tag{28}$$

Let $g_1^p(t)$ be the channel impulse response of R to S_1 respectively. The receive information at the source nodes are demodulated to detect, $\mathbf{s_3}$. Assuming perfect timing and frequency offset correction, the DFT output vector $\tilde{\mathbf{y}}_1$ and $\tilde{\mathbf{y}}_2$ at the source nodes S_1 and S_2 respectively perturbed by I/Q Imbalances due to relay transmitter and source node receiver.

2.2.2 Signal Reception at Source Node:

The receive signal at the source nodes S_1 and S_2 are given by,

$$y_1^p(t) = \left\{g_1^p(t) * s_3^p(t)\right\} + w_1^p(t) \tag{29}$$

$$y_2^p(t) = \left\{g_2^p(t) * s_3^p(t)\right\} + w_2^p(t) \tag{30}$$

The baseband equivalent of $y_1^p(t)$ and $y_2^p(t)$ is given by,

$$y_1(t) = \left\{g_1(t) * s_3^b(t)\right\} + w_1(t) \tag{31}$$

$$y_2(t) = \left\{g_2(t) * s_3^b(t)\right\} + w_2(t) \tag{32}$$

Let $a_{1,r}^{I}, a_{1,r}^{Q}$ and $\theta_{1,r}^{I}, \theta_{1,r}^{Q}$ be the amplitude and phase mismatches during signal reception at the destination node S_1. Further, let $\mu_{1_r} = \frac{1}{2}\left(a_{1,r}^{I} e^{-j\theta_{1,r}^{I}} + a_{1,r}^{Q} e^{-j\theta_{1,r}^{Q}}\right)$, $v_{1,r} = \frac{1}{2}\left(a_{1,r}^{I} e^{j\theta_{1,r}^{I}} - a_{1,r}^{Q} e^{j\theta_{1,r}^{Q}}\right)$. Let $\varepsilon_{1_r}^{I}(t)$ and $\varepsilon_{1_r}^{Q}(t)$ be the FD I/Q Imbalance at the destination node, S_1 during reception. The baseband equivalent output at the imbalance filter is given by,

$$\begin{aligned} z_{3,s_1}(t) &= z_{3,s_1}^{I}(t) + jz_{3,s_1}^{Q}(t) \\ &= \frac{1}{2}\left[\left(\mu_{1_r} + v'_{1_r}\right)\varepsilon_{1_r}^{I}(t) + \left(\mu_{1_r} - v'_{1_r}\right)\varepsilon_{1_r}^{Q}(t)\right] * h_{1_r}(t) * y_1(t) \\ &\quad + \frac{1}{2}\left[\left(\mu'_{1_r} + v_{1_r}\right)\varepsilon_{1_r}^{I}(t) + \left(v_{1_r} - \mu'_{1_r}\right)\varepsilon_{1_r}^{Q}(t)\right] * h_{1_r}(t) * y'_1(t) \end{aligned} \tag{33}$$

Let $\lambda_{1_r}(t) = \frac{1}{2}\left[\left(\mu_{1_r} + v'_{1_r}\right)\varepsilon_{1_r}^{I}(t) + \left(\mu_{1_r} - v'_{1_r}\right)\varepsilon_{1_r}^{Q}(t)\right]$ and $\phi_{1_r}(t) = \frac{1}{2}\left[\left(\mu'_{1_r} + v_{1_r}\right)\varepsilon_{1_r}^{I}(t) + \left(v_{1_r} - \mu'_{1_r}\right)\varepsilon_{1_r}^{Q}(t)\right]$.

Then, (33) can be simplified as

$$z_3(t) = \lambda_{1_r}(t) * h_{1_r}(t) * y_1(t) + \phi_{1_r}(t) * h_{1_r}(t) * y'_1(t) \tag{34}$$

Using (28) and (30) , (33) is written as

$$\begin{aligned} z_{3,m}(t) &= \sum_{m=1}^{2} g_m(t) * \left\{\lambda_{r_t}(t) * p_3(t) + \phi_{r_t}(t) * p'_3(t)\right\} * h_{m_r}(t) * \lambda_{m_r}(t) \\ &\quad + \sum_{m=1}^{2} g'_m(t) * \left\{\lambda_{r_t}(t) * p_3(t) + \phi_{r_t}(t) * p'_3(t)\right\}' * h_{m_r}(t) * \phi_{m_r}(t) \\ &\quad + \left\{\left(\lambda_{m_r}(t) * w_m(t)\right) + \left(\phi_{m_r}(t) * w'_m(t)\right)\right\} * h_{m_r}(t) \qquad m = 1,2 \end{aligned} \tag{35}$$

where, $m = 1,2$ represents the signal received at the source node S_1 and source node S_2 respectively.

Upon rearranging (35), it can be written as

$$\begin{aligned} z_{3,m}(t) &= \left(\sum_{m=1}^{2}\left[\lambda_{m_r}(t) * g_m(t) * \lambda_{r_t}(t) + \phi_{m_r}(t) * g'_m(t) * \phi'_{r_t}(t)\right] * p_3(t) * h_{m_r}(t)\right) \\ &\quad + \left(\sum_{m=1}^{2}\left[\lambda_{m_r}(t) * g_m(t) * \phi_{r_t}(t) + \phi_{m_r}(t) * g'_m(t) * \lambda'_{r_t}(t)\right] * p'_3(t) * h_{m_r}(t)\right) \\ &\quad + \left\{\left(\lambda_{m_r}(t) * w_m(t)\right) + \left(\phi_{m_r}(t) * w'_m(t)\right)\right\} * h_{m_r}(t) \quad ; \quad 0 \le t \le T + T_{CP} \end{aligned} \tag{36}$$

The output of the imbalance filter $z_r(t)$ is sampled and the cyclic prefix is removed. Assuming $\left\{h_{r_t}(t) * h_{m_r}(t)\right\}$, $m = 1,2$ satisfies Nyquist criterion for pulse shaping, the discrete time samples after removing the cyclic prefix can be represented as

$$\begin{aligned} z_{3,m}(n) &= \left(\sum_{m=1}^{2}\left[\lambda_{m_r}(n) \circ g_m(n) \circ \lambda_{r_t}(n) + \phi_{m_r}(n) \circ g'_m(n) \circ \phi'_{r_t}(n)\right] \circ p_3(n)\right) \\ &\quad + \left(\sum_{m=1}^{2}\left[\lambda_{m_r}(n) \circ g_m(n) \circ \phi_{r_t}(n) + \phi_{m_r}(n) \circ g'_m(n) \circ \lambda'_{r_t}(n)\right] \circ p'_3(n)\right) \\ &\quad + \left(\lambda_{m_r}(n) \circ w_m(n)\right) + \left(\phi_{m_r}(n) \circ w'_m(n)\right) \quad ; \quad 0 \le n \le N-1 \end{aligned} \tag{37}$$

Let the sampled version of $\lambda_{r_t}(t)$ be $\boldsymbol{\lambda}_{r_t} = \left[\lambda_{r_t}(0), \ldots, \lambda_{r_t}\left(L_{\lambda_{r_t}} - 1\right), \mathbf{0}_{1\times\left(N - L_{\lambda_{r_t}}\right)}^{T}\right]^{T}$ be a $N \times 1$ vector, where, $L_{\lambda_{r_t}}$ is the number of non zero elements in $\boldsymbol{\lambda}_{r_t}$. Similarly, the vectors $\boldsymbol{\lambda}_{\mathbf{m_r}}, \boldsymbol{\varphi}_{\mathbf{r_t}}, \boldsymbol{\varphi}_{\mathbf{m_r}}, \boldsymbol{\varphi}_{\mathbf{r}}, \mathbf{g_1}, \mathbf{g_2}$ and $\mathbf{p_3}$ are $N \times 1$ vector representation of samples from $\lambda_{m_r}(t), \phi_{r_t}(t), \phi_{m_r}(t), g_1(t), g_2(t)$ and $p_3(t)$ respectively. Moreover, $\boldsymbol{\Lambda}_{\mathbf{r_t}}, \boldsymbol{\Lambda}_{\mathbf{m_r}}$, $\boldsymbol{\Phi}_{\mathbf{r_t}}, \boldsymbol{\Phi}_{\mathbf{m_r}}, \mathbf{G}_1$ and $\mathbf{G}_2$ be the $N \times N$ circulant matrices with first columns as $\boldsymbol{\lambda}_{\mathbf{r_t}}, \boldsymbol{\lambda}_{\mathbf{m_r}}, \boldsymbol{\varphi}_{\mathbf{r_t}}, \boldsymbol{\varphi}_{\mathbf{m_r}}, \mathbf{g_1}$ and $\mathbf{g_2}$ respectively. The matrix representation of (37) is represented as

$$\mathbf{z}_{3,m} = \sum_{m=1}^{2}\left(\mathbf{\Lambda}_{\mathbf{m_r}}\mathbf{G_m}\mathbf{\Lambda}_{\mathbf{r_t}} + \mathbf{\Phi}_{\mathbf{m_r}}\mathbf{G_m^H}\mathbf{\Phi}_{\mathbf{r_t}}^{\mathbf{H}}\right)\mathbf{p_3} + \sum_{m=1}^{2}\left(\mathbf{\Lambda}_{\mathbf{m_r}}\mathbf{G_m}\mathbf{\Phi}_{\mathbf{r_t}} + \mathbf{\Phi}_{\mathbf{m_r}}\mathbf{G_m^H}\mathbf{\Lambda}_{\mathbf{r_t}}^{\mathbf{H}}\right)\mathbf{p}_3' + \sum_{m=1}^{2}\left[\mathbf{\Lambda}_{\mathbf{m_r}}\mathbf{w_m} + \mathbf{\Phi}_{\mathbf{m_r}}\mathbf{w}_{\mathbf{m}}'\right] \quad (38)$$

where, $\mathbf{\Lambda}_{\mathbf{m_r}}$ is a $[N\times(N+L_{CP})]$ matrix and $\mathbf{w_m}$ is a $(N+L_{CP})\times 1$ vector of noise. After taking DFT of (38), it is given by,

$$\tilde{\mathbf{z}}_{3,m} = \sum_{\mathbf{m=1}}^{\mathbf{2}}\left(\tilde{\mathbf{\Lambda}}_{\mathbf{m_r}}\tilde{\mathbf{G}}_{\mathbf{m}}\tilde{\mathbf{\Lambda}}_{\mathbf{r_t}} + \tilde{\mathbf{\Phi}}_{\mathbf{m_t}}\tilde{\mathbf{G}}_{\mathbf{m}}^{\#}\tilde{\mathbf{\Phi}}_{\mathbf{r_t}}^{\#}\right)\tilde{\mathbf{p}}_{\mathbf{3}} + \sum_{\mathbf{m=1}}^{\mathbf{2}}\left(\tilde{\mathbf{\Lambda}}_{\mathbf{m_r}}\tilde{\mathbf{G}}_{\mathbf{m}}\tilde{\mathbf{\Phi}}_{\mathbf{r_t}} + \tilde{\mathbf{\Phi}}_{\mathbf{m_r}}\tilde{\mathbf{G}}_{\mathbf{m}}^{\#}\tilde{\mathbf{\Lambda}}_{\mathbf{r_t}}^{\#}\right)\tilde{\mathbf{p}}_{\mathbf{3}}^{\#} + \sum_{m=1}^{2}\left(\tilde{\mathbf{\Lambda}}_{\mathbf{m_r}}''\tilde{\mathbf{w}}_{\mathbf{m}} + \tilde{\mathbf{\Phi}}_{\mathbf{m_r}}''\tilde{\mathbf{w}}_{\mathbf{m}}^{\#}\right) \quad (39)$$

where the frequency domain parameters, $\tilde{\mathbf{\Lambda}}_{\mathbf{1_r}}$, $\tilde{\mathbf{\Lambda}}_{\mathbf{2_r}}$, $\tilde{\mathbf{\Lambda}}_{\mathbf{r_t}}$, $\tilde{\mathbf{G}}_{\mathbf{1}}$, $\tilde{\mathbf{G}}_{\mathbf{2}}$, $\tilde{\mathbf{\Phi}}_{\mathbf{1_r}}$, $\tilde{\mathbf{\Phi}}_{\mathbf{2_r}}$, $\tilde{\mathbf{\Phi}}_{\mathbf{r_t}}$ are the diagonal matrices. In (39), the noise term $\tilde{\mathbf{\Lambda}}_{\mathbf{m_r}}\tilde{\mathbf{w}}_{\mathbf{m}}+\tilde{\mathbf{\Phi}}_{\mathbf{m_r}}\tilde{\mathbf{w}}_{\mathbf{m}}^{\#}=\tilde{\mathbf{n}}_{\mathbf{m_r}}$ is a complex Gaussian colored noise with covariance matrix given by

$$\mathbf{R}_{\tilde{\mathbf{n}}_{\mathbf{m_r}}\tilde{\mathbf{n}}_{\mathbf{m_r}}} = E\left\{\tilde{\mathbf{n}}_{\mathbf{m_r}}\tilde{\mathbf{n}}_{\mathbf{m_r}}^{\mathbf{H}}\right\} = \sigma^2\left(\mathbf{F}\mathbf{\Lambda}_{\mathbf{m_r}}\mathbf{\Lambda}_{\mathbf{m_r}}^{\mathbf{H}}\mathbf{F}^H + \mathbf{F}\mathbf{\Phi}_{\mathbf{m_r}}\boldsymbol{\varphi}_{\mathbf{m_r}}^{\mathbf{H}}\mathbf{F}^H\right) \quad (40)$$

where $\tilde{\mathbf{\Lambda}}_{\mathbf{m_r}}''$ and $\tilde{\mathbf{\Phi}}_{\mathbf{m_r}}''$ are the matrices of size $N\times(N+L_{CP})$. Let $\boldsymbol{\psi}_{\mathbf{11}} = \tilde{\mathbf{\Lambda}}_{\mathbf{1_r}}\tilde{\mathbf{G}}_{\mathbf{1}}\tilde{\mathbf{\Lambda}}_{\mathbf{r_t}} + \tilde{\mathbf{\Phi}}_{\mathbf{1_t}}\tilde{\mathbf{G}}_{\mathbf{1}}^{\#}\tilde{\mathbf{\Phi}}_{\mathbf{r_t}}^{\#}$, $\boldsymbol{\psi}_{\mathbf{12}} = \tilde{\mathbf{\Lambda}}_{\mathbf{1_r}}\tilde{\mathbf{G}}_{\mathbf{1}}\tilde{\mathbf{\Phi}}_{\mathbf{r_t}} + \tilde{\mathbf{\Phi}}_{\mathbf{1_r}}\tilde{\mathbf{G}}_{\mathbf{1}}^{\#}\tilde{\mathbf{\Lambda}}_{\mathbf{r_t}}^{\#}$, $\boldsymbol{\psi}_{\mathbf{21}} = \tilde{\mathbf{\Lambda}}_{\mathbf{2_r}}\tilde{\mathbf{G}}_{\mathbf{2}}\tilde{\mathbf{\Lambda}}_{\mathbf{r_t}} + \tilde{\mathbf{\Phi}}_{\mathbf{2_t}}\tilde{\mathbf{G}}_{\mathbf{2}}^{\#}\tilde{\mathbf{\Phi}}_{\mathbf{r_t}}^{\#}$, $\boldsymbol{\psi}_{\mathbf{22}} = \tilde{\mathbf{\Lambda}}_{\mathbf{2_r}}\tilde{\mathbf{G}}_{\mathbf{2}}\tilde{\mathbf{\Phi}}_{\mathbf{r_t}} + \tilde{\mathbf{\Phi}}_{\mathbf{2_r}}\tilde{\mathbf{G}}_{\mathbf{2}}^{\#}\tilde{\mathbf{\Lambda}}_{\mathbf{r_t}}^{\#}$. Then, (39) can be simply written as

$$\tilde{\mathbf{z}}_{3,m} = \boldsymbol{\psi}_{\mathbf{11}}\tilde{\mathbf{p}}_{\mathbf{3}} + \boldsymbol{\psi}_{\mathbf{12}}\tilde{\mathbf{p}}_{\mathbf{3}}^{\#} + \boldsymbol{\psi}_{\mathbf{21}}\tilde{\mathbf{p}}_{\mathbf{3}} + \boldsymbol{\psi}_{\mathbf{22}}\tilde{\mathbf{p}}_{\mathbf{3}}^{\#} + \tilde{\mathbf{n}}_{\mathbf{r}} \quad (41)$$

Using the receive signal at at source node 1, $\tilde{z}_{3,1}[k]$ the signal from S_2 is determined as

$$\hat{x}_2[k] = \tilde{x}_1[k].\tilde{z}_{3,1}[k] \quad (42)$$

Similarly at source S_2, the signal from source S_1 is decoded as

$$\hat{x}_1[k] = \tilde{x}_2[k].\tilde{z}_{3,1}[k] \quad (43)$$

III. PERFORMANCE ANALYSIS

The OFDM symbol at k^{th} subcarrier at the relay node, R can be rewritten from eqn. (18) as below,

$$\tilde{z}_r[k] = \sum_{m=1}^{2}(a_m + b_m)\tilde{s}_m[k] + c_m\tilde{s}_m^*[k] + n_r[k] \quad (44)$$

where $a_m = \tilde{\lambda}_r[k]\tilde{\lambda}_m[k]\tilde{g}_m[k] + \Phi_r[k]\Phi_m^*[k]\tilde{g}_m^*[k]$ $a_m = \tilde{\lambda}_r[k]\tilde{\lambda}_m[k]\tilde{g}_m[k] + \tilde{\Phi}_r[k]\tilde{\Phi}_m^*[k]\tilde{g}_m^*[k]$ is in-phase branch I/Q parameters, $b_m = \tilde{\lambda}_m[k]\tilde{g}_m[k]\tilde{\Phi}_r[k] + \tilde{\Phi}_m[k]\tilde{g}_m^*[k]\tilde{\lambda}_r^*[k]$ is quadrature branch I/Q parameters. The interference due to I/Q imbalance is defined as

$$c_m = \tilde{\lambda}_r[k]\tilde{\lambda}_m[N-k]\tilde{g}_m[N-k] + \tilde{\Phi}_r[k]\tilde{\Phi}_m^*[N-k]\tilde{g}_m^*[N-k]$$
$$+\tilde{\lambda}_m[k]\tilde{g}_m[N-k]\tilde{\Phi}_r[N-k] + \tilde{\Phi}_m[k]\tilde{g}_m^*[N-k]\tilde{\lambda}_r^*[N-k]$$

Using (44), the instantaneous Signal to Interference plus Noise Ratio (SINR)'s $\gamma_1[k]$ and $\gamma_2[k]$ at the relay node in time slot I and the SINRs $\gamma_3[k]$ and $\gamma_4[k]$ at S_1 and S_2 in time slot II, at k^{th} subcarrier, are determined as

$$\gamma_m^{IQ}[k] = \frac{\left|\tilde{g}_m[k]\right|^2}{A_m\left|\tilde{g}_m[N-k]\right|^2 + B} \quad m = 1,2,3,4 \quad (45)$$

where $A_m = \left(\dfrac{c_m}{a_m + b_m}\right)$ and $B = \dfrac{\sigma^2}{\varepsilon_x}$

In this subsection, the effect of I/Q Imbalance on the outage performance at the relay and at the destinations is characterized. The cumulative distribution function (CDF) of $\gamma_1[k]$ can be expressed as $P(\gamma_1[k] < \gamma)$. Let, the signal to interference noise ratio $\gamma_1[k]$ is further simplified as $\gamma_1[k] = \dfrac{X_1}{A_1Y_1 + B_1}$, where, $X_1 = \left|\tilde{g}_1[k]\right|^2$ and $Y_1 = \left|\tilde{g}_1[N-k]\right|^2$. The CDF of $\gamma_1[k]$ can be written as

$$\begin{aligned} F_{\gamma_1[k]}(\gamma) &= \int_0^\infty \left(X < \gamma\left(A_1 y + B_1\right) \middle| Y = y\right) f_Y(y)\,dy \\ &= \int_0^\infty \int_0^{\gamma(A_1 y + B_1)} f_X(x)\, f_Y(y) dx dy = \int_0^\infty \left[1 - e^{-\gamma(A_1 y + B_1)}\right] f_Y(y)\,dy \\ &= \left(1 - e^{-\gamma B_1} + \gamma A_1 e^{-\gamma B_1}\right) \end{aligned} \tag{46}$$

where the threshold SINR, $\gamma = 2^R - 1$. Similarly, CDF of $\gamma_2[k]$ can be expressed as

$F_{\gamma_2[k]}(\gamma) = \left(1 - e^{-\gamma B_2} + \gamma A_2 e^{-\gamma B_2}\right)$. Let the I/Q imbalance parameters $A_1 = A_2 = A,\ B_1 = B_2 = B$

Therefore, the upper bound outage probability at the relay node is determined as

$$P^U_{out,relay}(\gamma) = \left(2 - (1 - 2A\gamma)\exp\left(-2\gamma / SNR\right)\right) \tag{47}$$

The outage probability at the relay node for the lower bound is calculated as follows,

The CDF of $\min\left(\gamma_1[k], \gamma_2[k]\right)$ is expressed as $F_{\gamma_{\min}(k)}(\gamma) = 1 - \left(1 - F_1(\gamma)\right)\left(1 - F_2(\gamma)\right)$. It is further simplified and the lower bound outage probability is obtained as

$$\begin{aligned} P^L_{out,\mathrm{Re}lay}(\gamma) &= 1 - \exp\left(-\gamma/(2SNR)\right) + \gamma A \exp\left(-\gamma/(2SNR)\right) \\ &\quad + \gamma A \exp\left(-\gamma/(2SNR)\right) - \gamma^2 A^2 \exp\left(-\gamma/(2SNR)\right) \end{aligned} \tag{48}$$

The outage probability at the destination node S_1 during the second time slot is given by

$$P_{out,dest}(\gamma) = \left(1 - (1 - \gamma A_3)\exp\left(-\gamma / SNR\right)\right) \tag{49}$$

The end-to-end outage probability is defined as $P_{out,end}(\gamma) = P_{out,relay}(\gamma) + \left\{1 - P_{out,relay}(\gamma)\right\} P_{out,dest}(\gamma)$

Let $A_3 = A$, substituting (48) and (49), the End-to-End outage probability is determined as

$$\begin{aligned} P_{out,E-to-E}(\gamma) &= 1 + (1 - \gamma A)\exp\left(-\gamma/SNR\right) \\ &\quad - 2(1 + \gamma^2 A^3 - \gamma A - \gamma A^2)\exp\left(-\gamma/(3SNR)\right) \end{aligned} \tag{50}$$

IV. Numerical Results And Discussion

In this section, the outage performance of OFDM based bidirectional relay network in the presence of I/Q Imbalance is analyzed. The analytical expressions are derived for the outage probabilities at both the time slots. The number of subcarriers is assumed to be 64 and bandwidth is 20 MHz. Figure 4 shows the outage performance of OFDM based bidirectional relay network at $1\,b/s/Hz$ in the relay node. At the amplitude and phase mismatch of $a = 1.1, \theta = 3^0$ the outage probability is increased and further at the IQ Imbalance of $a = 1.4, \theta = 8^0$ the system reaches deep fade.

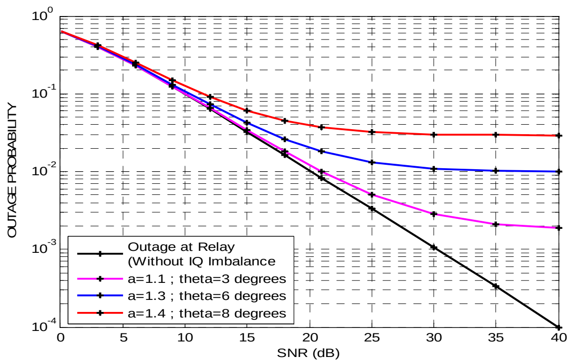

Fig 4. Outage Performance at the relay node with I/Q Imbalance at 1 b/s/Hz

The outage performance of End– to – End transmission for different levels of I/Q Imbalances are presented in Figure 4. At the amplitude and phase mismatches of $a = 1.1, \theta = 2^0$ the outage probability is increased to 8 dB at 10^{-2} and further at the IQ Imbalance of $a = 1.4, \theta = 8^0$ the system reaches deep fade.

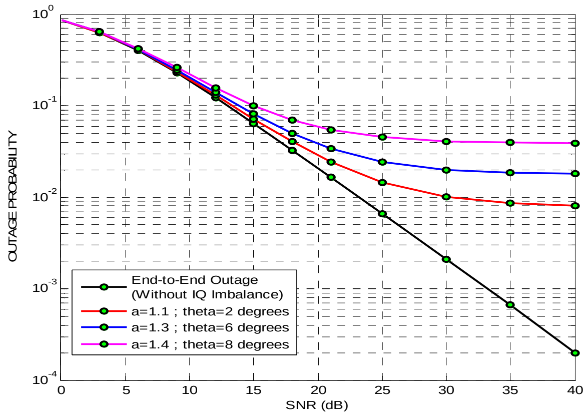

Fig 5. End-to-End Outage probability Performance with I/Q Imbalance at 1 b/s/Hz

V. Conclusion

The impact of I/Q Imbalance on OFDM based bidirectional relay network has been analyzed in terms of outage probability. Analytical expressions are derived for outage probabilities at both the time slots. The outage performance is calculated for individual as well as for combined levels of amplitude and phase mismatches. The outage analysis in the presence of I/Q imbalance would be very much helpful in developing compensation algorithms and to improve the performance of the OFDM based relay networks in the presence of amplitude and phase mismatches.

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