Service demand forecasting through the systemability model: a case study

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***Abstract*—Companies competing in an increasingly competitive market must ensure the production of goods with excellent performance, able to satisfy their customers and which have low manufacturing and management costs. It is in this context that companies have, in recent years, invested in research and development and have upgraded their reliability and maintenance functions. In many cases, the maintenance engineers have attempted to predict the reliability of the products, at least for evaluating the number of warranty repairs to be performed. This approach is on the one hand, extremely appropriate but, on the other, must face the difficulties of making laboratory test in conditions often radically different from those that the products meet during their normal operation. Frequently, the reliability estimation, coming from experimental test (in-house) are different from those obtained by the analysis of the service data (in-field). The former are executed in laboratory with standardized, controlled and repeatable conditions, while the latter are affected by random environmental and operating conditions. In the field of household appliances, this is so true that the conditions of use may vary even from country to country. There are some approaches that allow to assess the reliability performance of a system starting from the results of experimental tests performed in a laboratory. One of these was proposed some years ago and is called systemability. In this study, it was applied, for the first time, this approach to the field of household appliances. In addition, we wanted to try to identify the parameters that allowed to distinguish two different European markets. In fact the in-field data come from two different countries and could be considered a great opportunity to validate the correlation model. In fact, it was possible to investigate the effects of two different environmental condition sets (costumer behaviours, , market issues, logistics, etc.) on the reliability performances of a product population that has been manufactured in the same industrial plant. One of the most important outcomes of the Systemability model was the capacity to predicts two different in-field reliability performances relative to two different markets in contrast with the classic methodology that uses the same in-house reliability data without considering environmental effects. The initial stage of modeling was followed by a second validation phase, which gave satisfactory results. The overall outcomes were very positive and they have allowed us to focus some improvements in maintenance management that will lead to greater effectiveness of the method in the coming years.**

Keyword- Reliability, random environmental factor, Systemability, Availability, Maintainability engineering.

I. INTRODUCTION

In the last few years, the changes in the global markets have led to an increasing level of competition among companies to be able to satisfy increasingly demanding customers but with fewer economic resources. As a matter of fact the strong integration of international trade business has, on the one hand, significantly increased the number of potential customers but, on the other, the economic recession affecting most of the marketplace is making the business environment more complex and risky.

With the diffusion of new media and social network, customers are becoming more sensitive and well informed about the intrinsic characteristics of a product, such as its features, its performances, the level of the after sales services, whether the price is really competitive, and so on.

To have a competitive advantage over contenders, manufacturers must focus their attention on those aspects of the product that are tangible for customers as its performances (how well the product accomplishes its functions) and its duration (which is its success probability or, in other words, its reliability). Being able to obtain a high degree of reliability, means that you have to develop a product that, from the earliest design stages, focuses its attention on maintaining the highest level of the quality perceived by the customer, throughout the entire lifecycle. Although reliability estimation requires a large financial commitment, it is indispensable to manufacture products that can keep their performance and always meet the quality requirements established during the design stage.

The reliability function can be estimated analytically when historical data are available (for example using the already known reliability values of the different components of a system) or performing experimental tests.

The purpose of the test is to assess the mathematical function that expresses the probability that the system will fulfil its mission during a certain interval of time, reproducing in the laboratory the usage conditions under which the system is likely to meet during its useful life. Testing activities are then used to estimate the system reliability in that environment in which it will perform its function. The external setting, obviously, plays an important role on the reliability function estimation as it can significantly affect it [1] [2] [3]. Moreover, the product can perform its function in very dissimilar contexts than those in which it was tested. Users also change, as well as the way they use the tool. There can be different climatic conditions, variable and unpredictable interactions with other components, etc.. The variability of the environmental conditions, can influence the product reliability performance and, in particular, it can introduce differences between laboratory test (called *in-house* tests) and real world applications (called *in-field*).

According to the authors experience in the field of household appliances manufacturing, to which also this study refers, the difference between the reliability values evaluated with a laboratory test and those derived from the field record (warranty data and repair intervention) is often not negligible. This difference arises, once again, from the heterogeneous environmental conditions that the system meets during the laboratory tests and during the in-field product use. Environmental conditions may vary, on a large scale, according to the different nations in which could be different customs and habits, dissimilar behavioural habits, varied climates, different education, services etc. These conditions are difficult, if not impossible, to evaluate and recreate in laboratory [1]. For this reason it is of great interest to find a correlation function able to evaluate the in-field reliability once the reliability in-house (obtained with laboratory tests) is known.

The aim of the study is to implement a quite new methodology introduced by Pham [4] (called "systemability") in order to relate the reliability values of a domestic household appliance, obtained through laboratory tests, to those obtained with the field data, collected during the warranty period. The main advantage of the application of Systemability lies in the relative straightforwardness of calculating the factor η that models the randomness of the environment in which the domestic household appliance operates. Furthermore its application is very suitable for manufacturing systems [5] and, in our knowledge, there aren't application of this methodology in the household appliances. In this work we'll also try to quantify the influence of two different European marketplaces on the in-field reliability performances.

In particular, in this research we applied systemability to analyse two years of production data of a household appliance in order to give to the manufacturer a methodology that allows to correlate the reliability performances obtained through laboratory test to the in-field performance. The difference between performances will be identified by some environmental factors, which affect the reliability of the product when it operates in the field.

The decision to use the systemability with household appliances came from the availability of historical data; this allowed the authors not only to apply the model but also to validate it. Furthermore, in literature, there are no similar studies where the systemability is applied to household appliances.

The remainder of this paper is organized as follows: section II presents a literature review of the in-house in-field correlations, and in section III the systemability methodology is described. Section IV describes the experimental results and the last one presents a discussion of the results and the main conclusions.

II. Literature review

As mentioned in the previous section, if only the reliability performances obtained with laboratory test are used, it can be very difficult to evaluate the real lifetime distribution of the product, when used in the field. An accurate prediction of the reliability function would be extremely useful for the manufacturer when he has to predict the number of faults that will occur in a certain period of time, or when he needs to assess the goodness of the design stage [6]. Even the other fundamental availability and maintenance performance [7] can be deduced from a confident reliability estimation [8]. An example in the automotive sector of how link the test results to the field operation, is discussed by Kleyner and Sandborn [9]. Understanding the expected number of faults during the product life (its failure probability) is essential when the manufacturer has to organize the service [10], [11], [12] and when he needs to define its extents [13].

In addition, the in-field reliability can be used as a reference for the various releases of the product in order to verify if the new design has led to an enhancement of the reliability performances [14], [15] or to manage the production nonconformities [16]. For these reasons, it is essential to know, with the best possible precision, both the in-house reliability (obtained by laboratory tests) and the reliability in the field, as it will be crucial in defining the service organization. Since the costs related to warranty claims are paid by the manufacturer, wrong decision about service plans can generate very high costs for the company [17], [18].

In the scientific literature, there are many analytical approaches to assess the influence of environmental factors in the evaluation of field reliability. All the reviewed papers consider environmental factors as a set of parameters and external influences that can alter the product reliability values respect to the base conditions defined by laboratory tests. According to the literature, there are essentially three main research streams that model the correlation between in-house and in-field reliability:

- the first and oldest is the proportional hazard model (PHM) developed by Cox [19], [20], [21] in which the effect of the in-field use on product reliability is defined by the presence of one or more covariates;
- the second considers the effect of environmental factors with a single coefficient;
- the third uses accelerated life test (ALT) or accelerated degradation test (ADT) to simulate the effect of field use.

The PHM is usually spent in survival analysis, to understand how different covariates (which explain environmental factors, operating conditions, treatments etc.) affect the failure rate. PHM already had a significant development in reliability analysis [22]. Badia et al. [23] and Finkelstein [24], use additive and proportional hazard models to take into consideration a more risky environment respect to the baseline hazard function (the one that describe the standard environment). Jardine [25] uses the PHM to evaluate the failure rate incorporating explanatory variables from an engine's operational environment. Some other examples of application of the PHM in engineering are reported in Heng et al. [26].

The second stream gathers together models that use just one single factor to assess the environmental effects. Other authors [27] consider an environmental factor, estimated with a classical or a Bayesian approach, to convert the laboratory reliability into the correspondent failure information in other environments, while Kumar and Tiwari [28] suppose that the environmental effect is modelled through the multiplication of the reliability function by a positive factor η. The environment factor η is used to define the uncertainty of the operating environment, and it has effect directly on failure rate. Pham [4] not long ago introduced a new mathematical function called Systemability that takes into account the system failures under a randomly varying environment once that the parameters related to the in-house reliability function are known. In this case η is a random variable that represents the system operating environment with a certain distribution function. Persona et al. [5] applied the systemability approach to two different industrial cases (packaging machines and motorcycle system) where in-house reliability was known. Some other applications of the systemability regard the age replacement policy for preventive maintenance [29] and the cost estimation (testing activity, costs of fault removal, warranty costs, etc.) of a general industrial system [30].

The third way to quantify the environmental effects on reliability is to perform the in-house tests with altered stress parameters (load, temperature, voltage, vibration, usage rate etc.) respect to the standard use [1], [3], for example applying ALT or ADT. The purpose is to simulate the effect of the field use through different stress parameters acting on the product. This procedure requires that the field effects on the product should be well understood and clearly measurable in order to conveniently alter the operating parameters, with the right intensity. Pan [31] and Wang et al. [32] integrated data from ALT and from previous field observation in order to predict the product real field reliability using a calibration factor. Liao [33] and Elsayed [27] replaced constant-stress ADT with variable stress in order to reproduce the existent field condition. A special kind of semi-constant stress ADT to evaluate field reliability is described in the work of De Carlo et al. [34].

III. Methods

The definition of reliability $R(t)$ as a time function expressed by the failure rate is given by:

$$R(t) = \int_t^{\infty} f(s)ds = e^{-\int_0^t h(s)ds} \quad (1)$$

In equation (1) $f(t)$ and $h(t)$ are respectively the failure probability density function (p.d.f.) and the hazard rate. Equation (1) defines the probability that the system will perform its intended function for a specific time-period t when operating under normal environmental conditions.

Systemability is defined as the probability that the system will perform its intended function for a specified mission time, under the random operating environments (Pham), and can be written as:

$$R_s(t) = \int_{\eta} e^{-\eta \int_0^t h(s)ds} dG(\eta) \quad (2)$$

In Equation (2) was introduced a random variable η with a distribution G, which represents the uncertainty of the operating environments respect to the laboratory test failure rate. It is assumed that η has a gamma distribution with parameters α and β, so it is $\eta \sim Gamma\ (\alpha, \beta)$ with the p.d.f. defined as:

$$f_{\eta}(x) = \frac{\beta^{\alpha} x^{\alpha-1} e^{-\beta x}}{\Gamma(\alpha)} \quad for\ \alpha, \beta > 0; x \geq 0 \quad (3)$$

If the product failure p.d.f. is distributed according to a Weibull function, using the Laplace transform, Pham obtained the Systemability function R_s given by:

$$R_s(t) = \left[\frac{\beta}{\beta+\left(\frac{t}{\lambda}\right)^{\gamma}}\right]^{\alpha} \qquad (4)$$

where α and β represent the unknown parameters of the gamma distribution, while λ and γ are respectively the scale factor and the shape factor of the Weibull distribution (see Equation 5) that represent the in-house reliability:

$$R_t(t) = e^{-\left(\frac{t}{\lambda}\right)^{\gamma}} \qquad (5)$$

λ and γ can be easily calculated analytically or using a common statistical software, starting from in-house tests.

α and β represent the uncertainty of the environment and must be estimated using the in-house and in-field data, covering the same period of time. If we determine α and β and assume that they remain unvaried over time (constant ambient conditions), than $R_s(t)$ depends only on the test parameters (λ and γ). With the hypothesis of constant environmental conditions even if we consider different products, the systemability function can be used to make a prediction of the in-field reliability performance of a new product (about which no field data are available) before it is launched on the market.

Basically, to estimate α and β, a non-linear regression for equation 4 should be used, where $R_s(t)$ is the dependent variable and the loss function is the sum of the squares of the residuals [5]. Through an iterative procedure, the values of α and β are changed at each iteration in order to minimize the value of the error sum of squares (SSE) that is defined as:

$$SSE = \sum_{i=1}^{n}(R_{f_i}(t) - R_S(t))^2 \qquad (6)$$

where R_f is the known field reliability and R_s is the value of the systemability.

It stands to reason that, to use this methodology, the analyst must have the data of both in-field and in-house failures of at least one common period. Then, to make some inferences on the in-field reliability concerning further periods, he must take into account the hypothesis of unvaried environmental conditions (same α and β of the past period).

The main steps to apply the systemability approach in a problem similar to the one introduced here, are the following:

1) Evaluate Weibull parameters: identify the Weibull parameters λ and γ of the failure p.d.f. relative to the laboratory test;

2) Reliability function: calculate the reliability function $\mathrm{R_f}$ with the field data relative to the same period of the one evaluated at the previous step;

3) Evaluate Gamma parametrs: find the values of α and β of the Systemability function that minimize the SSE calculated as in equation 6

4) Sistemability function: use the Systemability to make inference on the field reliability performances relative to a following period.

IV. Results

To apply systemability approach to the domestic appliances, the authors started from the analysis of the Year 2007 test data, identifying the parameters of the Weibull p.d.f. which best represented laboratory the reliability function. The same was done for the warranty returns of the Year 2007. Systemability function was built using the laboratory Weibull parameters. To obtain α and β, parametres of the Gamma distribution of the systemability random variable η, we minimizes the value of the SSE, as stated in equation 6. After that the systemability function was completely defined it could be used to make a prediction of the Year 2008 in-field reliability.

All the household appliances considered are produced in the same factory, which is located in Italy. The analysed data, instead, come from two different marketplaces: Italy and United Kingdom. We decided to try to quantify the systemability environmental factors α and β for each one of these two markets, to analyse the differences and to understand where they originate. The repair actions during the warranty period are considered as "perfect maintenance" in order not to overcomplicate the problem [35].

Each household appliance was tested into the manufacturer's laboratory and was tested for 500 usage cycles that correspond to approximately two years use. The test followed a sequence of ten usage cycle sessions that were repeated 50 times. The household appliances that reached 500 cycles were considered as right censored of type I [36]. Laboratory test results for the Year 2007 and 2008 are summarized in Table I which also lists the corresponding Weibull parameters.

The field data relative to the warranty period, conversely, were obtained from the technical reports drawn up by the technicians at the end of their maintenance service.

It is important to note that there were some differences in the service organization of the two countries analysed. The most important is that in Italy the service was managed by private centres, while in the UK the service was operated by the manufacturer. This can lead to significant differences considering both how the data are registered and how the maintenance is carried out. For example, in Italy, if the service call was made during the warranty period, then the technician would have been more inclined to replace the piece to better protect the customer and because he would be refunded in any case by the manufacturer. In UK, on the other hand, the technician could visit a customer for the same call over and over again, as a threshold matter, without making any intervention.

TABLE I
Laboratory tests results relative to 2007 and 2008. Weibull p.d.f. parameters are also indicated

Year	Failed	Censored	Weibull parameters	
			γ	λ
2007	70	68	0,67	451,714
2008	12	40	0,62	2809,49

Both management approaches have their pros e cons but, for statistical analysis this inhomogeneity leads to different reliability performances, even if the product is the same. Since this difference will impact the in-field reliability evaluation, the systemability methodology will take into account its effect with different outcomes of the environmental factor η. For this reason, we had to study the two markets separately in order to define, for each of them, its random environment variable η. The results of the failed and censored items of both markets are summarized in Table II where there are indicated also their Weibull parameters (data belonging to Year 2007 were used to build the systemability parameters, while data from Year 2008 were used to validate the previsions made).

TABLE II
Year 2007 and Year 2008 in-field results for the two countries: Italy (IT) and United Kingdom (UK). In the two right columns, Weibull p.d.f. parameters are also indicated

Year	Failed	Censored	Weibull parameters	
			γ	λ
2007 (IT)	10413	61973	0,68	7539,4
2008 (IT)	8440	76923	0,57	25237,9
2007 (UK)	7698	38745	0,85	1851,11
2008 (UK)	6471	47182	0,88	2557,7

Using the data from 2007 it was possible to calculate the parameters α and β of the random variable η. This was necessary to define the systemability function. As stated in section 3, the determination of these parameters is carried out by an iterative non-linear regression, implemented through the use of the statistical software IBM SPSS Statistics 20. The results of this procedure for the Italian and for the English markets are summarized in Table III. It is quite clear that, since the two systemability functions come from the same laboratory test, they share the same Weibull parameters γ and λ.

TABLE III
Systemability parameters of the Italian and English markets. It's well worth to notice that γ and λ are identical for both Countries because the laboratory test was the same.

Country	α	β	γ	λ
Italy	31,337	219,182	0,621	2809,49
United Kingdom	216,378	881,697	0,621	2809,49

For the Italian market, the outcome of the iterative procedure implemented with data from Year 2007, gives a very good fit of the in-field reliability as it can be seen in Figure 1.

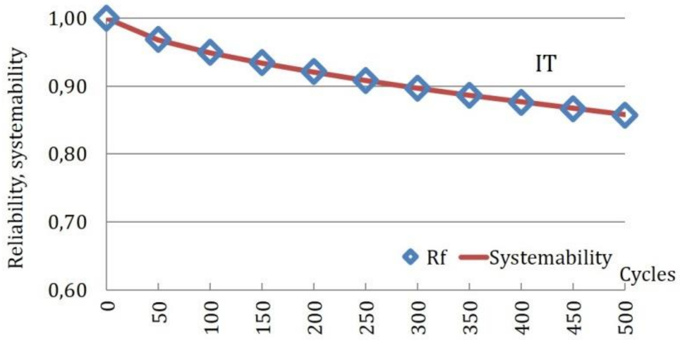

Fig. 1. Year 2007 field reliability (R_f) and Systemability function relative to the Italian market. As it can be seen the systemability function (continuous line) well fits the in-field reliability function of the items (the highlighted points).

For the UK's market, the comparison between the in-field reliability data and its estimate obtained by means of the systemability function, is shown in Figure 2. The fit is excellent for the first 250 cycles and good for the rest of the cycles (the maximum difference is visible at 500 cycles and its value is of less than 5%)

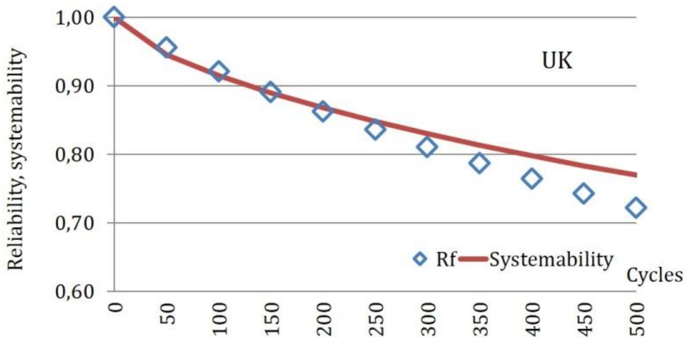

Fig. 2. 2007 field reliability (R_f) and Systemability function relative to the UK market. In this case, the systemability estimate of the in-field reliability is less accurate than before.

V. Discussion and conclusions

As you can see from the experimental results, the systemability methodology was applied with success. The outcome of the calculation is more accurate in the case of the Italian market, with respect to what occurred in the English case.

A possible verification of the goodness of the method, is to validate the methodology. That is, you should check how the model is able to predict situations likely to arise. We decided to check the results described in the previous section, using the parameters previously derived to predict the reliability values for the year 2008. The two previously obtained systemability functions, were used to estimate the field reliability for year 2008 on the two marketplaces. The results were be compared with the real data collected in-field in the two countries during the same year (2008) as it can be seen in Figure 3 for the Italian market.

The same results , but for the English market, are shown in Figure 4.

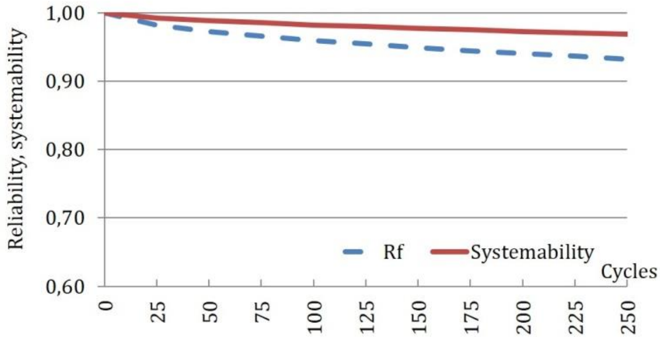

Fig. 3. Differences between the field reliability predicted with the systemability function (red continuous line) and the real field reliability (dashed blue line) for year 2008 on the Italian marketplace.

The same results , but for the English market, are shown in Figure 4.

In both cases, it is evident that the systemability estimation tends to overestimate the actual value of the field reliability. To quantify the differences between values predicted by the systemability function and the values actually observed, the root mean square error (RMSE, see equation 7), and the relative error (equation 8) calculated at t_i=500 cycles (maximum differences), were calculated for both marketplaces.

$$RMSE = \sqrt{\frac{\sum_{1=1}^{n}\left(R_S - R_f\right)^2}{n}} \quad n = \text{number of failure points} \qquad (7)$$

$$e_i = \frac{\left|R_S(t_i) - R_f(t_i)\right|}{R_S(t_i)} \qquad (8)$$

For the Italian market the RMSE is 0.036 and the relative error is 5.41 % while for the UK the RMSE is 0.077 and the relative error is 14.10 %.

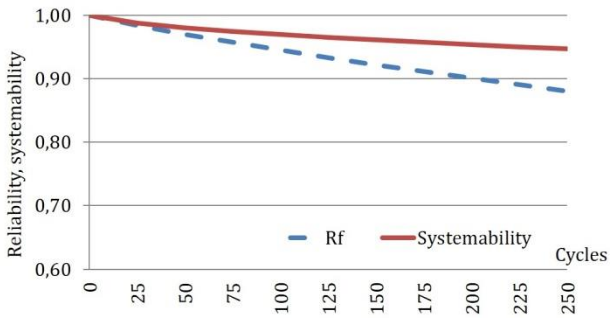

Fig. 4. Differences between the field reliability predicted with the systemability function (red) and the real field reliability(blue) for year 2008 on the UK marketplace.

In this case the difference between the estimated and the observed values increases with the number of usage cycles performed. To determine the organization of the service, the manufacturer will be interested primarily in the estimated reliability at 500 cycles as this datum will provide an indication of the number of machines that are expected to fail. The errors calculated above are not alarming but, in the field of household appliances, where production is very high, small errors can cause high discrepancy in terms of number of failures. In the present case study these errors come from the difficulties of interpreting the field failure data registered by the two different maintenance services. Furthermore, the lack of a systematic testing methodology for the maintainers crew, generate errors in data interpretation and, therefore, causes an incorrect assessment of the in-house reliability. All of these factors were highlighted and it was possible to design corrective actions that would hopefully allow to apply the methodology in a more effective way.

The aim the study was to determine a correlation between the in-house and the in-field reliability, that is how the random environmental conditions affect the reliability performance of a domestic household appliance.

This work is quite innovative, since it applies, for the first time, the systemability approach to an household appliance. Moreover, for the first time, the sales market was considered as an environmental factor. The estimated parameters α and β highlight the effect that different environmental conditions, service organization and other similar factors have on the product reliability performances.

If the environmental conditions were not defined or if the hypothesis of their being constant was violated, the systemability method could still be used to make a preliminary analysis of the in-field performance.

Errors on the systemability estimation could not be neglected but some corrective actions have been identified to standardize testing activity and the registration of in-field failures during the service operations. This will surely lead to improve the data analysis and, as a consequence, the accuracy of this new methodology results.

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