Simulation of a Drivetrain of a Vehicle comprising Continuously Variable Transmission

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***Abstract*—With the constant hike in fuel prices on a day to day basis, maximum performance with minimum compromise on the front of fuel economy and emissions is highly desirable and expected from a vehicle's drivetrain. In this context, a drivetrain comprising a Continuously Variable Transmission (CVT) plays the role to the best extent possible. CVT facilitates a continuous change in gear ratios between the driver and the driven shaft. CVTs are superior to automatic transmissions with a fixed number of gear ratios in that it offers greater acceleration and more efficient fuel economy.**

In this paper, we propose to model a drivetrain of a vehicle comprising CVT. A detailed mathematical model based on the geometry of the components has been formulated and also, equations of their respective motions representing their behaviour have been derived. Furthermore, a dynamic model of a drivetrain, using bond graph method, is prepared which captures the behaviour of the CVT during the transient shift ratio condition and the complete simulation is implemented in MATLAB Simulink. The results substantiate the effect of mass of flyweights and spring constant of torsion springs on the performance of CVT.

Keywords-Continuously Variable Transmission, Drivetrain, Simulation

I. INTRODUCTION

There are different types of transmissions available today & each has its pros and cons. One of which is CVT. A Continuously Variable Transmission, as the name suggests, has a continuous band of gear ratios between the drive and the driven shaft. It transmits an uninterrupted power to the wheels and mostly runs engine at constant power as shown in Fig. 1.

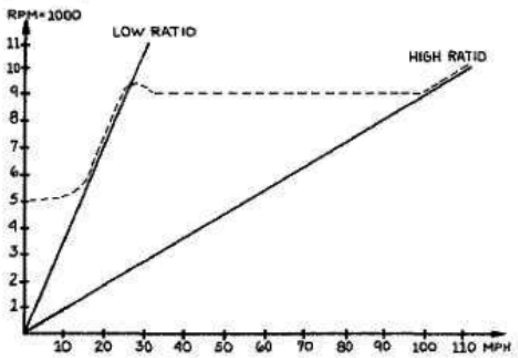

Fig. 1. CVT Curve for Engine Speed and Velocity of the Vehicle

Different types of CVT are: Pulley based CVT, Toroidal CVT, and Hydrostatic CVT. Among these, Pulley based CVT is the most affordable and simple one. It has two pulleys: (1) Primary Pulley connected to the Engine (2) Secondary Pulley connected to the Transaxle. These two pulleys are connected by a rubber V belt. Variation in the pitch radii of these pulleys lead to the change in the gear ratio.

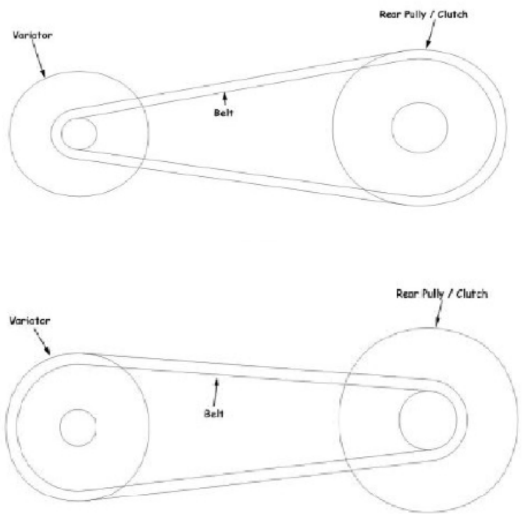

Fig. 2. CVT at low & high RPM respectively

The primary pulley consists of Flyweights- Spring assembly which, due to centrifugal force moves out causing the movable sheave to shift in the direction of increasing pitch radius. This causes the belt to move up on the drive pulley and down on the driven pulley. The secondary pulley consists of a Ramp-Spring assembly which varies the side force on the belt according to the torque provided. The ratio of different pitch radii on both the pulleys will give the gear ratios.

The goal of this paper is to prepare a mathematical model of a vehicle driveline system, which is quite suitable to the power-rpm curve of the engine, and provide maximum power and torque output. A dynamic model is then simulated in MATLAB Simulink to get various performance characteristics like acceleration, velocity etc. A deterministic optimization is also used to get the optimised results.

II. POWERTRAIN

Components of the powertrain are chosen on the basis of the calculations of lower & higher gear ratio by taking traction limited acceleration & maximum velocity of 60kmph respectively. The CAD model of the selected components and their orientation in the system is shown in Fig. 3. Specifications are as follows:

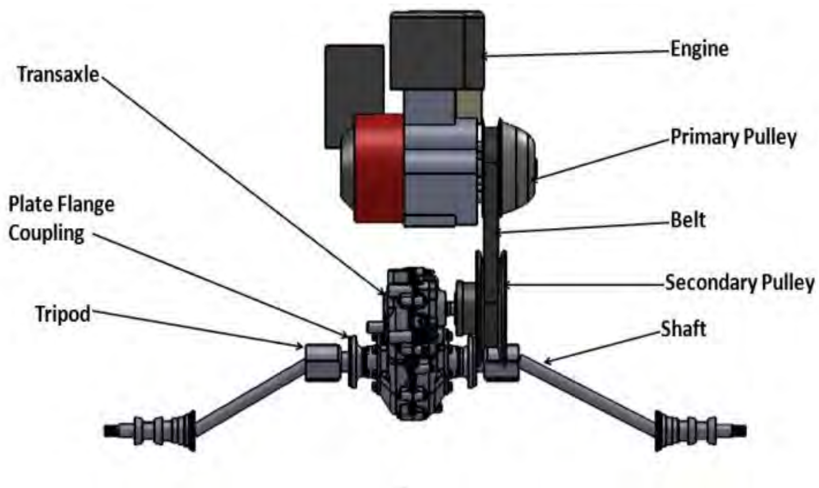

Fig. 3. CAD Model of the Powertrain

A. Engine

B&S 305cc M20 Intek OHV Engine is chosen which has Max. Torque of 18.6Nm @2600 rpm & Max. Power of 10hp @3800 rpm. Torque and Power characteristics of Engine with RPM are shown in Fig. 4.

B. CVT and Belt

Comet 790series CVT is selected as it is very much compatible with the Engine used & has an excellent power transmission capability. It has lower gear ratio of 3.3 & higher ratio of 0.5. It uses a Rubber V belt with centre to centre distance as 10.4” and length as 39.13”. Real time picture of CVT with belt is shown in Fig. 5.

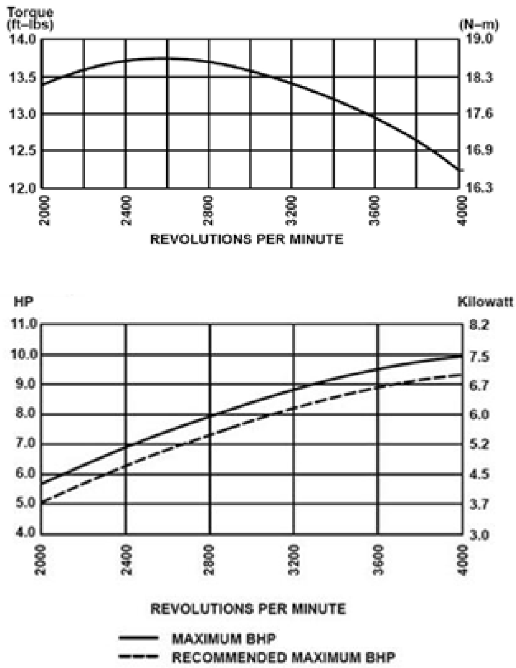

Fig. 4. Torque & Power Map of the Engine

Fig. 5. CVT & Belt

C. *Transaxle*

Chosen is a H12 Model FNR Transaxle from Dana Transmissions. It has an overall ratio of 10.15 in front and 11.15 in reverse. It also has a Limited Slip Differential which has its own benefits over Open Differential. It is shown in Fig. 6.

D. *Driveshafts*

Driveshafts are OEM taken from Mahindra Alfa Champion. Maximum angle and travel are 40° and 5cm respectively.

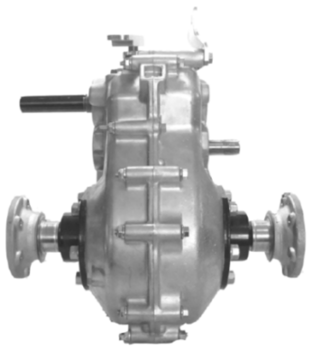

Fig. 6. DANA H12 Transaxle

III. MATHEMATICAL MODELLING OF A CVT

Idea was to model all the components in a much simpler way and a better way to do this is to represent each & every component of the CVT mathematically. This way they can be solved easily & can serve as a base for more complex models. Some of the assumptions were made to simplify the problem. These are:

- The entire CVT system consisting of Driven and Driving Pulleys and the Belt is considered conservative
- The Belt is assumed to be non-elastic and rigid
- Slippage is neglected
- Friction of the belt and pulley in radial direction is neglected

The goal is to get the relationship between the angular velocity and the gear ratio for further simulation. And for the same a simple mathematical model is made by studying the motion of the components inside CVT. Dependence of one parameter on other forms the basis here. Structure of CVT is shown in Fig. 7.

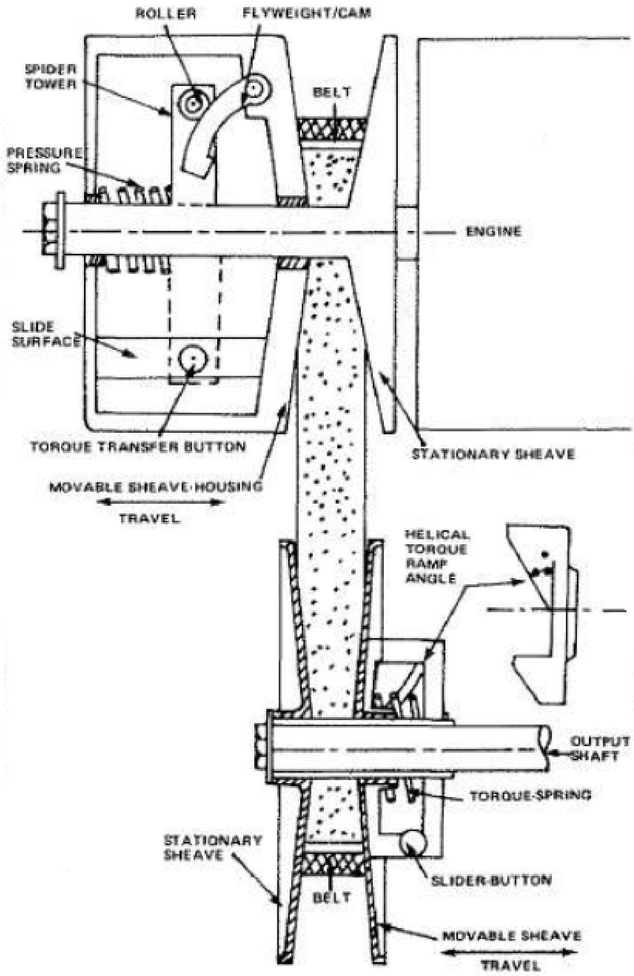

Fig. 7. Structure of CVT

A. Motion study of Primary Pulley:

As described earlier, drive pulley consist of a Flyweight-Spring assembly which when given an angular velocity (ω), moves out due to centrifugal force. Motion causing element here, a flyweight has a certain mass is connected to the movable sheave through a pivoted link. It needs certain rpm to engage & this rpm is known as engagement rpm. At this point only it overcomes the initial pretension of the torsion spring and engages the belt. As belt is engaged now, secondary pulley starts to rotate with the primary but has the same pitch radius as initial. At the shift out point only it has enough energy to overcome the pretension in the torsion spring of the driven pulley and change the pitch radii. As the dome of the primary pulley is fixed flyweight pushes the pivot point and thereby pushes the belt up. Geometry of this assembly is shown in Fig. 8.

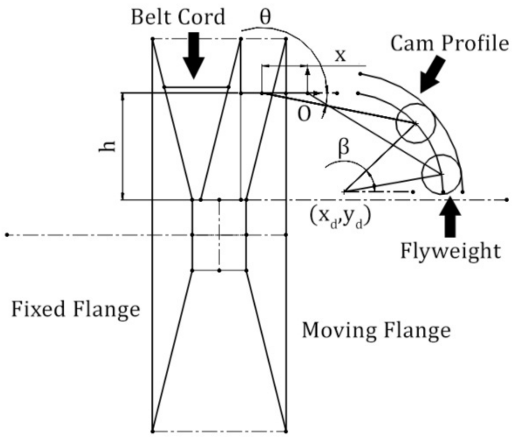

Fig. 8. Driving Pulley Cross Section

Approach, used to get the equations here, is to write the coordinate of the roller flyweight parametrically. This is done to get the relation between the distance (x) moved by the pivot point and the angle of the flyweight θ. These equations are:

$$x - r \cdot \cos\theta = x_d - R \cdot \cos\beta \tag{1}$$

$$y_d = R \cdot \sin\beta + r \cdot \sin\theta \tag{2}$$

where θ & β are the angles as shown in Fig. 8.

B. Motion study of Belt:

As the used V belt is rigid, increasing pitch radius on one pulley will decrease on other. This phenomenon will be according to the equation:

$$L = \; 2C + \frac{2}{\pi}\,(r_2 + r_1) + \frac{(r_2 - r_1)^2}{4C} \tag{3}$$

where L is the total circumferential length of the belt & C is the centre distance between the pulleys. r_1 & r_2 are the pitch radii of primary and secondary pulley respectively.

$$r_1 = \frac{x \cdot \tan\alpha}{2} \tag{4}$$

where α is the half apex angle of the belt.

C. Motion study of Secondary Pulley:

As the belt forces the sheave of the driven pulley to shift, torsion spring in the ramp works against this force and thus helps in backshifting. It consists of a Ramp-Spring assembly which changes the side force on belt according to the force applied by the belt. This side force is equal to the force produced at the ramp divided by the tangent of the ramp angle. Geometry is shown in Fig. 9.

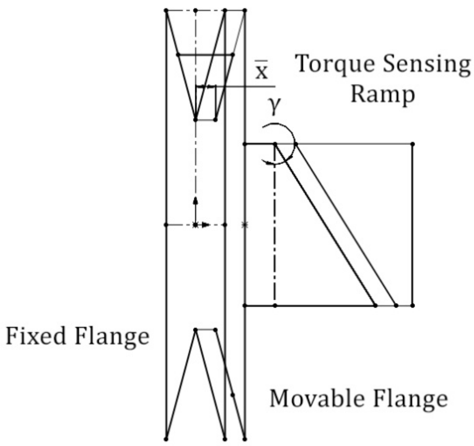

Fig. 9. Driven Pulley Cross Section

Motion of the sheave ($\bar{x}$) in the direction of decreasing pitch radius is represented in the equations given below:

$$\bar{x} = \frac{2 \cdot r_2}{\tan\alpha - \frac{2 \cdot r_2}{\tan\alpha}} \tag{5}$$

$$\emptyset = \frac{\bar{x} \cdot \tan\gamma}{r_{ramp}} \tag{6}$$

where γ & r_{ramp} are the angle & radius of the ramp respectively. $\emptyset$ is the angle of twist of the torsion spring.

According to Lagrange's Equation, for a conservative system

$$\frac{\partial L}{\partial x_i} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x_i}}\right) = 0 \qquad i = 1, 2, 3 \ldots\ldots n \tag{7}$$

where L is called the Lagrangian and is equal to the difference of Kinetic Energy and Potential Energy of the system. Hence, $L = PE - KE. x$ is called the generalized co-ordinate and the state of the system is defined with respect to this variable. The number of generalized co-ordinate is equal to the Degree of Freedom of the System. Of instance, the CVT system has one Degree of Freedom and θ is chosen as the variable.

$$PE = \frac{1}{2} \cdot K_p \cdot (\theta_o + \theta)^2 + \frac{1}{2} \cdot K_t \cdot (\emptyset_o + \emptyset)^2 \tag{8}$$

$$KE = \frac{3}{2} \cdot m \cdot \omega^2 \cdot {r_f}^2 \tag{9}$$

where K_p & K_t are the torsion spring constant and m, ω & r_f are the mass of flyweight, angular velocity & radius of the flyweight.

As the system does not have terms pertaining to $\dot{\theta}$, making second term of Eq (7) zero. Therefore, Eq (7) becomes

$$\frac{\partial L}{\partial \theta} = 0 \tag{10}$$

These equations are solved according to the flowchart shown in Fig. 10.

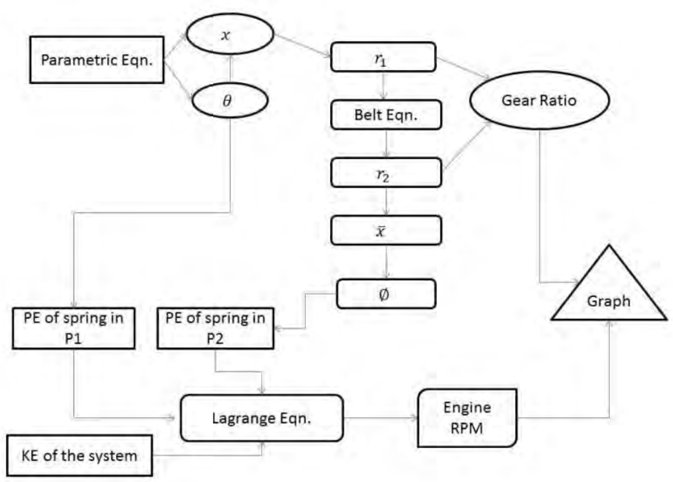

Fig. 10. Flowchart for solving equations

Graph for ω $vs. g$ is generated and hence the purpose is solved.

IV. MODELLING OF THE ENGINE

The results from the Torque & Power curve were directly taken into the model. It gave an approximate value of τ_e as a quadratic function of Engine RPM N_e. N_e is back calculated from v_v from the following equation:

$$N_e = \frac{30\omega_e}{\pi} = \frac{30 G_{cvt} G_{ex} \omega_v}{\pi} = \frac{30 G_{cvt} G_{ex} v_v}{\pi r_d} \tag{11}$$

where ω is the angular velocity and subscript e and v represent engine and vehicle respectively.

V. DYNAMIC MODELLING OF THE VEHICLE

Applying Newton's 2nd Law of Motion to a vehicle,

$$F_{te} = M_v a_v \tag{12}$$

where F_{te} is the Tractive Effort present at the wheels of the vehicle, M_v is the Mass of the vehicle and a_v is the acceleration produced by F_{te}. Further,

$$F_{te} = F_w - F_r \tag{13}$$

F_w is the Force at the Wheels due to the Engine and the Transmission and F_r is the net resistance on the vehicle due to Road Roughness, Gradient and Drag. Once there variables are found, the motion of a vehicle can be completely defined.

$$x_v = \dot{v_v} = \ddot{a_v} \tag{14}$$

where v_v is the velocity and x_v is the displacement of the vehicle.

$$F_w = \frac{r_d \tau_e}{G_{cvt} G_{ex}} \eta \tag{15}$$

Here, r_d is the dynamic radius of the vehicle, τ_e is the engine torque produced, G_{cvt} and G_{ex} is the Gear Ratios of CVT and additional Gear Reduction. η is the transmission efficiency.

$$F_r = M_v g c_r cos\, \psi + \ M_v g sin\, \psi + \frac{1}{2} c_w A_e \rho_l v_v^2 \tag{16}$$

where the first term is the rolling resistance, second term is the gradient resistance and third term is the aerodynamic drag resistance.

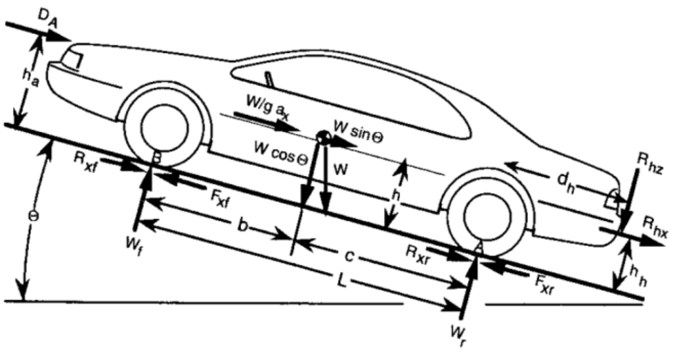

Fig. 11. Various resistances on the Vehicle

VI. SOLUTION ROUTINE

All the Equations are solved using MATLAB Simulink expect for the equations of CVT, which are solved in MATLAB for faster processing. The Engine Block supplies the initiating Torque according to an input RPM. The Transmission Block contains two sub divisions: the CVT block and the External Reduction Block. A MATLAB module solves Eqs (1) to (10) and formulates a relationship between the Gear Ratio and the Output RPM. The relation is brought into the Simulink environment where according to the instantaneous Gear Ratio, the Output RPM is calculated. Then the New Gear Ratio is recalculated and a multiplied Torque is given as output. The External Transmission block multiplies the Torque further according to the set Gear Ratio.

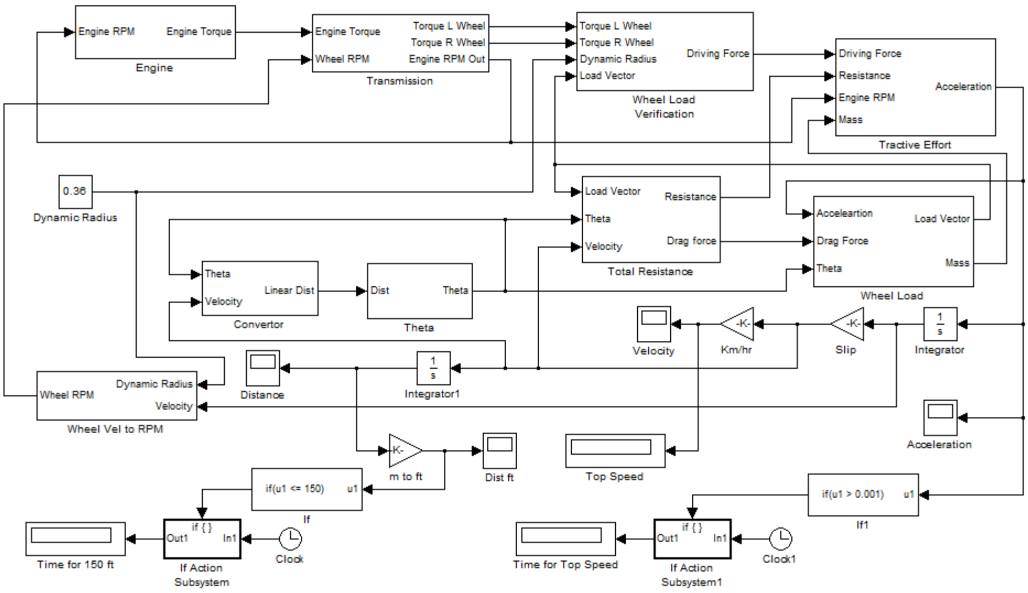

Fig. 12. Simulink Model for Full Transmission System

The torque obtained after Transmission Block is converted into Force present at the wheels by dividing with dynamic radius. Hence, F_w is obtained. Resisting Torque is found according to the empirical formula. Hence, F_{te} is calculated according to Eq (13).

VII. RESULT AND DISCUSSION

The equations from the modelling were formulated in the Matlab code and the graph for Gear Ratio vs. Engine RPM was plotted. The result is given in Fig. 13.

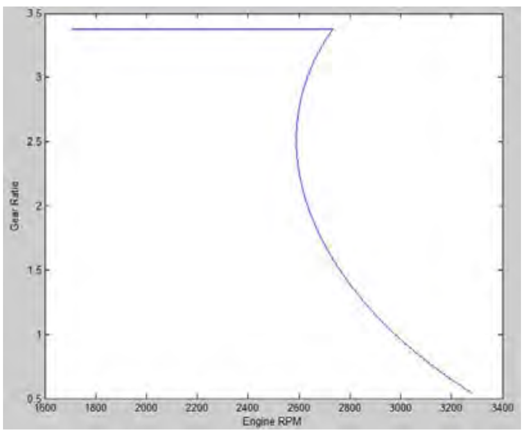

Fig. 13. Variation of Gear Ratio with Engine RPM

It can be concluded from the above shown result that CVT needs certain Engagement RPM to engage the belt, and the Gear Ratio remains constant until the shift out point. After this point only Gear Ratio gradually decreases to the higher ratio at higher RPMs.

This dynamic model is simulated for a vehicle of mass 320 kg. and wheel radius as 25 inches for 65sec & graphs for acceleration & velocity vs. time are plotted. Acceleration vs. Time & Velocity vs. Time graphs are shown in Fig. 14 & Fig 15 respectively.

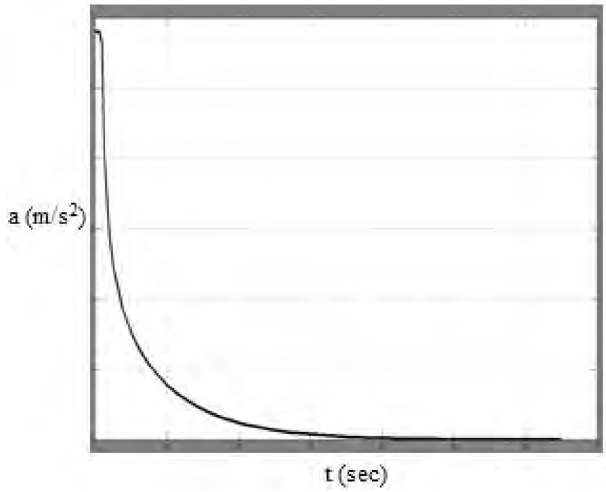

Fig. 14. Acceleration vs. Time

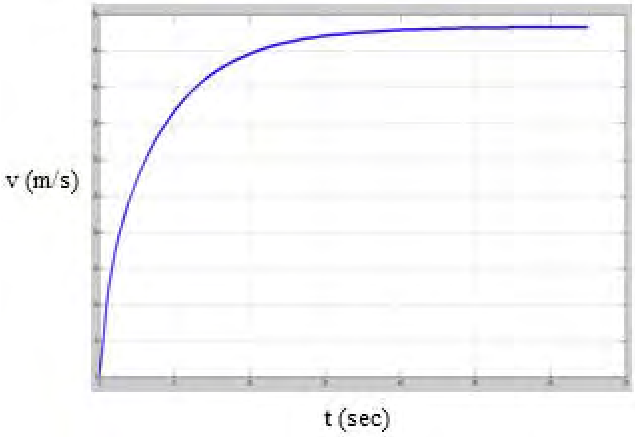

Fig. 15. Velocity vs. Time

In the results obtained, acceleration is at its maximum value at starting & then decreasing to 0 as the time passes. This case exists in reality also, as the resistance at t=0 is at its least value and the increases to a constant value where it equals the tractive force. At this point only acceleration becomes 0. For velocity the case is opposite, it increases from 0 to a certain constant value as the acceleration decreases as $a \propto \frac{1}{v}$.

The parameters obtained from the simulation are listed in Table 1.

TABLE I
Performance Parameters

S. No.	Parameters	Values
1.	Time for 150ft	7.08s
2.	Gradeability	33.64deg
3.	Max. Speed	47.2kmph
4.	Max. Tractive Effort	1372.5

VIII. CONCLUSION

A drivetrain comprising CVT and a FNR transaxle has been presented for the simulation as CVT alone cannot provide the wider torque range needed for the smooth functioning of an All-Terrain Vehicle. The simulation has been useful in estimating the performance characteristics of the vehicle. It also helped to understand the behaviour of each and every component of CVT and enabled a deeper insight in the operations of a CVT. Equation of motion of the components in Primary and Secondary Pulleys are coupled to give a combined effect in the final equation. A Lagrange's equation was formed using different parameters and solved in MATLAB. The results obtained were very close to the ideal curve exist and helped in determining the performance characteristics of a vehicle.

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