Bamboo Taper Effect on Third Point Loading Bending Test

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***Abstract*—Geometrical shape of bamboo usually assumed as tapered hollow pipe. This study proved that the dimensional changes along the bamboo stem significantly affected to its Modulus of Rupture (S_R) value which measured from third point loading bending test. Therefore if the bending test applied using third point loading configuration, the S_R value should be adjusted by strength ratio of taper (C_t). C_t is the ratio between (S_R) calculated in the center span and the maximum bending stress along the bamboo beam. This study resulted mathematical formulae to calculate the C_t value for overall range of bamboo taper based on six species namely Tali (*Gigantochloa apus* (Bl.Ex Schult.f) Kurz), Hitam (*Gigantochloa atroviolaceae* Widjaja), Andong (*Gigantochloa psedorundinaceae*), Ampel (*Bambusa vulgaris* Schrad), Gombong (*Gigantochloa verticillata* (Willd) Munro), and Mayan (*Gigantochloa robusta* Kurz). The first tree species were obtained from the Bogor market, while the others were harvested from bamboo clumps in Arboretum Bamboo – Bogor Agricultural University. Then the formula was applied to sketch the graphical style in order to simplify the result.**

Keywords-Bamboo taper, Third point loading bending test, Strength ratio

I. INTRODUCTION

Two point loading test (sometime called four-point loaded beam) is a beam resting on two supports and loaded at two points equally from each support. Third point loading bending test is a special cases in the method, which the position of load is a third length measured from each nearest support. Third point loading on bending test has commonly become major method to determine the flexural properties of full sized material. The method was designated in worldwide standards e.g.: ASTM D198-09[1], EN 408:2010[2], and ISO 8375:2009-02[3] for timber and structural wood products; ASTM E855-08[4] for metallic flat materials; EN 1288-3:2000[5] for glass in building, TAPPI T836 OM-2002[6] for paper, and ISO 22157-1:2004[7] for bamboo. All of those standards were applied within assumption that the cross sectional area of the beam were the same along the span. This assumption could not be fulfilled in bamboo stem bending test because it's tapered shape. Bamboo stem's diameter often tapers from basal to top, with differences between species [8,9]. Its top has smaller diameter than the basal. In order to resolve that natural condition, ISO 22157-1:2004 assign the average value of diameter used for Modulus of Elasticity (E) and Modulus of Rupture (S_R) calculation, while its cross section is assumed as hollow cylinder.

Ignoring the actual condition of sample in laboratory testing sometimes turn out dangerous when the result applied in the field. Strength Ratio as effect of bamboo's eccentricity have been studied [10], and the result shows that the flexural properties measurement will be over estimate if the major axis arranged vertically, while it will be under estimate if the major axis arranged horizontally. This study evaluated effect of bamboo stem's taper to the S_R value which is obtained from third point loading bending test. The ratio between S_R which considered the taper effect with the S_R of perfect cylindrical bamboo called strength ratio of bamboo taper (C_t). This strength ratio should be used as adjustment factor in order to design the better bamboo construction. It is expected that the application of Ct value will make the design closer to reality than non-taper assumption. Previous study about bamboo taper effect on center point bending test has been done [11] and the result showed that the maximum bending stress was still in the center span for overall taper range of four studied bamboo species, therefore it is reasonable to ignore the bamboo taper effect on center point bending test. However, the different result could be found in third point bending test, because the loads were not applied in the center but in the third length.

II. THEORETICAL BASIS

A. *Moment of inertia*

The geometrical shape of samples plays important rule on the mechanical testing to define the material elasticity and strength. There exist exact relationship between geometrical shape e.g. length and cross sectional shape with its elasticity and strength. Cross sectional shape properties of beam which measured bending resistance could be determined by its moment of inertia. Moment of inertia is a property of a shape that is usually used to predict its resistance to bending and deflection.

If the bamboo stem is assumed as a hollow tube, the moment of inertia formulae could be defined as Equation 1:

$$I_x = \frac{\pi\left(d_o^4 - d_i^4\right)}{64} \quad (1)$$

Bamboo diameter commonly is not the same size along the stem, but the basal diameter (d_{bo}) is higher than the top (d_{to}). We define taper (t) as ratio between diameter difference and its length (Equation 2 and 3 for outer and inner tapper respectively).

$$t_o = \frac{d_{bo} - d_{to}}{L} \quad (2)$$

$$t_i = \frac{d_{bi} - d_{ti}}{L} \quad (3)$$

According to taper formulae (Equation 2 and 3), the outer and inner diameter of the stem will changes gradually. The outer and inner diameter at a distance *x* from the top, could be defined as:

$$d_{xo} = d_{to} + t_o x \quad (4)$$

$$d_{xi} = d_{to} + t_i x \quad (5)$$

So, the moment of inertia of bamboo stem at a distance *x* from the top could be calculated by:

$$I_x = \frac{\pi}{64}\left((d_{to} + t_o x)^4 - (d_{ti} + t_i)^4\right) \quad (6)$$

B. Normal Stress on Beam

Bamboo stem is composed from some amount of fine fibers which arranged longitudinally. If beam is loaded by bending moment, every fiber bellow neutral axis will be longer and the fibers above neutral axis will be shorter. Therefore the tension stress arises on fiber bellow neutral axis, and compression stress arises on fiber above neutral axis. Both tension and compression stresses are usually called normal stress in beam which could be calculated by Equation 7:

$$\sigma = \frac{My}{I} \quad (7)$$

On third point loading bending test, two similar loads applied at a third point from each nearest support (Figure 1). Bending moment (M) through the length of bamboo beam on third point loading bending test could be defined as Equation 8.

$$M_x = \begin{cases} \frac{Px}{2}; \text{for } 0 \le x \le \frac{L}{3} \\ \frac{PL}{6}; \text{for } \frac{L}{3} \le x \le \frac{2L}{3} \\ \frac{P(L-x)}{2}; \text{for } \frac{2L}{3} \le x \le L \end{cases} \quad (8)$$

While y is maximum distance from neutral axis, which could be calculated as:

$$y = \frac{d_{to} + t_o x}{2} \quad (9)$$

Substituting Equation 6, 8, and 9 into Equation 7, we get formulae for normal stress in bamboo stem:

$$\sigma_x = \begin{cases} \frac{16Px(d_{to} + t_o x)}{\pi\left((d_{to} + t_o x)^4 - (d_{ti} + t_i x)^4\right)}; \text{for } 0 \le x \le \frac{L}{3} \\ \frac{16PL(d_{to} + t_o x)}{3\pi\left((d_{to} + t_o x)^4 - (d_{ti} + t_i x)^4\right)}; \text{for } \frac{L}{3} \le x \le \frac{2L}{3} \\ \frac{16P(L-x)(d_{to} + t_o x)}{\pi\left((d_{to} + t_o x)^4 - (d_{ti} + t_i x)^4\right)}; \text{for } \frac{L}{3} \le x \le L \end{cases} \quad (10)$$

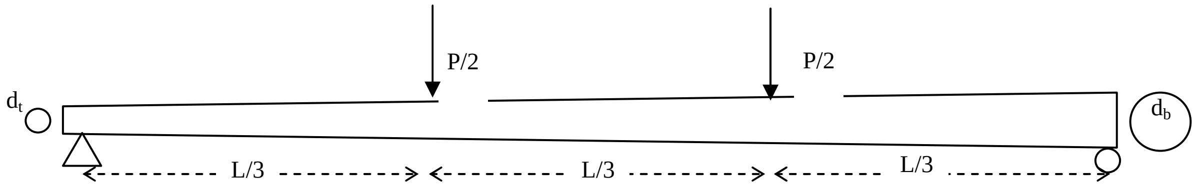

Fig. 1. Bamboo stem on bending test with center point loading configuration

The two loads equally and symmetrically applied in the third length from the nearest support, so the resultant of load is at the center length. Equation 15 doesn't always have maximum value in the center length, even though the load resultant is in the center. Meanwhile the measurement and calculation of Modulus of Rupture (S_R) is usually in the center of the length. This condition could be dangerous for building planning because the estimation of material properties could be higher than the actual maximum normal stress in bending test, especially for bamboo with high taper value. To avoid this condition, a strength ratio of taper (*Ct*) should be conducted to adjust the material properties in bending. The strength ratio of taper (*Ct*) is defined as the ratio of stress in the center length and maximum stress throughout the length (Equation 11).

$$C_t = \frac{\sigma_{\left(L/2\right)}}{\sigma_{\max}} \qquad (11)$$

Figure 2 shows the sketch of bending stress in every taper value for bamboo stem in third point loading bending test configuration. The Figure 2 is built within assumption that the inner and outer tapers are the same value. As seen on Figure 2, the maximum bending stress moves to the top of stem. The peak is farther from the center length and closer to the top if the taper value is higher. The normal stress in the center of the span has maximum value only if the bamboo stem is not tapered (t=0). If the taper value is lower than 0.0269, the maximum bending stress is happened at the loading point with smaller stem diameter. Above 0.0269 taper value, the maximum normal stress moving to the top. The maximum bending stresses in every bamboo taper were showed as dot type line in Figure 3. Based on above description we classified two ranges of bamboo stem's taper value which give different effect to its bending stress on third point loading configuration i.e. $0<t\leq 0.0269$ and $0.0269<t<1$. We need differential equation to find the stationer point which indicates the maximum bending stress for $0.0269<t\leq 1$. In that taper range, the stationer point is happened in the $0 \leq x \leq L/3$ range as seen on Figure 2, the point is characterized by zero value for its first differential (Equation 12).

$$\frac{d}{dx}\left(\frac{16Px\left(d_{to}+t_o x\right)}{\pi\left(\left(d_{to}+t_o x\right)^4-\left(d_{ti}+t_i x\right)^4\right)}\right)=0;\ \text{for } 0\leq x\leq \frac{L}{3} \text{ and } 0.0269\leq t\leq 1 \qquad (12)$$

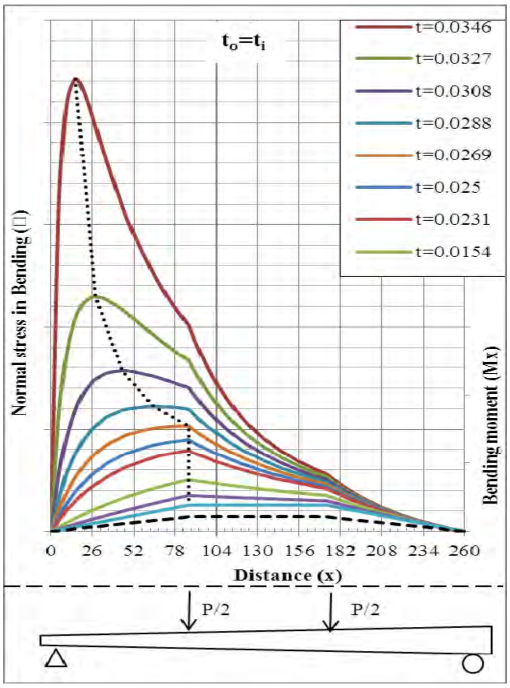

Fig. 2. Effect of bamboo taper on its bending stress (with restriction t_o=t_i)

The solution for x in Equation 12 and its relationship with maximum bending stress (Equation 10) could be plotted as a curve in Cartesian diagram. The curve is shown as dot line type in Figure 2. As seen on Figure 2, the (x, σ_{max}) value moving to the top of the bamboo stem if the taper is lower.

Meanwhile for bamboo stem's taper lower than 0.0269, the maximum normal stress always happened in the third length point which have smaller diameter. Therefore the maximum bending stress could be found exactly by solving Equation 10 for $x = L/3$ and the result is Equation 13:

$$\sigma_x = \frac{16PL\left(d_{to} + t_o L/3\right)}{3\pi\left(\left(d_{to} + t_o L/3\right)^4 - \left(d_{ti} + t_i L/3\right)^4\right)}; \text{for } 0 \le t \le 0.0269 \quad (13)$$

Dividing the maximum bending stress with the value in the center of the span ($x = L/2$), we got Figure 3. For $0 \le t \le 0.0269$ taper range, the exact taper strength ratio could be solved by substituting Equation 13 into Equation 11 become:

$$C_t = \frac{\left(d_{to} + t_o L/3\right)\left(\left(d_{to} + t_o L/2\right)^4 - \left(d_{ti} + t_i L/2\right)^4\right)}{\left(d_{to} + t_o L/2\right)\left(\left(d_{to} + t_o L/3\right)^4 - \left(d_{ti} + t_i L/3\right)^4\right)}; \text{for } 0 \le t \le 0.0269 \quad (14)$$

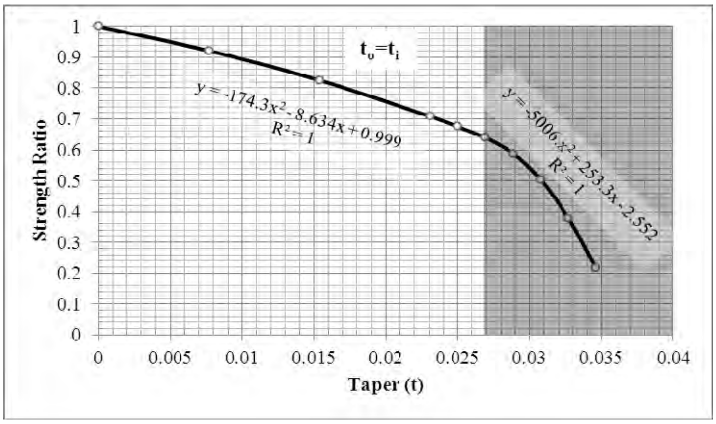

Fig. 3. Strength ratio (C_t) of bamboo taper on third point loading bending test (with restriction t_o=t_i)

Then in order to make it simpler, the polynomial regression was conducted. Finally we got Equation 15 with perfect coefficient of determination (R^2=100%) which indicated the strength ratio of bamboo stem's taper, and the graph was shown in Figure 3. It is simpler to use Equation 15 than Equation 14.

$$C_t = \begin{cases} -174.3t^2 - 8.64t + 0.999; \text{for } 0 \le t_o = t_i = t < 0.0269 \\ -5006t^2 + 253.3t - 2.552; \text{for } 0.0269 \le t_o = t_i = t < 1 \\ 1; \text{for } t_o = t_i = t = 1 \end{cases} \quad (15)$$

III. Bamboo Taper Measurement

We made a survey on bamboo market in Bogor, identified the species, and measured the dimension of some stem. During our survey we found there are three bamboo species sold in the Bogor market namely Tali (Gigantochloa apus (Bl.Ex Schult.f) Kurz), Hitam (Gigantochloa atroviolaceae Widjaja), and Andong (Gigantochloa psedorundinaceae). The sample quantity and dimensional properties are tabulated in Table 1. Previous study[11]on four bamboo species planted in Arboretum Bamboo - Bogor Agricultural University, namely Ampel (Bambusa vulgaris Schrad.), Tali (Gigantochloa apus (Bl.Ex Schult.f) Kurz), Gombong (Gigantochloa verticillata (Willd.) Munro), and Mayan (Gigantochloa robusta Kurz.) reported that the overall outer and inner taper ranges are 0 – 0.0127 and -0.0047 – 0.0088 respectively. Yu, et al [12] reported the typical dimension of two bamboo species, namely Kao Jue (Bambusa pervariabilis) and Mao Jue (Phyllotaschys pubescens) for buckling test. Kao Jue had almost straight cylindrical shape with similar diameter from the basal to the top. The average outer and inner diameters were 40 and 30 mm, respectively. On the contrary Mao Jue had tapering stem. The average outer and inner tapers of Mao Jue are 0.005 and 0.004. Table 2 showed the bamboo taper range reported on previous study.

TABLE 1
Survey result on bamboo sold in Bogor market

	Species		
	Tali	**Hitam**	**Andong**
Sample Measured	110	20	30
Dimension			
Length (m)	3.8 – 8.2	5.5 – 7.2	5.2 – 8.2
Outer Basal Diameter (cm)	4.5 – 10	4.2 – 7.8	8.7 – 11.1
Inner Basal Diameter (cm)	2.1 – 6.9	2.1 – 5.2	5.8 – 8.6
Outer Top Diameter (cm)	3.1 – 8.4	3.2 – 5.3	5.2 – 8.6
Inner Top Diameter (cm)	1.9 – 6.8	1.7 – 4.6	4.2 – 6.0
Basal Thickness (cm)	0.2 – 2.2	0.4 – 1.6	0.7 – 1.8
Top Thickness (cm)	0.1 – 1.4	0.2 – 1.1	0.4 – 1.6
Taper			
Outer taper	0.0002 – 0.0055	0.0001 – 0.0068	0.0025 – 0.0067
Inner taper	-0.0036 – 0.0045	0.0001 – 0.0043	0.0003 – 0.0060

TABLE 2
Taper of Bamboo Stem

Species	Taper	MIN	MAX
Ampel (*Bambusa vulgaris* Schrad.) [11]	Inner (t_i)	0.00239	0.0083
	Outer (t_o)	0	0.0079
Tali (*Gigantochloa apus* (Bl.Ex Schult.f) Kurz) [11]	Inner (t_i)	-0.0042	0.0085
	Outer (t_o)	0	0.0054
Gombong (*Gigantochloa verticillata* (Willd.) Munro) [11]	Inner (t_i)	0.0004	0.0087
	Outer (t_o)	0.0013	0.0127
Mayan (*Gigantochloa robusta* Kurz.) [11]	Inner (t_i)	-0.0047	0.0088
	Outer (t_o)	0.0008	0.0079
Kao Jue (*Bambusa pervariabilis*) [12]	Inner (t_i)	0	
	Outer (t_o)	0	
Mao Jue (*Phyllotaschys pubescens*) [12]	Inner (t_i)	0.004	
	Outer (t_o)	0.005	

Based on Table 1 and Table 2, the overall outer taper of bamboo stem is 0 – 0.0127 and inner taper is -0.0047 – 0.0088. Applying overall taper range of bamboo species, Figure 4 are created according to Equation 10. As seen on the Figure 4, all peaks are happened in the load point closer to the top of bamboo stem which had smaller diameter. Since the maximum stress always happen in that point, Equation 14 became the most suitable formula to determine the strength ratio of bamboo taper on third point loading bending test.

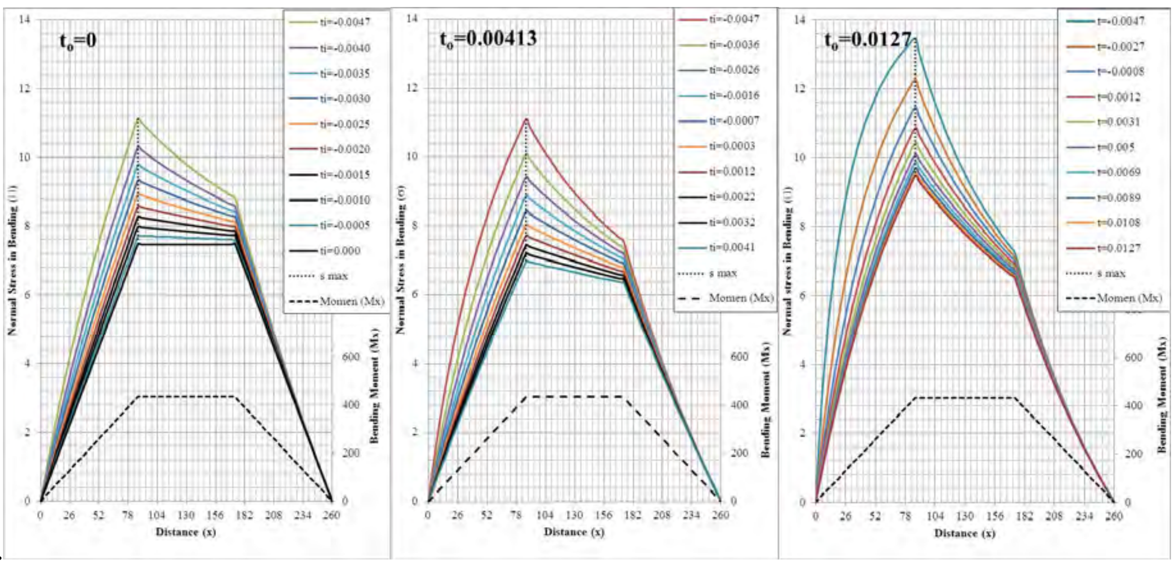

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Fig. 4. Normal stress in third point loading bending test for overall range of bamboo taper

Applying Equation 14 into graphical sketch, we made Figure 5. Figure 5 showed the strength ratio (C_t) value for various bamboo tapers on third point loading bending test. C_t value indicates the ratio of maximum stress along the length with the stress in the center point position. Since the C_t value was not 1 (one), the Modulus of Rupture (S_R) of bamboo stem which calculated from third point loading bending test result, should be adjusted by the Ct value. The adjusted S_R value will be closer to actual S_R than the conventional one because the taper is ignored in the conventional S_R calculation.

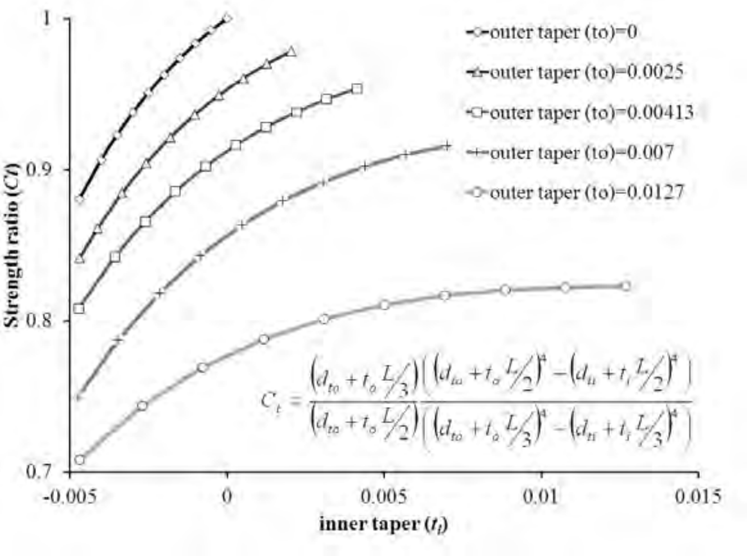

Fig. 5. Strength ratio of bamboo taper in third point loading bending test

IV. CONCLUSION

Dimensional changes along the bamboo stem significantly affected to its Modulus of Rupture (S_R) value which resulted from third point loading bending test. Therefore if the bending test applied for bamboo stem using third point loading configuration as designated by ISO 22157-1:2004, the S_R value should be adjusted by strength ratio of taper (*Ct*). *Ct* is the ratio between maximum bending stress and the bending stress on the center length. This study resulted mathematical formulae to calculate the *Ct* value for overall range of bamboo taper. Then the formula was applied to sketch the graphical style in order to simplify the result.

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