Semi-analytical solution for soliton propagation in colloidal suspension

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***Abstract-* We consider the propagation of soliton in colloidal nano-suspension. We derive the semi analytical solution for soliton propagation in colloidal nano-suspensions for both one and two spatial dimensions using variational method. This Variational method uses both Averaged Lagrangian and suitable trial functions. Finally we analyse about Rayleigh scattering loss in the soliton propagation through the colloidal nano-suspensions.**

Key words- Colloidal suspension, soliton propagation, semi-analytical solution, variational method Lagrangian density, nano-particles.

I. INTRODUCTION

Soliton propagation through the colloidal medium is studied in ref.[2],[3],[5],[8],[9] and [10]. We consider the suspension of dielectric spherical nano-particles as a colloidal medium. The interaction between light and low dimensional materials plays crucial role in the field of science, engineering and technology over the past 25 years. If we pass a laser light beam through a colloidal suspension which consists of spherical dielectric nano-particles, then the laser light will change the refractive index of the nano-particles. The refractive index increases near higher light intensity region. This change in refractive index will create an optical self trapped beam called Optical spatial soliton[1].

The governing equations for spatial soliton in colloidal suspension is derived in ref. [3],[5]. The colloidal nano-particles are considered as a hard-sphere gas and Carnahan-Starling compressibility formula are used and the numerical solution and the stability of the soliton in colloidal medium for both one and two spatial dimensions are given. Given in ref.[5], two different kinds of spherical dielectric nanoparticles are considered. On these two kinds of nano-particles one has refractive index higher and another one has lower than that of background medium.

Ref.[8] gives the semi analytical solution for propagation of soliton in colloidal medium for both one and two spatial dimensions using the Variational method and on neglecting the losses due to Rayleigh scattering. From ref.[3] the scattering loss coefficient is directly proportional to diameter of the nano-particles and inversely proportional to wavelength of the propagating light. The radiation loss in soliton propagation is studied in ref.[7],[11]. In this paper, we include the Rayleigh scattering loss of the soliton propagation in colloidal suspension. When a wave propagates through a colloidal suspension of spherical dielectric nanoparticles, scattering of light occur (Rayleigh scattering) and this scattering effect is considered in this paper. By using variational method [6],[12], we obtaine the semi-analytical solution for the soliton propagation in colloidal suspension.

This paper is organized as follows. The governing equation for soliton propagation is given in chapter 2. Chapter 3 gives the semi-analytical solution for soliton propagation in colloidal suspension for the one spatial dimension. Chapter 4 gives the same for two spatial dimensions and finally conclusion is given in chapter 5.

II. GOVERNING EQUATION

When an Optical beam propagates through a colloidal nano-suspension of spherical dielectric nano-particles, the governing dimensionless non-linear evolution equation can be written as follows [3],[5]

$$i\frac{\partial E}{\partial z}+\frac{1}{2}\nabla^2 \mathrm{E}+(\eta-\eta_0)\mathrm{E}+i\gamma\mathrm{E}=0,$$

$$|E|^2=g(\eta)-g(\eta_0)$$

and

$$g(\eta)=\frac{3-\eta}{(1-\eta)^3}+\ln\eta. \qquad (1)$$

In the above equation, 'E' is Envelope of electric field, 'η' is packing fraction of colloidal nanoparticles, η_0 is background packing fraction and γ is loss coefficient.

The Lagrangian density for the above colloidal equation is given as follows

$$\mathrm{L} = \mathrm{i}(E^* E_z - E E_z^*) - |\Delta E|^2 + 2(\eta - \eta_0)|E|^2 + 2i\gamma|E|^2 - \frac{4-2\eta}{(1-\eta)^2} + \frac{4-2\eta_0}{(1-\eta_0)^2} - 2\eta \ln\eta + 2\eta_0 \ln\eta_0 + 2(\eta - \eta_0)(1 + g_0). \quad (2)$$

Here, '*' denotes the complex conjugate, $E_z = \frac{\partial E}{\partial z}$ is the derivative of E with respect to z.

Equation (1) explains the soliton formation in colloidal suspension

III. ONE SPATIAL DIMENSION

The trial function for one spatial dimension is given as

$$E = \mathrm{a}\ \mathrm{sech}\left(\frac{x-\xi}{\omega}\right) e^{i(\sigma + U(x-\xi))} + ige^{i(\sigma + U(x-\xi))},$$

and

$$\eta = \eta_0 + \alpha\ sech^2 \frac{x-\xi}{\beta}. \quad (3)$$

Here, 'a' is the amplitude and 'ω' is the width of the electric field. 'α' is the amplitude and 'β' is the width of the colloidal fraction. 'σ' represents the propagation constant of the soliton. $\xi' = U$ is the soltion velocity. 'g' represents the radiation bed.

On substitutes these trial functions into Lagrangian density and on integrates it with respect to 'x' over infinite interval, the Averaged Lagrangian is obtained as

$$\mathcal{L} = -2(2a^2\omega + \Lambda g^2)\left(\sigma' + \frac{U^2}{2} - U\xi'\right) - 2\pi a g'\omega + 2\pi a' g\omega + 2\pi a g\omega'\omega - \frac{2a^2}{3\omega} + 4ia^2\gamma\omega + 4\alpha a^2\Omega_1(\omega,\beta) - \beta\Xi_1(\alpha) - 4\beta\Theta_1(\alpha) + 4\alpha\beta(1+g_0), \quad (4)$$

where,

$$\Omega_1(\omega,\beta) = \int_0^\infty sech^2\left(\frac{\zeta}{\omega}\right) sech^2\left(\frac{\zeta}{\beta}\right) d\zeta,$$

$$\Xi_1(\alpha) = \int_0^\infty \left(\frac{4 - 2\eta_0 - 2\alpha\ sech^2\varsigma}{(1 - \eta_0 - \alpha\ sech^2)^2} - \frac{4 - 2\eta_0}{(1-\eta_0)^2}\right) d\varsigma,$$

and $\Theta_1(\alpha) = \int_0^\infty (\eta_0(2\ln\eta_0 + \ln\alpha\ sech^2\varsigma + \alpha\ sech^2\varsigma \ln(\eta_0 + \alpha\ sech^2\varsigma))d\varsigma.$

To obtain the modulation equations, variations on the above Lagrangian density with respect to soliton parameters are taken as follows

$$\frac{d}{dz}(2a^2\omega + \Lambda g^2) = 0, \quad (5)$$

$$\pi\frac{d}{dz}(a\omega) = \Lambda g\left(\sigma' + \frac{U^2}{2} - U\varsigma'\right), \quad (6)$$

$$\pi\frac{dg}{dz} = \frac{2a}{3\omega^2} - \frac{2\alpha a}{\omega}(\Omega_1 - \omega\Omega_{1\omega}), \quad (7)$$

$$\frac{d\sigma}{dz} + \frac{U^2}{2} - U\frac{d\varsigma}{dz} = \frac{-1}{3\omega^2} + \frac{\alpha}{\omega}(2\Omega_1 - \omega\Omega_{1\omega}) + 2i\gamma, \quad (8)$$

$$\frac{d}{dz}(2a^2\omega + \Lambda g^2)U = 0, \quad (9)$$

$$\frac{d\varsigma}{dz} = 0, \quad (10)$$

$$4a^2\Omega_1 - \beta\Xi_{1\alpha} - 4\beta\Theta_{1\alpha} + 4\beta(1+g_0) = 0, \quad (11)$$

$$4\alpha a^2\Omega_{1\beta} - \Xi_1 - 4\Theta_1 + 4\alpha(1+g_0) = 0, \quad (12)$$

and

$$4ia^2\omega = 0. \quad (13)$$

Mass conservation is represented by equation (5) and the momentum conservation is represented by equation (9).
For the steady state the values of U and ξ become zero. Hence equations (5)-(13) can be rewritten as

$$g' = \frac{2a^2}{3\pi\omega^2} - \frac{2\alpha a}{\pi\omega}(\Omega_1 - \omega\Omega_{1\omega}), \tag{14}$$

$$\sigma' + \frac{1}{2\omega^2} - \frac{\alpha}{\omega}(2\Omega_1 - \omega\Omega_{1\omega}) - 2i\gamma = 0, \tag{15}$$

$$4a^2\alpha\left(\Omega_1 - \beta\Omega_{1\beta}\right) - \beta(\alpha\Xi_{1\alpha} - \Xi_1) - 4\beta(\alpha\Theta_{1\alpha} - \Theta_1) = 0, \tag{16}$$

$$4\alpha a^2\Omega_{1\beta} - \Xi_1 - 4\Theta_1 + 4\alpha(1 + g_0) = 0, \tag{17}$$

and

$$4ia^2\omega = 0. \tag{18}$$

In the above equations $g' = \frac{dg}{dz}$, $\sigma' = \frac{d\sigma}{dz}$, $\Omega_{1\omega} = \frac{d\Omega_1}{d\omega}$, $\Omega_{1\beta} = \frac{d\Omega_1}{d\omega}$ and $\Theta_{1\alpha} = \frac{d\Theta_1}{d\alpha}$. Above equations (14-18) are called variational equations or modulation equations of one spatial dimension. These equations explain how soliton propagates in colloidal suspension. We can see that from equation (14-18) the soliton propagation mainly depends on the loss parameter γ. So according to the value of γ, the propagation of the soliton in colloidal suspension will vary. So for smaller particle the scattering loss is less and the soliton will propagate with negligible scattering loss. Thus for smaller particle size and high wavelength laser light, the loss will be very small or negligible. Otherwise scattering loss will affect the propagation of soliton in colloidal suspension of dielectric nano-particles.

IV.TWO SPATIAL DIMENSION

Trial function for two spatial dimensions is given as

$$E = a\,\mathrm{sech}\left(\frac{\phi}{\omega}\right)e^{i\left(\sigma+V(x-\xi_x)+V(y-\xi_y)\right)} + ige^{i\left(\sigma+V(x-\xi_x)+V(y-\xi_y)\right)},$$

and

$$\eta = \eta_0 + \alpha\, sech^2\left(\frac{\phi}{\beta}\right). \tag{19}$$

where, ‘a’ is the amplitude and ‘ω’ is the width of the electric field. ‘α’ is the amplitude and ‘β’ is the width of the colloidal fraction. ‘σ’ represents the propagation constant of the soliton. ‘g’ represents the radiation bed and

$$\phi = \sqrt{(x-\xi_x)^2 + (y-\xi_y)^2}.$$

On substitutes these trial functions into Lagrangian density and on integrates it with respect to 'x' over infinite interval, the Averaged Lagrangian is obtained as

$$L = -2(2a^2\omega I_2 + \Lambda g^2)\left(\sigma' + \frac{U^2}{2} - U\xi_x' + \frac{V^2}{2} - V\xi_y'\right) - 2ag'\omega^2 I_1 + 2a'g\omega^2 I_1 + 2ag\omega\omega' I_1 - a^2 I_{22} + 2\alpha a^2\Omega_2(\omega,\beta) + 2ia^2\omega^2\gamma I_2 - \beta^2\Xi_2(\alpha) - 2\beta^2\Theta_2(\alpha) + a\alpha\beta^2(1+g_0)I_2, \tag{20}$$

where,

$$I_1 = \int_0^\infty \varsigma\, sech\varsigma\, d\varsigma,$$

$$I_2 = \int_0^\infty \varsigma\, sech^2\varsigma\, d\varsigma,$$

$$I_{22} = \int_0^\infty \varsigma\, sech^2\varsigma tanh^2\varsigma\, d\varsigma,$$

$$\Omega_2(\omega,\beta) = \int_0^\infty \varsigma\, sech^2\left(\frac{\varsigma}{\beta}\right) sech^2\left(\frac{\varsigma}{\omega}\right) d\varsigma,$$

$$\Xi_2(\alpha) = \int_0^\infty \varsigma\left(\frac{4-2\eta_0-2\alpha\, sech^2\varsigma}{(1-\eta_0-\alpha\, sech^{2)})^2} - \frac{4-2\eta_0}{(1-\eta_0)^2}\right) d\varsigma,$$

and $$\Theta_2(\alpha) = \int_0^\infty \varsigma\, (\eta_0(2\ln\eta_0 + \ln\alpha\, sech^2\varsigma + \alpha\, sech^2\varsigma \ln(\eta_0 + \alpha\, sech^2\varsigma))d\varsigma$$

Variational equations are obtain similar to one spatial dimension and the final equations are written as follows

$$g' = \frac{I_{22}a}{2I_1\omega^2} - \frac{\alpha a}{2I_1\omega^2}(2\Omega_2 - \omega\Omega_{2\omega}), \tag{21}$$

$$I_{22}\sigma' + \frac{I_{22}}{\omega^2} - \frac{\alpha}{2\omega^2}(4\Omega_2 - \omega\Omega_{2\omega}) - i\gamma I_2 = 0, \tag{22}$$

$$2a^2\Omega_2 - \beta^2\Xi_{2\alpha} - 2\beta^2\Theta_{2\alpha} + 2I_2\beta^2(1+g_0) = 0, \tag{23}$$

$$\alpha a^2\Omega_{2\beta} - \beta\Xi_2 - 2\beta\Theta_2 + 2I_2\alpha\beta(1+g_0) = 0, \tag{24}$$

and $$2ia^2\omega^2 I_2 = 0. \tag{25}$$

In the above equations $g' = \frac{dg}{dz}$, $\sigma' = \frac{d\sigma}{dz}$, $\Omega_{2\beta} = \frac{d\Omega_2}{d\omega}$ and $\Omega_{2\omega} = \frac{d\Omega_2}{d\omega}$. The above equations (21-25) are called variational equations or modulation equations. These equations explain how soliton propagates in colloidal suspension for two spatial dimensions. From the semi-analytical solutions it is clear that the Rayleigh scattering loss parameter γ affects the soliton propagation in colloidal suspension. So to reduce the scattering loss the selection of size of the nano-particle as well as the wavelength of the light is very important criteria.

V. CONCLUSION

Soliton propagation in colloidal suspensions is considered. We obtain the semi-analytical solution for soliton propagation in colloidal suspension using variational method for both the cases of one as well as two spatial dimensions. From these solutions Rayleigh scattering is analyzed. From the semi-analytical solution it is clear that the scattering loss directly affects the soliton propagation in colloidal suspension. For soliton propagation in colloidal suspension the size of the suspended particle as well as the wavelength of the input light is very important.

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