Multi Perspective Metrics for Finding All Efficient Solutions to Bi-Criteria Travelling Salesman Problem

Anuradha.D[#1], Bhavani.S[#2]
[#1]Department of Mathematics, School of Advanced Sciences,
[#2]School of Information Technology & Engineering,
[#]VIT University, Vellore-14, Tamil Nadu, India.
[1]anuradhadhanapal1981@gmail.com
[2]saras.bhava@gmail.com

Abstract -**The investigation of metrics in multiple perspectives is dealt in this paper for a bi-criteria travelling salesman problem (BTSP). By representing the problem in a graphical view, its corresponding metrics in terms of graph theory is estimated. With the programming viewpoint a program using Java programming language is implemented to solve a BTSP. The development of efficient software requires metrics, which is measured to highlight the performance of the software. The application can also be viewed in management perspective through which the solutions in reality are discussed. These approaches can be served as an essential device for the decision makers when they are dealing different varieties of logistics problems comprising two criterions.**

Keywords-Bi-criteria travelling salesman problem, Minimal spanning tree, Hamiltonian cycle, Object oriented metrics (OOM), Size metrics, efficient solution, level of satisfaction.

I. INTRODUCTION

The Travelling Salesman Problem (TSP) is a representative of a large category of problems called as combinatorial optimization problems. In the classical TSP, the salesman must visit all the cities only once and revisit the initial point of the city to end the tour. Each city can be arrived from every other city and for each couple of cities; there is metric that determines the cost/distance/time between them. The ultimate goal of the problem is to get a tour of minimal length in terms of cost/distance/time on a fully connected graph. In the classical TSP, Hamiltonian cycles are usually called tours. TSP has numerous applications in diverse engineering and optimization problems. For example, the order-picking problem in warehouses, drilling of printed circuit boards, computer wiring, and so on. Dantzig et al. [9] drafted a path of approaching the classical TSP and especially its case of finding a shortest route. In literature, many researchers [[2], [6], [7], [8]] have developed various algorithms to solve the classical TSP. In our day by day life, TSP may be designed more gainfully with the concurrent consideration of multi criterions, because a decision-maker is generally assumed to achieve multiple tasks. Fisher and Richter [10] designed a dynamic programming approach for solving a multi objective TSP. Sigal [17] introduced an algorithm for solving large-scale TSP. An E-constrained based procedure for bicriteria TSP was investigated by Melamed and Sigal [13]. Hansen [11] implemented tabu search algorithm to MOTSP. Jaszkiewicz [12] discussed genetic local search for multiple objective combinatorial optimization. Angel et al. [3] proposed a dynamic search algorithm which uses local search for bi-criteria TSP. Nicolas et al. [15] introduced and tested a new approach for the bi-objective routing problem known as the TSP with profits. Cottrell [18] concluded some performance measures that concerns vary according to the freight transport providers. Chaudhuri and De [5] discussed fuzzy multi-objective linear programming (FMLOP) for TSP. Fereidouni [16] designed a FMOLP model for solving the multi-objective TSP in the imprecise environment. Amit Kumar and Anila Gupta [1] illustrated new methods to solve fuzzy assignment problem and fuzzy TSP.

The primary focus of this paper is to investigate the metrics of the symmetric BTSP in multiple perspectives. This paper is organized as follows: Section 2 overviews the general BTSP with its terms and conditions. Section 3 projects the basics of the BTSP metrics. In section 4, a simple application is chosen to discuss the possibility of metrics using the graph theory models, programming and management structures. Finally the paper is concluded in section 5.

II. BI-CRITERIA TRAVELLING SALESMAN PROBLEM

Suppose a salesman has to trip n cities. Beginning from a specific city, he has to trip each city once and return to the beginning point. Our aim is to minimize the total travelling distance and total cost of travelling. Now, BTSP may be modeled into the following Linear Programming Problem.

(P) Minimize $Z_1 = \sum_{i=1}^{n}\sum_{j=1}^{n} d_{ij}x_{ij}$

Minimize $Z_2 = \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}x_{ij}$

Subject to

$$\sum_{i=1}^{n} x_{ij} = 1, \quad j = 1,2,...,n \text{ and } j \neq i \tag{2.1}$$

$$\sum_{j=1}^{n} x_{ij} = 1, \quad i = 1,2,...,n \text{ and } i \neq j \tag{2.2}$$

$$x_{ij} + x_{ji} \leq 1, \quad 1 \leq i \neq j \leq n \tag{2.3}$$

$$x_{ip_1} + x_{p_1p_2} + ... + x_{p_{(n-2)}i} \leq (n-2), \quad 1 \leq i \neq p_1 \neq ... \neq p_{(n-2)} \leq n \tag{2.4}$$

$$x_{ij} = \begin{cases} 1; & \text{salesman travels from city } i \text{ to } j \\ 0; & \text{otherwise} \end{cases} \tag{2.5}$$

where d_{ij} is the distance from city i to j; c_{ij} is the cost of traveling from city i to j; x_{ij} is the link from city i to j ; (2.1) and (2.2) make certain that each city is visited only once; (2.3) is subtour elimination constraint and eliminates all 2-city subtours; (2.4) eliminates all $(n-1)$ -city subtours. We consider a symmetric problem that is $(d_{ij}, c_{ij}) = (c_{ji}, d_{ji})$ for all pairs of cities i to j.

A set $X^\circ = \{\overset{\circ}{x}_{ij}, i = 1,2,...,n; j = 1,2,...,n\}$ is said to be feasible to problem (P) if X° satisfies the conditions (2.1) to (2.5).

A feasible solution X° is said to be an efficient solution to problem (P) if there exists no other feasible X of BTSP such that $Z_1(X) \leq Z_1(X^\circ)$ and $Z_2(X) < Z_2(X^\circ)$ or $Z_2(X) \leq Z_2(X^\circ)$ and $Z_1(X) < Z_1(X^\circ)$. Otherwise, it is called non-efficient solution to problem (P).

III. METRIC BTSP

Measuring the finite solutions of the symmetric BTSP is performed by evaluating various metrics. Thus the metrics BTSP evolve with the considerations of graphs, programming and managerial metrics. BTSP can also be modeled as graph by representing the cities as nodes and the roads connecting the cities as edges. The distance and costs are considered as weights assigned to the edges. Our aim is to obtain a tour of minimal weight. We need the following definitions which can be found in Narsingh Deo [14].

Definition A: Any graph $\mathrm{G} = (\mathrm{V,E})$ consists of two sets of objects namely vertex set V and edge set E where an edge $\mathrm{e_k} = (v_i, v_j)$ is identified by an unordered pair of vertices v_i and v_j.

Definition B: A connected graph without any circuit is termed as a tree.

Definition C: A sub graph T of a connected graph G is said to be a spanning tree of G if T is a tree containing all the vertices of G .

The BTSP can be implemented as a program that can be measured on the basis of its size and OOM. Size is measured using its Lines of Code (LOC), which is the count of the source code, and McCabe's Cyclomatic complexity (MCC) that directly evaluates the number of linearly independent paths through a program's source code. OOM measurements are used to calculate and analyze the quality of software. Weighted methods per class are the number of methods defined in a class and Coupling is a measure of interdependence of two objects.

IV. ILLUSTRATION

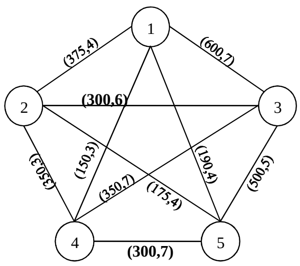
Consider a bi-criteria travelling salesman problem with five cities. Any travelling salesman has to visit all his business cities starting from his residential place and return back to the same place. Assume that there are two criterions under circumstance: (i) minimization of total distance travelled by a salesman and (ii) minimization of total travelling cost during travel. The distance (km) and cost ('000) between the cities are given in the following Table I.

Table I
(Distance, Cost) Matrix

City → ↓	1	2	3	4	5
1	-	(375,4)	(600,7)	(150,3)	(190,4)
2	(375,4)	-	(300,6)	(350,3)	(175,4)
3	(600,7)	(300,6)	-	(350,7)	(500,5)
4	(150,3)	(350,3)	(350,7)	-	(300,7)
5	(190,4)	(175,4)	(500,5)	(300,7)	-

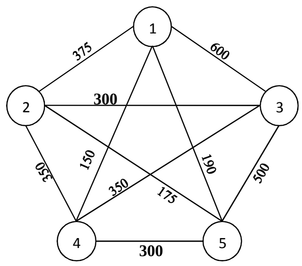
A. Metrics in Graphical BTSP

Any BTSP in Table I can be efficiently modeled as a graph G. The cities are represented as nodes. For each pair of nodes in graph G, there are metrics that define the distance and cost between them, which are represented as arcs.

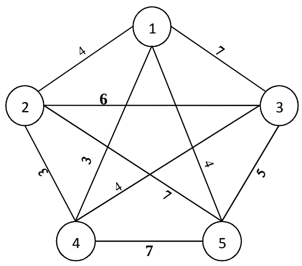


G

The above graph G is a complete graph as every pair of nodes are connected by an edge. Each edge of G has been associated with an ordered pair of weights which are non-negative real numbers. A weighted graph is considered with only one weight at each edge. So we split the graph G into two weighted graphs G_1 and G_2 with first and second weights respectively.



G_1



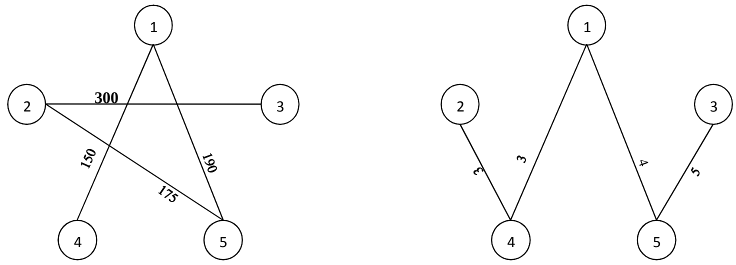
G_2

For weighted graphs, one or more minimal spanning trees can be created. In general, a spanning tree of a weighted connected graph G is said to be minimal spanning tree if its total weight is less than or equal to any other spanning tree of G. Though there are many algorithms to find a minimal spanning tree of a weighted connected graph, algorithm given by Kruskal seem to be simpler. According to Kruskal's algorithm, the edges are arranged in the ascending order of its weights for G_1 and G_2 as in Table II.

Table II
Arrangement of Edges and its Weights

G_1				G_2			
Edge	Weight	Edge	Weight	Edge	Weight	Edge	Weight
(1,4)	150	(2,4)	350	(1,4)	3	(3,5)	5
(2,5)	175	(3,4)	350	(2,4)	3	(2,3)	6
(1,5)	190	(1,2)	375	(1,2)	4	(1,3)	7
(2,3)	300	(3,5)	500	(1,5)	4	(3,4)	7
(4,5)	300	(1,3)	600	(2,5)	4	(4,5)	7

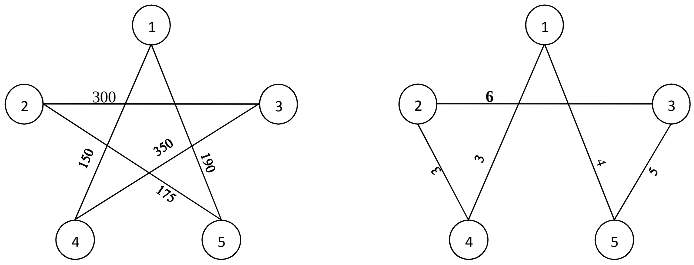
The construction of a minimal spanning tree is done by selecting the edges from lower to higher weights such that no edge forms a loop. Continue the selection process until all nodes are included in it. As a result minimal spanning trees T_1 and T_2 for G_1 and G_2 are obtained as follows:



T_1 T_2

T_1 and T_2 are two minimal spanning trees with weights 815 kms and 15 (‘000) respectively. Corresponding weights with respect to other criterions are 20 (‘000) and 1190 kms respectively.

The bi-criteria travelling salesman problem resembles like tracing a Hamiltonian cycle, which includes all the vertices of a graph.



H_1 H_2

Here H_1 and H_2 are the two minimal Hamiltonian cycles for graphs G_1 and G_2 with weights 1165 kms and 21 (‘000) respectively. Hence the corresponding cost of H_1 is 24 (‘000) and the corresponding distance of H_2 is 1490 kms.

B. Metrics in BTSP Program

Any bi-criteria travelling salesman problem can be well programmed. Software was developed using Java for the BTSP network. The flow of the program begins with the description of the branch and bound method (BBM) to solve any BTSP network. The algorithm considers two individual matrices (distance and cost) of the given illustration. By applying BBM, we obtain the solutions of the matrices. The solutions optimality is checked with the TSP condition. For each matrix, the cost along with the distance and the distance along with the cost is been calculated. Fig.1 depicts the empirical results of the program flow with its detailed processing statements.

Any program can be break down into several modules and for each module metrics can be measured. Measures are helpful data in all engineering streams in order to predict the quality of the final product. Numerical software measurements also called as software metrics continue to play a major role in the development, implementation, and maintenance of quality software. Measuring the software’s internal product attributes is of great concern. The essence of every software structure is its design. OOM have been identified for the intention of assessing the design of a software structure. An examination of the various metrics shows the complexity of the code increase with the number of decisions taken at each phase. For the generated LOC, the MCC is predicted as 23% to 26%. The software size in terms of length has the value of 1313 LOC. The OOM at the class level through the coupling between objects and weighted methods per class are estimated with the values 5 and 54 respectively.

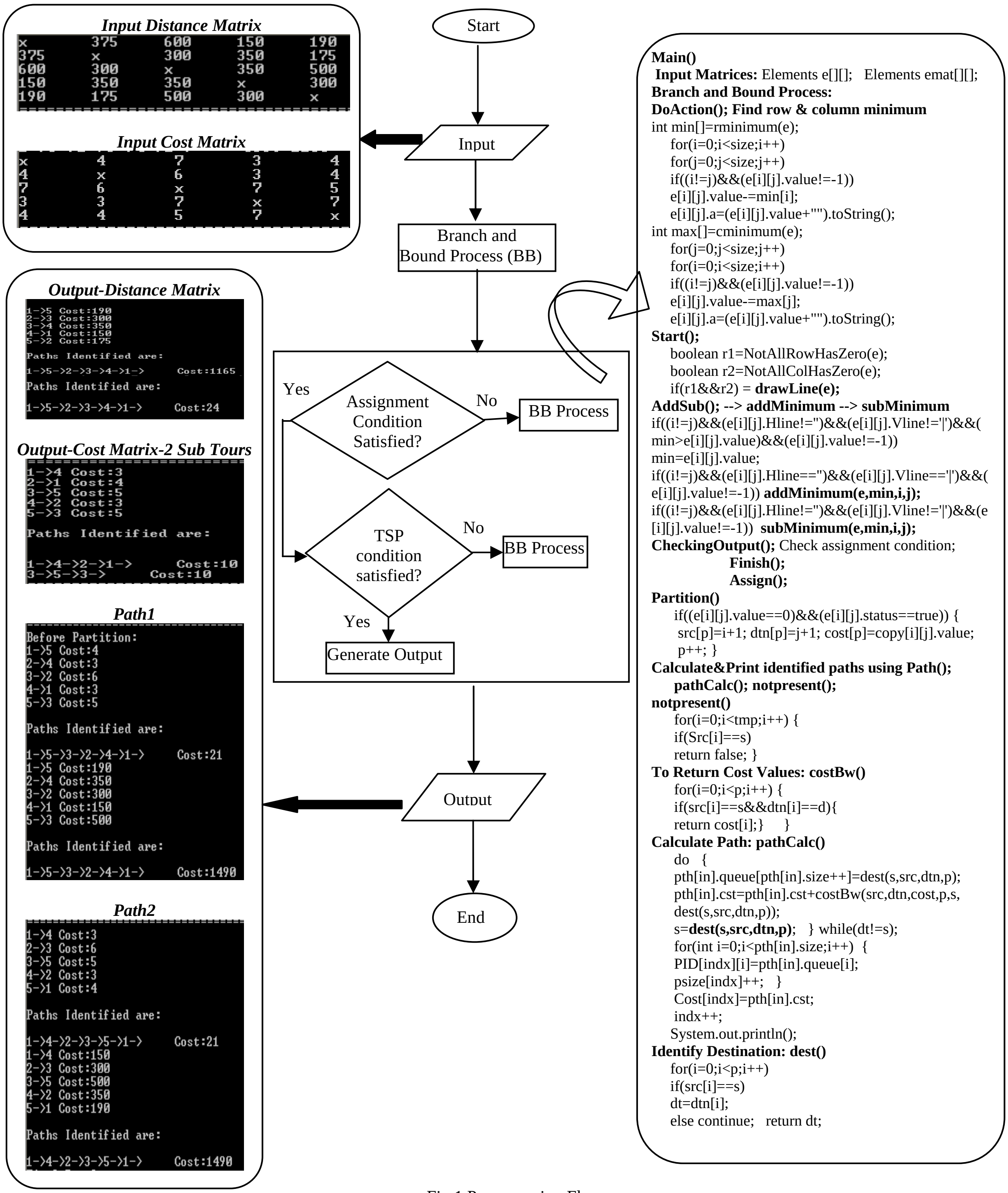


Fig.1 Programming Flow

C. Management Metrics

Metrics are an objective means of measuring performance and effectiveness. They are crucial in the strategic planning process for optimizing the application of resources to achieve the criterion. For the well-organized management of the BTSP, the cost and distance are considered as metrics. The given BTSP in Table I is viewed as two criterions with distance and cost as its corresponding factors. The problem is solved individually for the distance and cost factors using the BBM satisfying the route condition. For the first criteria BTSP, we obtain an optimal tour with the path P_1: 1-4-3-2-5-1 of 1165 kms. For the second criteria, we obtain two sub tours 1-4-2-1 and 3-5-3, which does not satisfy the TSP condition. Each sub tour is solved further and we found the path from each tour as the same. Now, the optimal tour for each sub tours are found as P_2: 1-5-3-2-

4-1 of 21('000) and P_3: 1-4-2-3-5-1 of 21('000). Therefore the ideal solution of the BTSP is (1165 kms, 21('000)). Now, we consider the optimal tour of the first criteria in the second as the feasible tour and vice-versa. Thus the feasible tour for the path P_1 is 24 ('000) and P_2 and P_3 are 1490 kms each. All efficient solution to the given BTSP is given in Table III.

Table III
Efficient Solutions

Sl. No	Tour	Bi-criteria value
1	1-4-3-2-5-1	(1165 kms, 24 ('000))
2	1-5-3-2-4-1	(1490 kms, 21('000))
3	1-4-2-3-5-1	(1490 kms, 21('000))

From Table III, the satisfaction level of the criterion [4] can be predicted for the beneficial of the decision makers at each efficient solution. In Table IV the satisfaction level of criterion of the BTSP is depicted.

Table IV
Satisfaction Level of Criterions

Sl. No	Bi-criteria value of BTSP	Satisfaction level	
		First criteria	Second criteria
1	(1165 kms, 24 ('000))	100	85.714
2	(1490 kms, 21('000))	72.103	100
3	(1490 kms, 21('000))	72.103	100

V. CONCLUSION

This paper provides the set of efficient solutions for bi-criteria travelling salesman problem by analyzing the metrics in various perspectives such as graphical view, programming methodology and management approach. The obtained solutions helps the decision makers to assess the economical activities with the satisfaction level of the criterion and make suitable managerial decision while they are dealing with various bi-criteria logistics problems.

REFERENCES

[1] Amit Kumar and Anila Gupta, "Assignment and travelling salesman problems with coefficients as LR fuzzy parameters", *International Journal of Applied Science and Engineering*, Vol. 10, pp. 155-170, 2012.

[2] T.Andreae, "On the travelling salesman problem restricted to inputs satisfying a relaxed triangle inequality", *Networks*, Vol. 38, pp. 59-67. 2001.

[3] E.Angel, E.Bampis, L.Gourvès, "Approximating the pareto curve with local search for bi-criteria TSP (1,2) problem", *Theor. Comp. Sci.*, Vol. 310, pp.135-146, 2004.

[4] D.Anuradha, and P.Pandian, "New method for finding all efficient solutions to bi-objective assignment problems", *Universal Journal of Mathematics and Mathematical Sciences*, Vol.3, pp.11-22, 2013.

[5] Arindam Chaudhuri and Kajal De, "Fuzzy multi-objective linear programming for travelling salesman problem", *African J. of Math. and Comp. Sci. Res.*, Vol. 4, pp-64-70, 2011.

[6] S.Bhide, N.John, M.R.Kabuka, "A Boolean Neural Network Approach for the Traveling Salesman Problem", *IEEE Transactions on Computers*, Vol. 42, pp. 1271-1278, 1993.

[7] M.Blaser, B.Manthey and J.Sgall, "An improved approximation algorithm for the asymmetric TSP with strengthened triangle inequality", *Journal of Discrete Algorithms*, Vol. 4, pp. 623-632, 2006.

[8] H.J.Bockenhauer, J.Hromkovi, R.Klasing, S.Seibert, and W.Unger, "Towards the notion of stability of approximation for hard optimization tasks and the travelling salesman problem", *Theoretical Computer Science*, Vol. 285, pp. 3-24, 2002.

[9] G.Dantzig, R.Fulkerson, S.Johnson, "Solution of a large-scale Traveling-salesman problem", *Journal of the Operations Research Society of America*, Vol. 2, pp.393-410, 1954.

[10] R.Fischer and K.Richter, "Solving a multi objective travelling salesman problem by dynamic programming", *Optimization*, Vol.13, pp.247-252, 1982.

[11] M.P.Hansen, "Use of substitute scalarizing functions to guide local search based heuristics: the case of MOTSP", *J. Heurisitics*, Vol. 6, pp.419-431, 2000.

[12] A.Jaszkiewicz, "Genetic local search for multiple objective combinatorial optimization", *European J. of Operational Research*, Vol.1, pp. 50-71, 2002.

[13] I.I.Melamed and I.K.Sigal, "The linear convolution of criteria in the bicriteria TSP", *Computational Mathematics and Mathematical Physics*, Vol. 37, pp. 902-905, 1997.

[14] Narsingh Deo, Graph Theory with Applications to Engineering and Computer Science, Eastern Economy Edition, 2011.

[15] Nicolas Jozefowiez, Fred Glover, Manuel Laguna, "Multi-objective meta-heuristics for the Traveling salesman problem with profits", *Springer Science+Business Media B.V*, 2008.

[16] Sepideh Fereidouni, "Solving TSP by using a Fuzzy multi objective linear programming", *African J. of Math. and Comp. Sci. Res.*, Vol. 4, pp.339-349, 2011.

[17] I.K.Sigal, "An algorithm for solving large-scale travelling salesman problem and its numerical implementation", *USSR Computational Mathematics and Mathematical Physics*, Vol.27, pp. 121-127., 1994.

[18] Wayne D. Cottrell, "Performance metrics used by freight transport providers", leonard transportation center, California State University, San Bernardino Project 2007-SGP-1011, 2008.