Effect of Four-wave Mixing on WDM System and its Suppression Using Optimum Algorithms

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***Abstract*-The phenomenon where an undesirable nonlinear effect gives significantly degraded system performance, and becomes the major drawback for optical communication systems is known as Four wave mixing. In order to achieve affordable BER and Q-factor, a comparison of a WDM system with equal and unequal channel spacing is performed. The result of the channel spacing can be verified using opt-sim. A channel allocation method, based on the optimal Golomb ruler, that allows the reduction of FWM effect while maintaining bandwidth efficiency, is presented. The two algorithms i.e. Exhaust algorithm and Search algorithm to construct the Golomb ruler sequences are presented here. The result of these two algorithms is compared using Matlab**.

Keywords: Four-wave mixing, DWDM System, equal-unequal channel spacing, Golomb ruler algorithm.

I.INTRODUCTION

Researchers have been developing different methods and techniques in order to reduce the effect of FWM crosstalk in optical communication systems. The FWM is one of the major limiting factors in DWDM systems. It is third-order nonlinearity in silica fibers, which is similar to inter-modulation distortion present in electrical systems. It is mainly caused due to change in the refractive index with optical power called the optical Kerr effect. New optical frequencies or FWM products are generated in FWM that may cause channel crosstalk. In WDM systems employing dispersion-shifted fibers this nonlinear effect is considered the main cause for crosstalk degradation. The impact of fiber nonlinearities on WDM/DWDM systems can be mitigated by using different methods, including different channel allocation techniques like equal-channel spacing and unequal-channel spacing techniques [1]. In this paper, we have evaluated BER for equal channel spacing and different values of unequal channel spacing, obtained from the Golomb ruler, in the presence of FWM. The eye diagrams are also analysed for different cases. Four-wave mixing (FWM) is defined as a nonlinear process in which three waves of frequencies f_a, f_b, and f_c ($c\neq a,b$) interact through the third-order electric susceptibility of the optical fiber [2] to generate a wave of frequency:

$$f_{abc} = f_a + f_b - f_c \tag{1}$$

As a result of this, three co-propagating waves give rise, by FWM, to nine new optical waves [2]. This process will take place for every possible choice of three channel waves in a WDM system, therefore, suppose if the system has only ten channels, hundreds of new frequencies are generated by FWM. The conventional WDM systems have channels that are usually assigned with centre frequencies (or wave- lengths) which are equally spaced from each other. As a result of this the FWM problem cannot be solved only by increasing channel spacing, which can only decrease the chance and magnitude of the spectral sidebands of unwanted FWM signals trying to enter the pre-assigned channels. Although severe crosstalk can be resulted since there is still very high probability that FWM signals may fall into the WDM channels. In order to reduce the four-wave mixing effect in WDM systems, many unequally spaced channel allocation methods [3] are proposed. However, an increase in the bandwidth requirement is observed as compared to equally spaced channel allocation. Using the concept of Optimal Golomb ruler (OGR) a bandwidth allocation algorithm is presented here to reduce the FWM effect resulting in the improvement of the performance of the WDM system without increasing any additional cost in terms of bandwidth. Section II describes the Optimum algorithm for WDM channel allocation for reducing the four wave mixing effects. Section IV describes the simulation setup of a DWDM system with equal and unequal channel spacing, along with detailed discussion on obtained simulated results.

II. GOLOMB RULER-BASED ALLOCATION

The term ‘‘Golomb ruler’’ [6] refers to a set of nonnegative integer values, such that any two different pairs of numbers from the set have not the same difference. It is similar to a ruler constructed in a way that no two pairs of marks measure the same distance. An example of the Golomb ruler is shown in Fig. 1. [3] An Optimal

Golomb Ruler is the shortest ruler possible for a given number of marks [7]. Therefore applying OGR to the channel allocation problem, it is possible to achieve the smallest distinct number to be used for the channel allocation. Since the difference between any two numbers is distinct, the new FWM frequencies generated would not fall into the one already assigned for the carrier channels.

An n-mark Golomb ruler is a set of n distinct non negative integers (a_1, a_2, .., a_n) called marks, such that the positive differences $|a_i-a_j|$, computed overall possible pairs of different i, j = 1,… n with i≠j are distinct. Let a_n be the largest integer in an n-mark Golomb ruler. Then an OGR with n marks (0,...., an) is an n-mark Golomb ruler if

1. There exists no other n-mark Golomb ruler having smaller largest mark a_n, and

2. The ruler is written in a canonical form as the'smaller' of the equivalent rulers (0, a_2, …, a_n) and(0,...a_n–a_2, a_n), where smaller means the first differing entry is less than the corresponding entry in the other ruler.

The unequal-spaced channel allocation design begins with the division of the available optical bandwidth into equal frequency slots of width Δf [7]. Let f_0 be

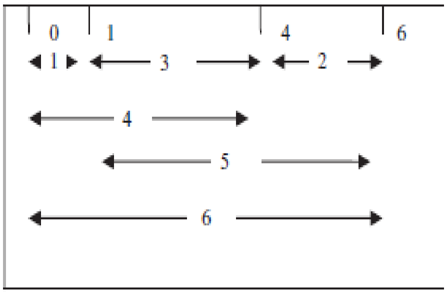


Fig. 1. A Golomb ruler.

the center frequency of the first channel and $f_i = f_0+n_i\,\Delta f$ be the center frequency of the ith channel (orslot), where the integer n_i represents the slot number of the ith channel and N is the total number of channels .In addition, $m_i = n_{i+1}-n_i$ is defined as the channel spacing (in integer) between the ith and(i+1)th channels for i ={1, 2,3,.., N-1}. Therefore, the new frequencies f_{ijk}'s created by FWM inEq.(1)[2]can equivalently be written in terms of slot number n_{ijk} so that

$$n_{ijk} = n_i + n_j - n_k \quad (2)$$

for i, j, k [1, N] and k≠{i,j}. In other words, to ensure that no FWM signals can fall on the pre-assigned WDM channels, the channel-allocation problem can be treated as finding a set of distinct slot numbers so that $n_{ijk}\neq$ {n1, n2, n3,…, n_N}. To further formulate the allocation problem, we consider the physical system parameters. The slot width Δf should be large enough to accommodate the optical signal in a channel with minimum distortion, even with some instability in channel frequencies. On the other hand, to reduce unwanted spectral sidebands entering into a desired channel, the channel frequency-separation Δf_c should also be large enough. For example, to have reasonable system performance, Forghieri [2] suggested the required minimum values of slot width(i.e., $\Delta f\geq 2R$) and channel-to-channel separation (i.e., $\Delta f_c\geq 10R$) as an integer multiple of bitrate in order to avoid significant crosstalk created by FWM spectra and adjacent WDM channels, respectively. These two requirements impose a constraint that relates the minimum channel separation to the slot width(i.e., $\Delta f_c = n\Delta f$) in terms of an integer multiple n.

Constraint1: Since

$$m_i = n_{i+1}-n_i \quad (3)$$

denotes the integer channel spacing between the ith and(i+1)th channels, the inequality $m_i\geq n$ must be satisfied for all i= {1, 2,3, .., N-1}. Furthermore, to minimize the total optical bandwidth occupied by the WDM channels, an additional constraint on the total number of slots is needed while solving the channel-allocation problem.

Constraint2: The total number of slots

$$S = \sum m_i = n_N \quad (4)$$

Must be as small as possible. A lower bound to the total optical bandwidth required B_{un} can be found just from the condition that the m_i's must be different from each other(and larger than n). It follows that [2]

$$B_{un} \geq \left[1 + \left((N/2) - 1\right)/n\right]B_{eq} \quad (5)$$

Where B_{eq} =(N-l) Δf_c is the total optical bandwidth of a conventional WDM system with the channels equally spaced. Fig. 2 [3] shows the bandwidth expansion factor, defined as B_{un}/B_{eq}, versus the number N of channels in the WDM system for various values of the minimum separation parameter n. It can be observed that for n≥5 and up to 10channels the lower bound is achievable. In general, for any value of N and n there are several optimum solutions

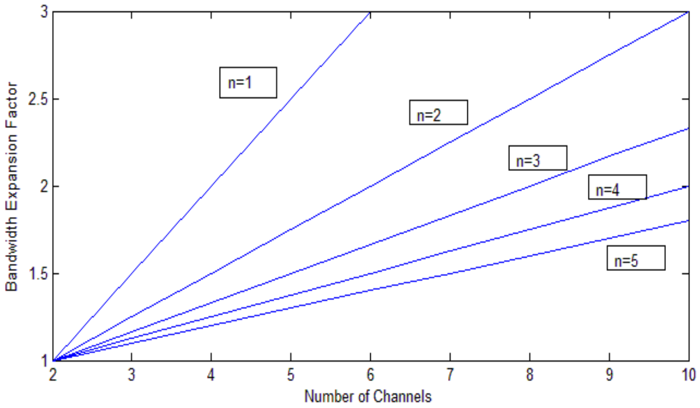


Fig.2. Bandwidth expansion factor (Bun/Beq) vs number of channels for various values of N = $\Delta F_c/\Delta F$.

A. Golomb Ruler Algorithms

The two algorithms to construct the Golomb ruler sequences are described here, which have been totally implemented in Matlab.

1) Exhaust Algorithm: The generation procedure using Exhaust algorithm requires two parameters. First is the number of marks contained in the desired Golomb ruler and the second parameter sets an upper bound on the length of the Golomb Ruler. This procedure is recursive in nature. Here an existing N-mark Golomb ruler is taken and a new mark is appended to the right side of the ruler resulting in N+1 mark ruler. This procedure does not keep track of the mark position but it keeps a track of the spaces between the adjacent marks stored in arrays called spaces. These values represent the first row of the difference triangle for the ruler. The algorithm begins by initialising the first elements in the spaces to the distance. Then it proceeds to the next distance in the spaces and starting at a value of 1, increments this value until the distances measured by the first two entries are unique. Then it repeats this process for the next element and so on. If at any point the total distance measured by these elements in the spaces exceeds the maximum ruler length then the algorithm will back up one element and increment that element and add new distances from there. When the procedure places its last mark and finds a ruler of the desired length it prints this information and continues the search. For the ruler verification procedure, a checker is used to check the series of marks fulfils the requirement of the Golomb ruler. The checker consists of two nested loops which compute every possible distance measures by the first N-elements of the space array. It uses the distance computed as index into an array of Boolean values. If the array element indexed by the distance is already set true, then the distance being checked has already been measured by the set of marks and sequence is not a Golomb ruler. The procedure stops at this point, returning the result as false. If the distance array element is clear then the procedure sets that element to be true and goes on to process the next pair of marks. If there are no conflicts after all the distances have been computed, the checker returns a value of true. A Golomb ruler can be constructed by using the equations as follows;

$$d1_x=M_{x+1}-M_x \tag{5}$$
$$d2_x=d1_x+d1_{x+1} \tag{6}$$
$$d3_x=d2_x+d2_{x+1}-d1_{x+1} \tag{7}$$

The equation for higher order differences is simply extensions of third order difference equation. The first order differences are the distances measured between every pair of adjacent marks in the ruler. The second order differences are the distances measured between marks placed too apart on the ruler. Ruler with m marks will have m-1 first order differences, m-2 second order difference and so on up to single m-1 order difference. The sequences generated by this algorithm does not yield the optimum Golomb sequence as the sequences result in containing large value of marks than necessary. So another algorithm, i.e search algorithm is used to get the optimum result.

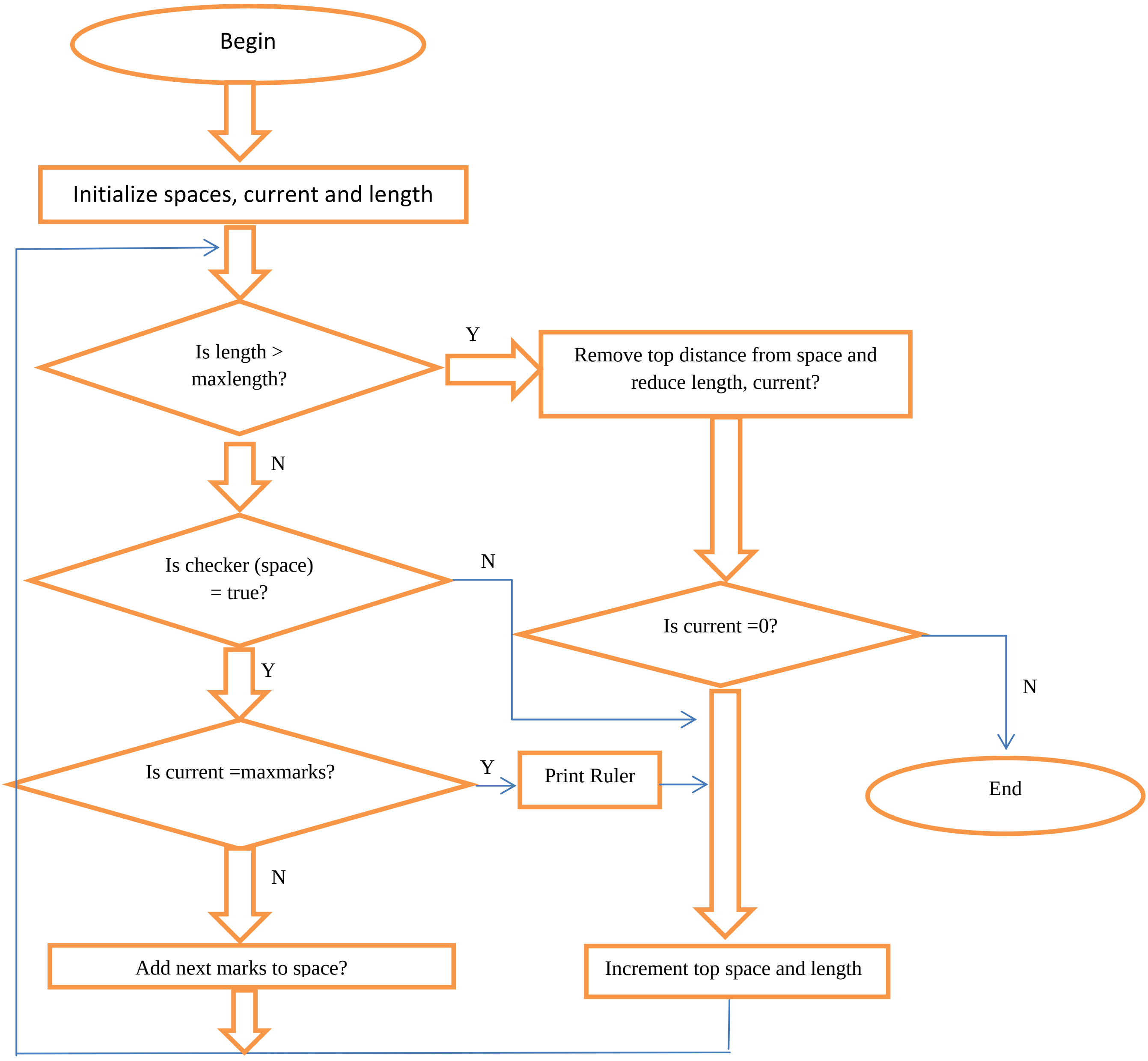


Fig. 3.Flow Chart of Exhaust Algorithm

2) Search Algorithm: The search algorithm is used to generate optimal sequences for a given prime number P and minimum pulse separation n

N = P+1 and S = (n+(P-1)/2)P,

where N is the number of terms in the sequence and S is the maximum length of the slot vector

1.If a prime number is denoted by P and minimum pulse separation n, the first delay vector(or channel spacing vector) $m_1 = [m_0, m_1,.., m_{P\_1}] = [n, n+1, y, n+P-1]$ is constructed.

2. The jth delay vector $m_j = [l_0, l_1 ,…, l_k,.., l_{P-1}]$ for j ={1, 2,3,.., P-1} are generated with $l_k = m_{j\otimes k}$, where ⊗ denotes modulo-P-multiplication.

3. The jth code word $s_j = [s_0, s_1, s_2,.., s_q,…, s_P]$ with weight P+1 are created from m_j, according to the rule $s_q = l_{q-1}+s_{q-1}$ where q = {1,2, 3,.., P} and $s_0 = 0$.

4. Finally, find the code words s_j's with aperiodic correlation constraint one.

TABLE I Simulation results obtained from the search algorithm

N	S	n	Example of slot vector	Nun	Neq	Bun/ Beq
4	15	4	[0,4,9,15]	28	12	2.33
6	45	7	[0,7,15,24,34,45]	125	35	3.6
8	91	10	[0,10,21,33,46,60,75,91]	336	70	4.8
12	231	16	[0,16,33,51,70,90,111,133,156,180,205,231]	1276	176	7.3
14	325	19	[0,19,39,60,82,105,129,154,180,207,235,264,294,325]	2093	247	8.5
18	561	25	[0,25,51,78,106,135,165,196,228,261,295,330,366,403,441,480,520,561]	4641	425	10.9
20	703	28	[0,28,57,87,118,150,183,217,252,288,325,363,402,442,483,525,568,612,657,703]	6457	532	12.1

III. SIMULATION SETUP OF EQUAL AND UNEQUAL CHANNEL SPACING

The simulation setup to evaluate the impact of varying channel spacing between the input channels of a DWDM system in presence of four wave mixing is shown in Fig. 4 [1]consisting of four CW lasers externally modulated by 10 Gbps NRZ data for each channel with equal channel spacing varying in the range of 0.2nm for equal spacing.

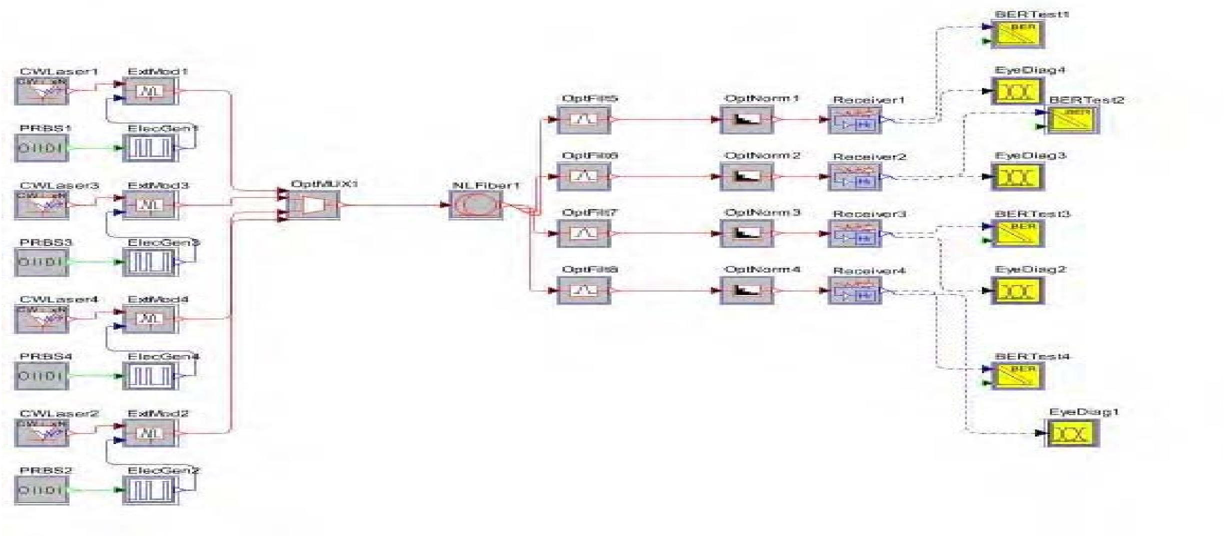


Fig. 4. Simulation setup of four channels with different equal channel spacing with optical span of 100 km in the presence of FWM.

The results are calculated for an optical span of 100 km of 0.25 dB/km attenuation factor and fiber. An electrical scope is kept at the receiver output to examine the eye diagram to compute BER, Q-factor, eye opening and jittering effect. In Fig. 5, eye diagrams of 4-channel DWDM system with equal channel 0.2 nm after optical span of 100 km in the presence of FWM has been observed. We have measured bit error rate for all the simulated channels under the impact of equal and unequal channel spacing as shown in Fig. 4. [1] The observation reveals out that unequal channel spacing improves the Q factor for DWDM systems.

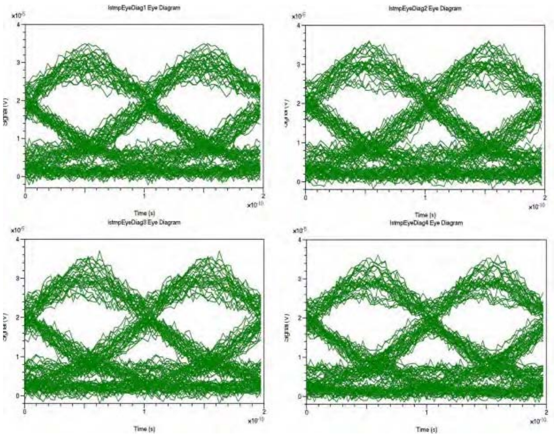


Fig. 5. Eye diagrams of DWDM system with equal channel spacing with optical span of 100 km in the presence of FWM

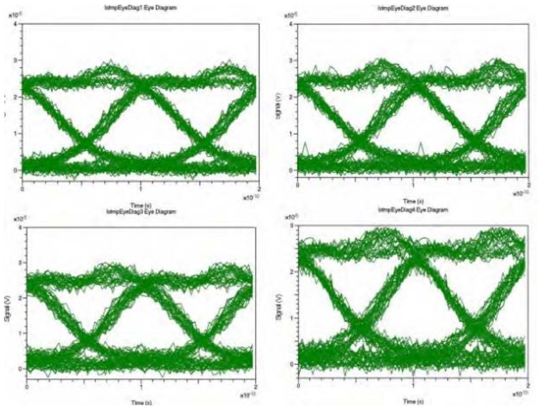


Fig. 6. Eye diagrams of DWDM system with unequal channel spacing with optical span of 100 km in the presence of FWM.

TABLE II
BER for equal channel spacing in the given DWDM system

Channel frequency (nm)	BER
1550	2.5871 EXP-14
1550.20	5.9130EXP-14
1550.40	9.4565EXP-14
1550.60	2.8558EXP-15

TABLE III
BER for unequal channel spacing in the given DWDM system

Channel frequency (nm)	BER
1550	1.452 EXP-23
1550.20	1.783EXP-21
1550.45	1.490EXP-24
1550.75	4.654EXP-25

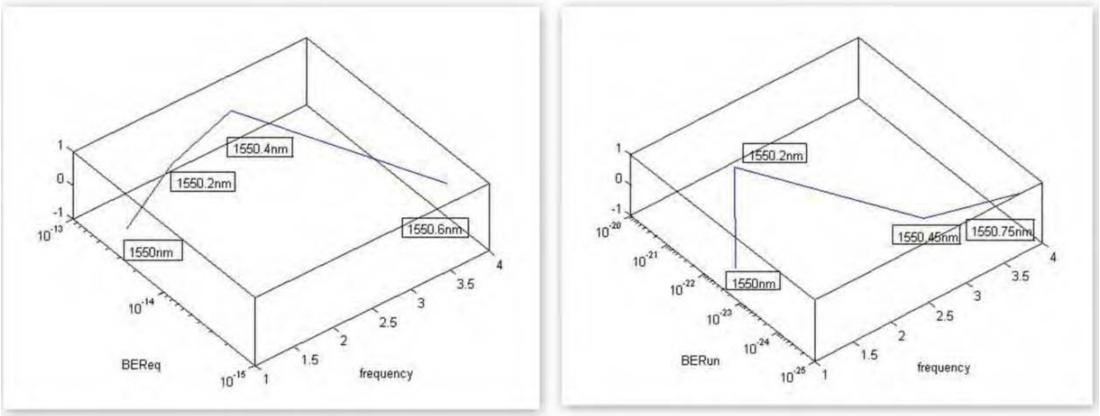


Fig7. Plot of BER vs frequency for equal and unequal channel spacing respectively

IV. FWM REDUCTION USING MODULATION TECHNIQUES

The unequal channel spacing techniques resulted in an increase of bandwidth requirement, compared with equally spaced channel allocation. The dispersion compensation technique on the other hand is a linear phenomenon whereas FWM is a nonlinear effect; therefore dispersion compensation techniques have less effect. Therefore some hybrid modulating techniques can be used to reduce the effect of FWM.

The hybrid modulation techniques can be divide into two techniques:-

1. Low level FWM Reduction and,
2. High Level FWM Reduction

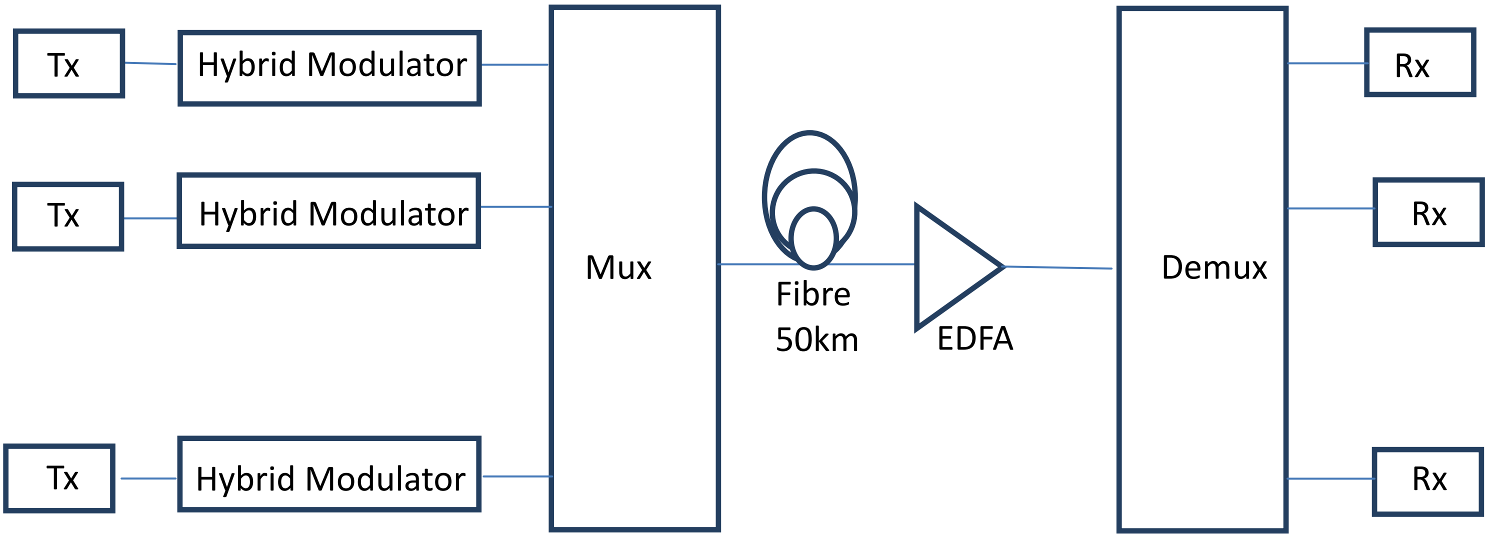


Fig 8 Block Diagram for Hybrid Modulated System

Experimental values:-

8-channel 2.5 Gbps WDM system, sample rate of 160 GHz, sequence length of 128 bits, 64 samples per bit, bit rate of 2.5 Gbps, operating at normal mode, NRZ pulse generator with hybrid modulation schemes. Fibre:-non-zero dispersion fibre, length of 50-100 Km, dispersion value of 16.75 ps/nm/km and a reference wavelength of 1550 nm, Optical amplifier:- EDFA with operating wavelength of 1550 nm. The filter used on the receiving side is a Low Pass Bessel Filter with a cut-off frequency of 0.75*Bit rate.

A) Low Level FWM Reduction

In this stage the hybrid modulator portion is the combination of optical PM modulator followed by an optical AM modulator. The optical PM modulator introduces the phase mismatch in each wavelength which then adds constructively or destructively by the AM modulator.

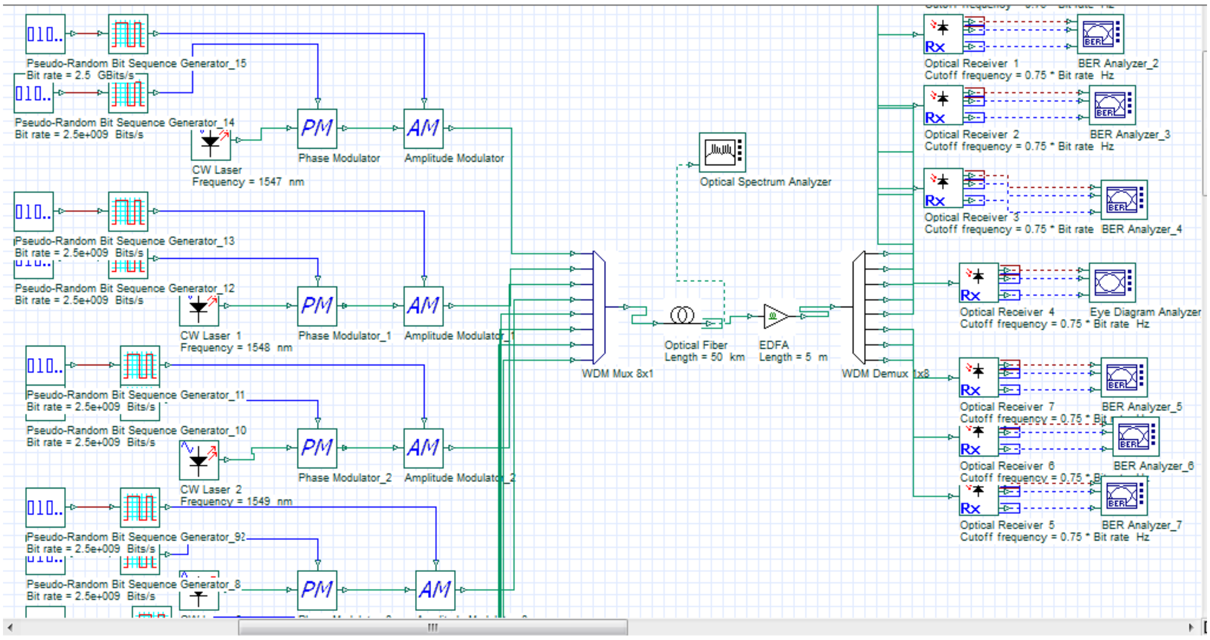
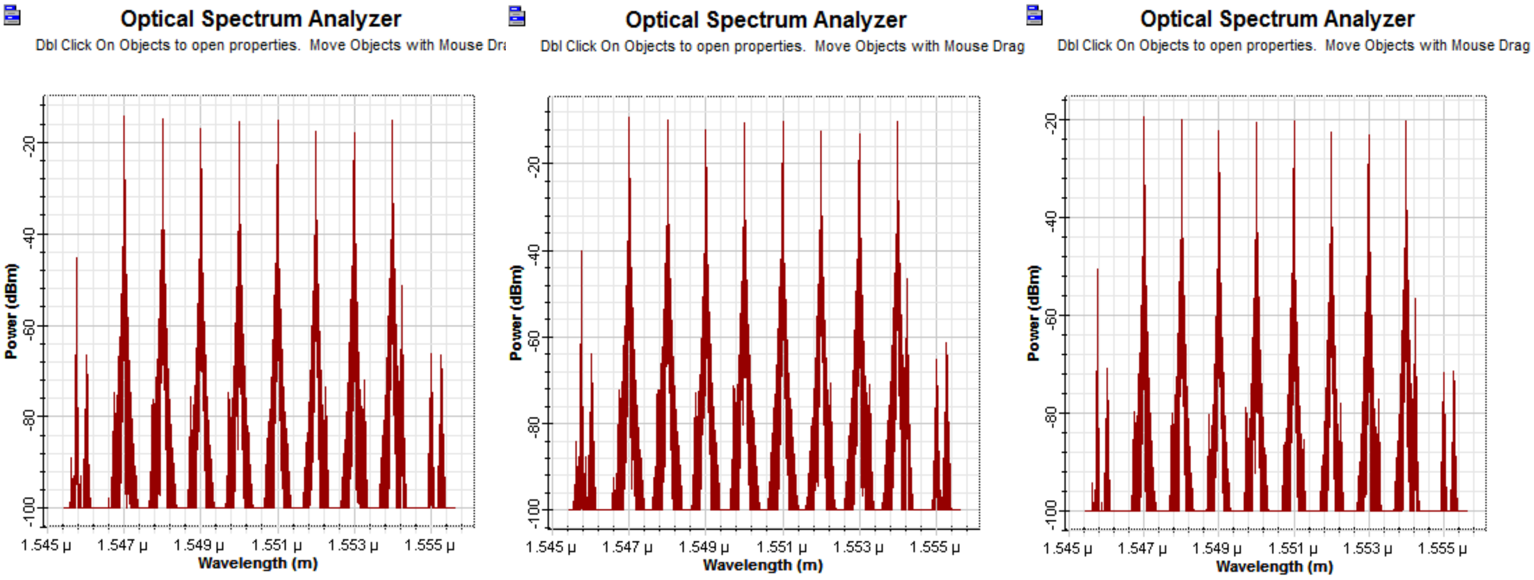
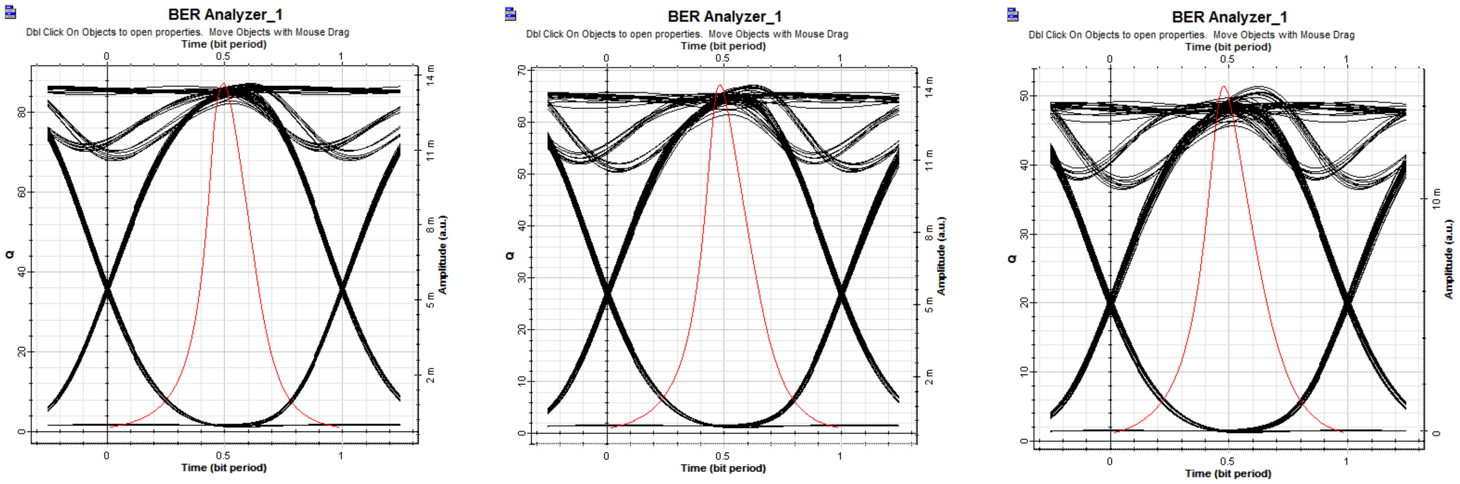


Fig.9. A 8 channel Low level FWM reduction using Optisystem (only 4 receiver section shown in diagram)



(a) (b) (c)

Fig10. Low level FWM reduction (power = 10dBm) for fiber length 50km (a), 75km (b), 100km (c) Respectively



(a) (b) (c)

Fig11. Eye diagram Low level FWM reduction (power = 10dBm) for fiber length 50km (a), 75km (b), 100km (c) Respectively

Table IV

Low level FWM Factors affected by Input Power at 50km (i) ,75km (ii),100km (iii)

Distance	50Km			75Km			100Km		
Power(dBm)	Ber	q-factor	P(fwm)(dBm)	Ber	q-factor	P(fwm)(dBm)	BER	Q-factor	P(fwm)(dBm))
-10	0	68.626	-94	0	42.0321	-95	5.634e-184	28.8836	-95
0	0	84.0025	-68	0	60.7003	-74	0	44.6795	-80
5	0	81.361	-54	0	61.884	-60	0	46.4302	-66
10	0	87.2527	-40	0	67.218	-44	0	51.3797	-50
15	0	62.1779	-26	0	45.5821	-32	1.842e-243	33.2855	-36
20	4.601e-59	16.131	-20	1.422e-57	15.9143	-24	2.661e-44	13.8727	-28

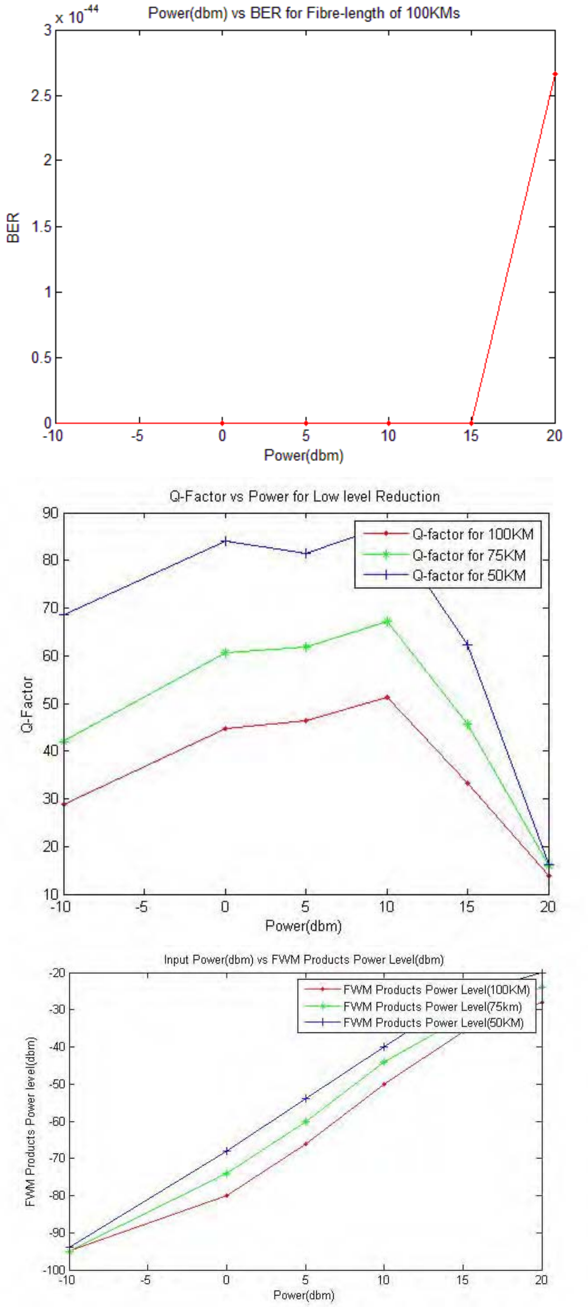


Fig 12 (a) BER 100Km, (b) Q-Factor, (c) FWM Products Power Level vs. Input Power

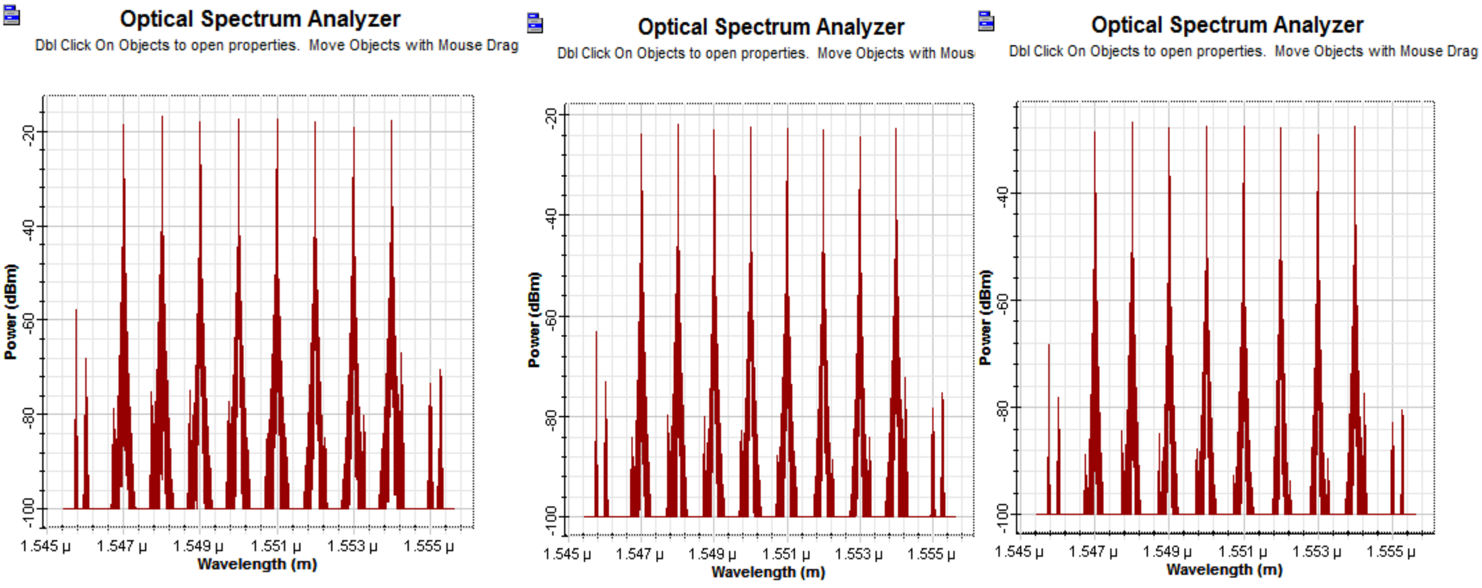
The simulation results for different input power levels shows that the BER decreases to zero, Q-factor increases as the input power varies from -10dBm to 20dBm with a fiber length of 50km with dispersion 16.75ps/nm/km.

1) Drawbacks: With low level FWM reduction scheme:-

The FWM products reduces at lower levels but, The BER analyser parameters give efficient results i.e. Min BER decreases, eye height increases. And quality factor also increases.

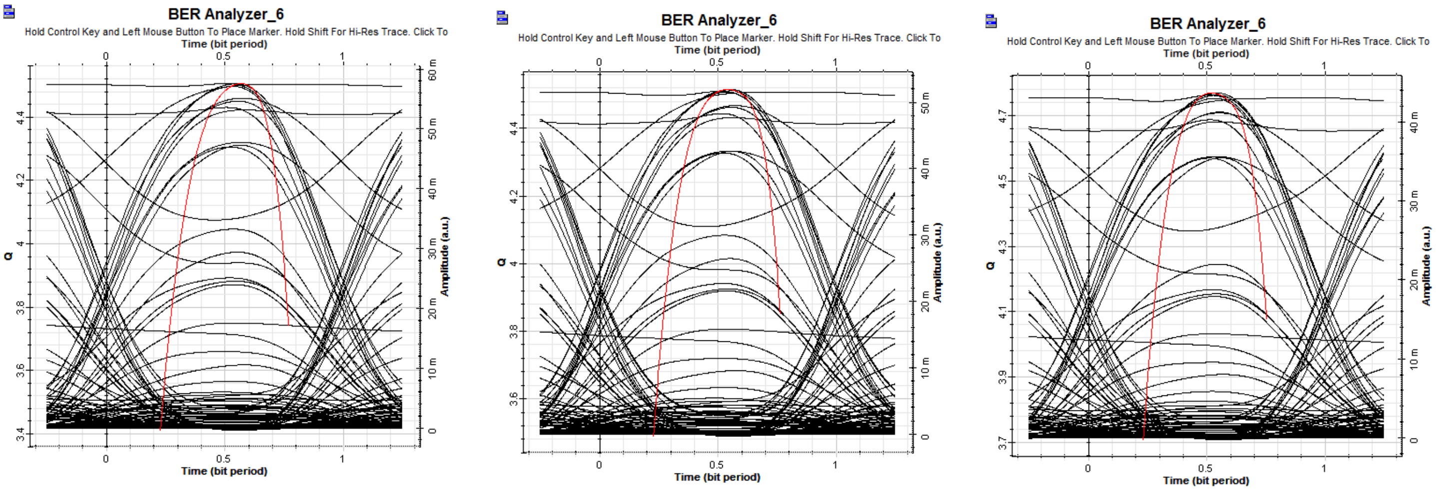
B) High Level FWM Reduction

The high level FWM reduction method uses the hybrid combination of one electrical and three optical modulators. The electrical modulator is CPFSK whose output is connected with optical Dual Port Dual Drive Mach Zehnder modulator followed by Dual Drive Mach Zehnder and AM optical modulators respectively. The electrical CPFSK modulator is responsible for generating the distortion in the signal. This technique can be employed for short distances and low power consuming systems.



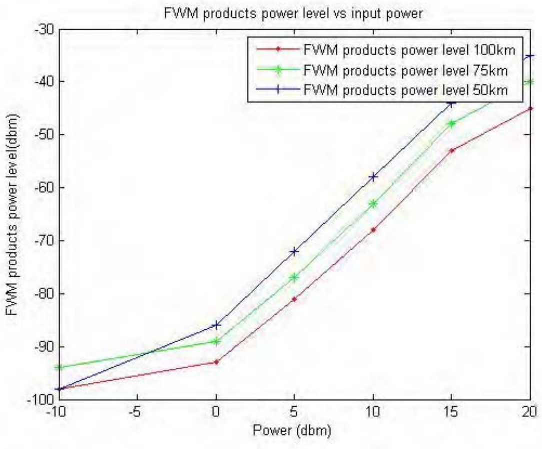
(a) (b) (c)

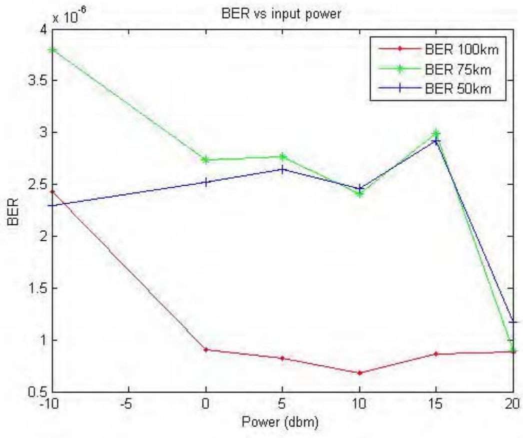
Fig13. High level FWM reduction (power = 10dBm) for fiber length 50km (a), 75km (b), 100km (c) Respectively



(a) (b) (c)

Fig14. Eye diagram High level FWM reduction (power = 10dBm) for fiber length 50km (a), 75km (b), 100km (c) Respectively





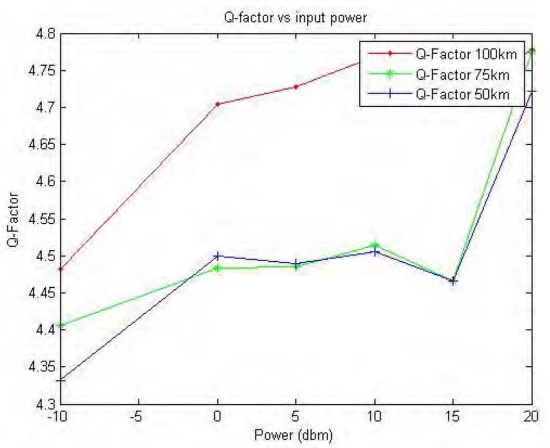


Fig 15 (a) FWM Products Power Level vs. Input Power for high level reduction (b) BER 100Km, (c) Q-Factor

Table V
High level FWM Factors affected by Input Power at 50km (i),75km (ii),100km (iii)

Distance	50Km			75Km			100Km		
power	BER	Q	p	BER	Q	p	BER	Q	p
-10	2.291e-6	4.332	-98	3.801e-6	4.4061	-94	2.432e-6	4.48163	-98
0	2.52e-6	4.49931	-86	2.740e-6	4.484	-89	9.064e-7	4.70493	-93
5	2.6476e-6	4.489	-72	2.761e-6	4.4847	-77	8.216e-7	4.727	-81
10	2.460e-6	4.505	-58	2.411e-6	4.514	-63	6.768e-7	4.76863	-68
15	2.925e-6	4.46641	-44	2.991e-6	4.466	-48	8.614e-7	4.718	-53
20	1.166e-6	4.722	-35	8.889e-7	4.776	-40	8.836e-7	4.77796	-45

The simulation results shows that the minimum BER increases and Quality factor decreases with the increase of input power from -10dBm to 20dBm with a fibre length of 50km with dispersion 16.75ps/nm/km. The height of eye is decreased due to which the quality of the signal is reduced. These distortions will wrap the eye if the power is further increased to 20dBm or 25 dBm. CPFSK offers less side lobes but on the other hand it also introduces the distortions in the signal.

1) Drawbacks: The FWM products are greatly reduced with this scheme up but the results of BER analyser are not satisfactory i.e. there are distortions in the eye diagram

C) Comparative Analysis.

The high level reduction decreases the FWM products to minimum level hence inducing very limited cross-talk but it results in high distortion and non-negligible BER. This causes a limitation to the length of fiber. Also the quality factor is not too good for long distance communication. On the contrary, the low level reduction technique partially affects the FWM products but promises good quality factors and low dispersion effects. Also the BER is approximately zero. Hence this technique is more suitable for long distance data transfer using longer lengths of fibers.

The figure 16, Shows the behaviour of BER for the two techniques as we increases the value of input power. It is observed that the BER of high level reduction have higher value while low level reduction has the minimum value.

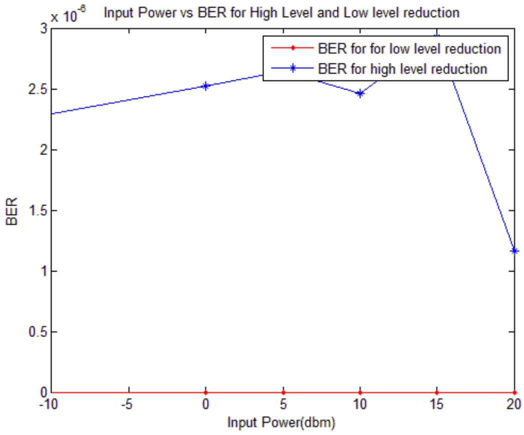


Fig.16. BER vs Input Power

The figure 17 shows the behaviour of FWM products power level for the two techniques as we increases the value of input power. It is observed that the P (FWM) of high level reduction have lower value while low level reduction have higher values

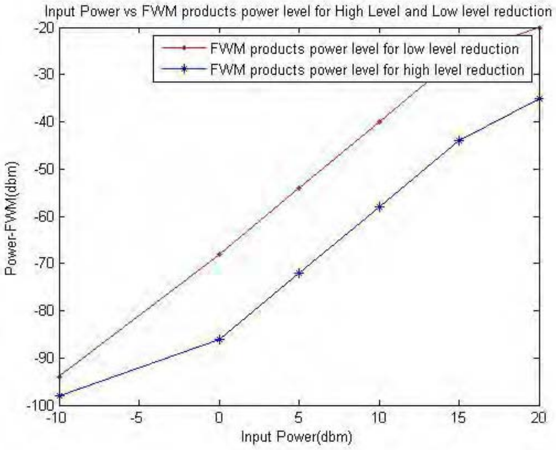


Fig.17. FWM Products Power Level vs Input Power

The Figure 18 shows the effect of input power on the quality factor behaviour. By increasing the value of input power from -10 dBm to 10 dBm the quality factor of low level reduction technique gradually increases while there is a sudden decrease onwards as we increase the input power further from 10 dBm to 20 dBm. The quality factor for high level reduction nearly remains constant as we increase the value of input power from -10 dBm to 20 dBm.

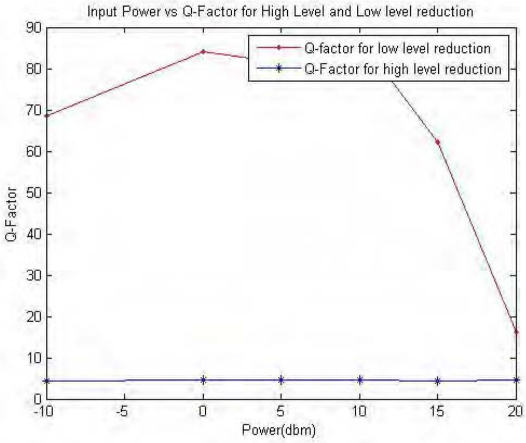


Fig.18. Q-Factor vs Input Power

V.CONCLUSION

An implementation of DWDM system has been investigated to evaluate BER in the presence of FWM under the impact of equal and unequal-channel spacing. It has been observed that reduction in channel spacing to accommodate more optical channels, in the presence of FWM, results in degradation of the performance of dense-optical links. Our simulative results show an efficient improvement in Q-factor and received power with unequal channel spacing in high speed DWDM system. We have illustrated a novel channel allocation method, based on the optical Golomb ruler (OGR) that allows reduction of the FWM effect while maintaining bandwidth efficiency along with the algorithms has been presented in this paper. This channel allocation method generates unequal channel allocation in wave- length division multiplexing (WDM), resulting in reduction in the four-wave mixing (FWM) effect.

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