

Performance Analysis of FWM Efficiency and Schrödinger Equation Solution

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Abstract-The main objective of this paper is to analyze the Four Wave Mixing efficiency (FWM) for different fiber lengths and channel spacing and nonlinear Schrödinger equation solution. In this document, we have simulated the FWM design for two and three waves. We have also analyzed the influences of non-linear refractive index on the four-wave mixing (FWM) characteristics in Semiconductor Optical Amplifiers (SOAs). It has been shown that the generated FWM signal characteristics can be modified due to the variation of non-linear refractive index of the SOA's medium. The wave propagation in the SOA has been modeled using the nonlinear Schrödinger equation. Simulation of Schrödinger equation is carried out using split step Fourier method. The FWM design was simulated with the OPTSIMv5.0 tool and analyzed using the eye pattern method with respect to bit error rate and Q factor. In this paper, FWM efficiency is analytically simulated for two wave and three wave fiber transmission. The results obtained show that the efficiency decreases with the increase of both the frequency spacing and the fiber length, which can be explained using the phase-matching condition. All the efficiency and power equations have been numerically simulated in MATLAB.

Keywords: Wavelength division multiplexing, Four wave mixing, Schrödinger equation, Split step algorithm, Nonlinear refractive index, Phase matching.

I. INTRODUCTION

FWM is one of the important nonlinear effects in optical fibers, so it becomes very important to study its effects in fibers and minimize them to improve the system performance([1],[2],[3],[4]). This paper talks about the nonlinear FWM effect in optical fibers which plays a very important role in today's WDMs (Wavelength Division Multiplexing systems). FWM effect affects the transmission performance of optical link in WDM. In this paper we try to analyze Schrödinger equation solution([1],[2],[5],[6]). and the effect of fiber length and channel spacing on FWM efficiency and FWM power([7]). The efficiency depends on the phase mismatch condition([7-10]) in which dispersion parameters play a significant role. When three waves of different frequencies travel through an optical fiber, a new frequency will be generated [11] depending on the channel separation and this new generated wave may coincide with the original ones leading crosstalk phenomenon. So it becomes important to avoid and minimize such disturbances for error free transmission of signals[3].

II. THEORY

A. Schrodinger Equation

FWM in SOAs is an attractive phenomenon for wavelength conversion. The wave propagation in fiber transmission is studied through Schrödinger equation.([5],[6],[12]) Nonlinear Schrödinger equation is used to analyze nonlinear behavior of signal propagating in an optical fiber as it includes the physical effects, dispersion and nonlinearity of propagating signal.

The basic form of NLSE is

$$\frac{\partial A}{\partial z} + \frac{i\beta_2}{2} \frac{\partial^2 A}{\partial t^2} = i\gamma |A|^2 A - \frac{\alpha}{2} A \quad (1)$$

Where $A = A(z, t)$ is amplitude of Gaussian input pulse and is associated with electric field E of an optical signal in fiber. Propagation of Gaussian in pulse in fiber is considered by taking initial amplitude of that pulse, which is given by

$$A(0, t) = A_0 e^{\left[-\frac{(1+ic)}{2} \left(\frac{t}{T_0} \right)^2 \right]} \quad (2)$$

1) Split Step Algorithm

The numerical analysis of these nonlinear effects is done using nonlinear Schrödinger equation. The equation is solved using an algorithm called “Split-step algorithm” which is coded in Matlab.[2-3] Split step algorithm separates linear and nonlinear parts of the equation as shown below and solves it separately.

The nonlinear Schrödinger equation is given by

$$\frac{\partial A}{\partial z} = -\frac{i\beta_2}{2} \frac{\partial^2 A}{\partial t^2} - \frac{\alpha}{2} A + i\gamma |A|^2 A \tag{3}$$

Representation of above equation after dividing into linear and nonlinear parts is

$$\frac{\partial A}{\partial z} = -\frac{i\beta_2}{2} \frac{\partial^2 A}{\partial t^2} - \frac{\alpha}{2} A + i\gamma |A|^2 A = [D + N]A \tag{4}$$

When $\gamma=0$ results in linear part of nonlinear schrodinger equation

$$\frac{\partial A_D}{\partial z} = -\frac{i\beta_2}{2} \frac{\partial^2 A}{\partial t^2} - \frac{\alpha}{2} A = DA \tag{5}$$

Consider $\alpha=0, \beta_2=0$ results in nonlinear part of nonlinear Schrödinger equation

$$\frac{\partial A_N}{\partial z} = i\gamma |A|^2 A = NA \tag{6}$$

Added a small step “h” simulation parameter is added to separate the linear and nonlinear terms of equation in time domain will result as

$$A_N(t, Z + h) = e^{i\gamma |A|^2 h} A(t, Z) \tag{7}$$

In the same way , solving the equation of linear part gives us

$$A(\omega, Z + h) = e^{[\frac{i\beta_2}{2}(\omega)^2 h - \frac{\alpha}{2} h]} A_D(\omega, Z) \tag{8}$$

The linear function is solved in Matlab. Then took inverse Fourier transform of the linear function and multiplied with the nonlinear function as both are in time domain. A Fourier transform is taken to generate the spectrum for combined equation and inverse Fourier transform ensures the propagation of signal.

B. FWM Efficiency

Four wave mixing effect is also performed with two or more WDM channels. In this design model, there are two WDM channels. Signals generated from these channels are used to generate new signal with the new frequency. Design model of FWM is shown below

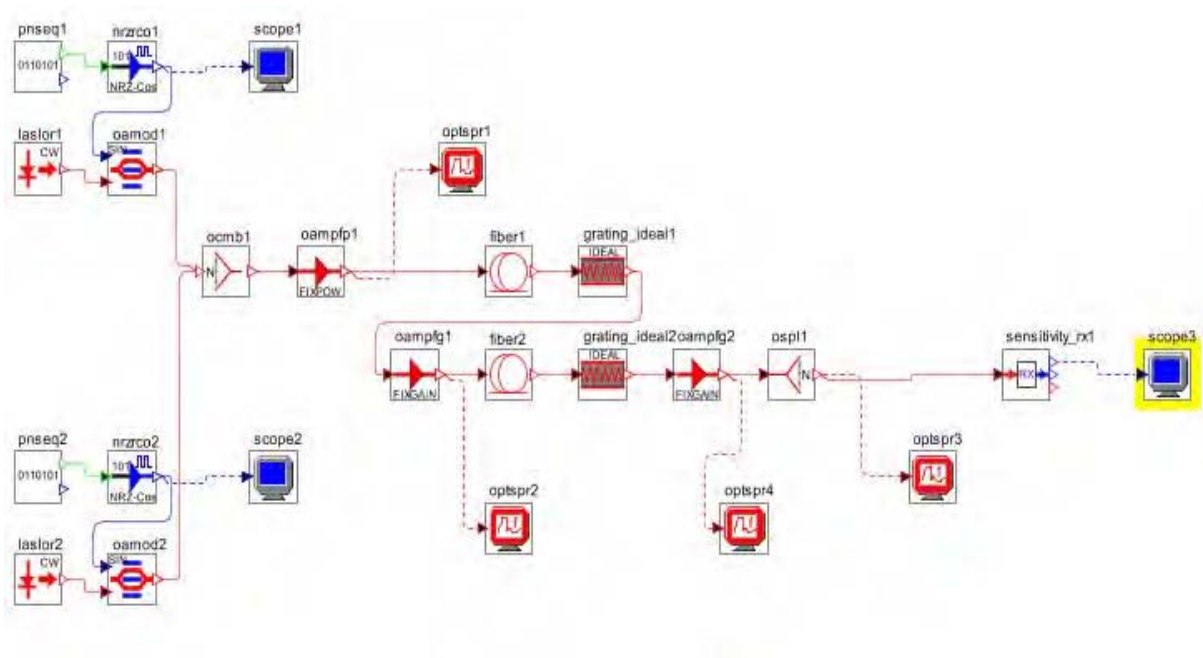


Fig 1. FWM design model for two waves transmission

For any three co-propagating optical signals with frequencies f_i, f_j, f_k the new frequencies f_{ijk} generated by FWM are represented by $f_{ijk} = f_i + f_j - f_k$ for i,j,k . [5] Considering all the possible permutations, N co-propagating optical signals will give rise to M new optical signals as

$$M = N2(N-1) / 2$$

Some of these new frequencies fall on to the N original channels, while others are found in other new frequency locations. Those FWM signals, which overlap with the original ones, are considered as crosstalk and will interfere with the normal operation of the WDM channels. This crosstalk between neighboring channels places a lower limit on the wavelength separation between adjacent channels and an upper limit on the input power in each channel.

Through an FWM process, three waves of frequencies $f_i, f_j, f_k (j \neq k)$ generate the frequency $f_{ijk} = f_i + f_j - f_k$ (subscripts i, j and k select 1, 2, and 3). Fig. 2 & 3 illustrates schematically three different signal frequencies f_1, f_2 and f_3 , and nine new frequencies f_{ijk} . Three signal frequencies and the frequencies generated through the FWM are overlapped for the situation of equal frequency separation $\Delta f = f_2 - f_1 = f_3 - f_2$, as shown in Fig3

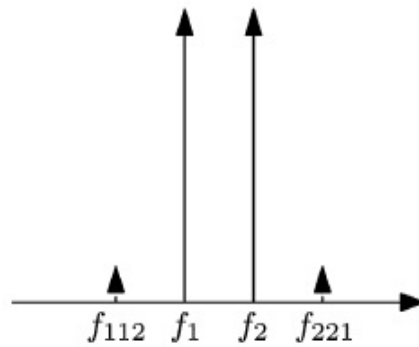


Fig 2. Frequency signals generated through four wave mixing for two wave transmission

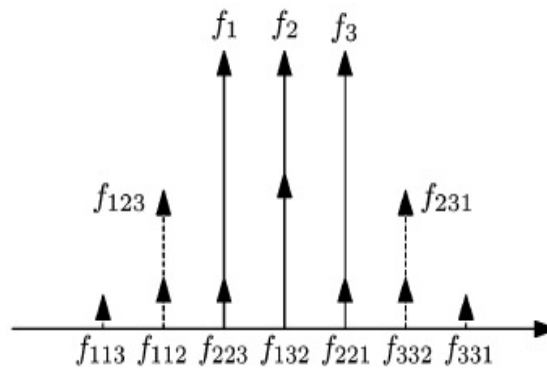


Fig 3. Frequency signals generated through four wave mixing for three wave transmission

This leads to the crosstalk problem in frequency multiplexed coherent communication systems. The time averaged optical power $P_{ijk}(L, \Delta\beta)$ generated through the FWM process for the frequency component f_{ijk} is written as

$$P_{ijk}(L) = (1024\pi^6/n^4\lambda^2c^2)(DX_{1111})^2(L_{eff}/A_{eff})^2 \times P_iP_jP_k e^{(-\alpha L)} \tag{9}$$

where

$$= \{\alpha^2/(\alpha^2 + \Delta\beta^2)\} \{1 + 4e^{(-\alpha L)} \times \sin^2(\Delta\beta L/2)/[1 - e^{(-\alpha L)}]^2\} \tag{10}$$

$$\Delta\beta = (2\pi\lambda_k^2/c)\Delta f_{ik}\Delta f_{jk} \{D_c + (\lambda_k^2/2c) \times (\Delta f_{ik} + \Delta f_{jk})[dD_c(\lambda_k)/d\lambda]\} \tag{11}$$

$$\Delta f_{mn} = |f_m - f_n|, (m, n = i, j, k) \tag{12}$$

L is the fiber length; η is FWM efficiency; n is the refractive index of the core; λ is the wavelength; c is the velocity of light in free space; D is the degeneracy factor having values 3 and 6 for degenerate and non-degenerate cases respectively; X_{1111} is the third-order nonlinear susceptibility; A_{eff} is the effective area of core; α is the fiber attenuation; P_i, P_j, P_k are the input pump powers; D_c is the chromatic dispersion parameter and L_{eff} is the effective fiber length given as

$$L_{eff} = (1 - e^{(-\alpha L)})/\alpha$$

III. RESULTS DISCUSSION

A. Schrodinger equation solution

Here we try to examine the output spectrum of the received signal after transmission through the fiber using Schrödinger equation for two conditions without dispersion and with dispersion. Pulse broadening ratio is also calculated which shows us how much broadened is the input pulse at the output with respect to the distance travelled. Ideally, there is no pulse broadening and change in optical spectrum waveform, when input pulse transmitted through zero dispersion and zero chirp factor in the fiber calculated by numerical analysis but when an input pulse is sent through the fiber, simulation results show dispersion. Thus, numerical analysis of the nonlinear effects ideally shows how the input pulse looks like at the receiver and practical implementation gives us virtual experience of the dispersion due to fiber seen in the received signal.

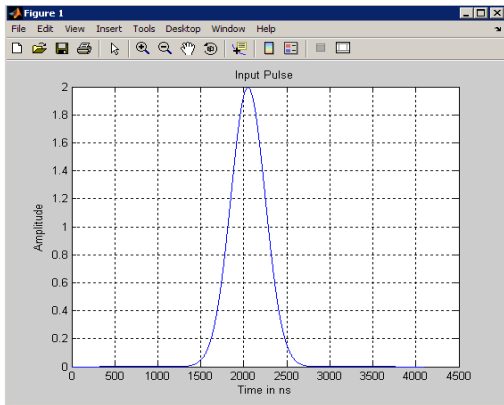


Fig 4. Input signal for chirp factor=0

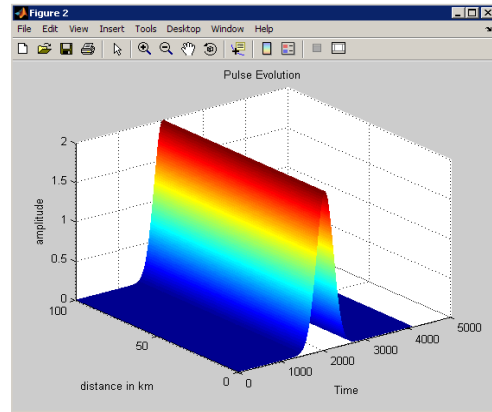


Fig 5. Output spectrum for input signal (without dispersion and chirp factor=0)

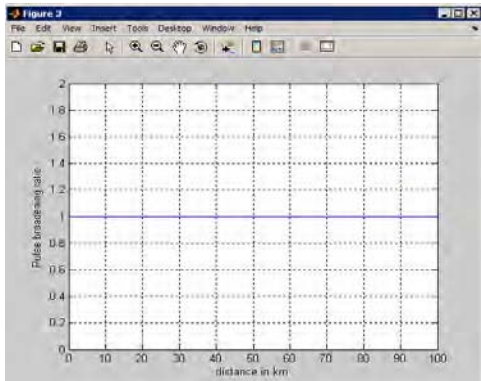


Fig 6. Pulse broadening ratio (without dispersion and chirp factor=0)

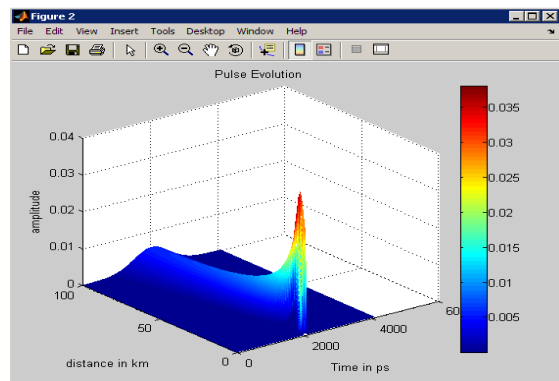


Fig 8. Output spectrum for input signal (with dispersion and chirp factor = -2)

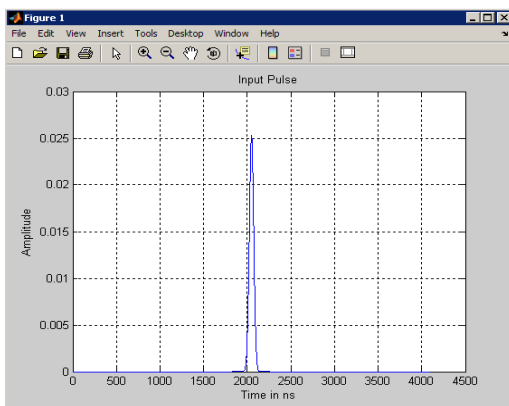


Fig 7. Input signal for chirp factor = -2

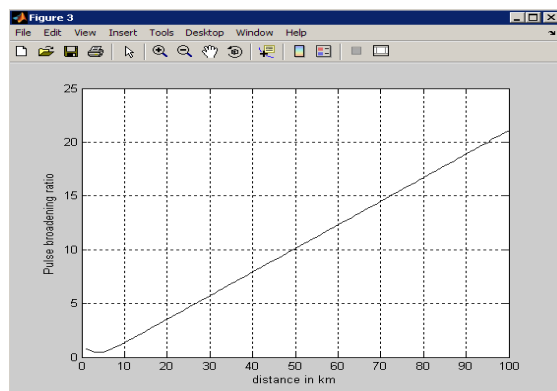


Fig 9. Pulse broadening ratio (with dispersion and chirp factor = -2)

B. Influence of Frequency Spacing

The variations of FM efficiency with respect to frequency variations is shown in figs 10 and 12 for two wave and three wave fiber transmissions respectively. The fiber length is fixed to be 100km. it can be seen that the efficiency decreases with the increase of frequency spacing. The power of the newly generated frequencies is also shown in fig 10 respectively. The power also decreases with the increase of frequency separation.

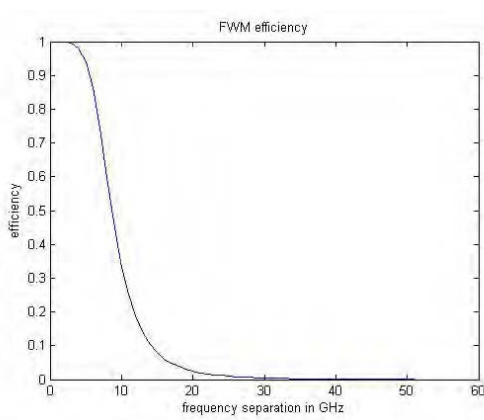


Fig 10. FWM efficiency for two wave transmission for $D_c=16\text{ps/nm/km}$ and fiber

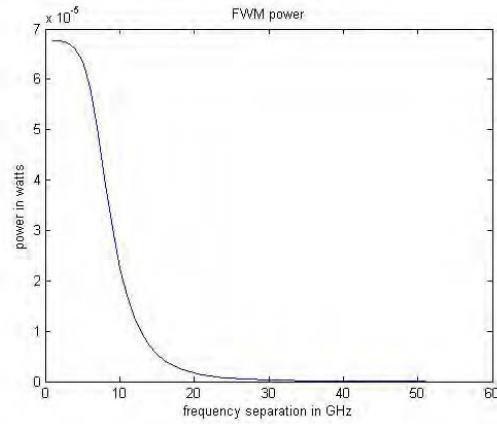


Fig 11. FWM power for two wave transmission for $D_c=16\text{ps/nm/km}$ and fiber length=100km

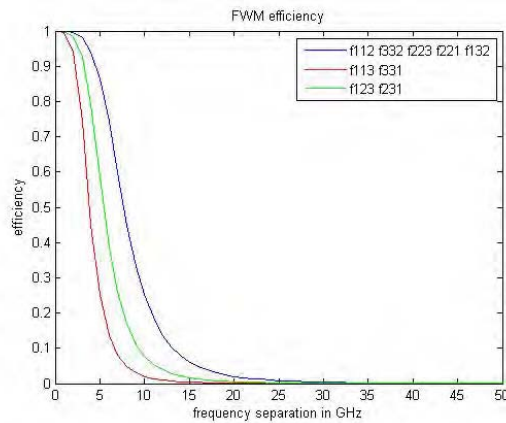


Fig 12. FWM efficiency for three wave transmission for $D_c=16\text{ps/nm/km}$ and fiber length=100km

C. Influence of Fiber Length

The FWM efficiency for both two wave and three waves fiber transmission also decreases with the increase of fiber length. The power also decreases with increase in length as the attenuations and distortions become more prominent with the increase in length of the fiber. The results are shown below

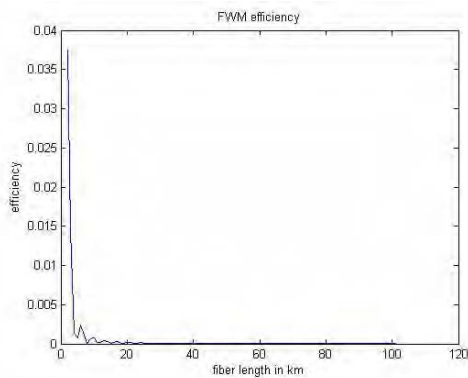


Fig 13. FWM efficiency for two wave transmission for and frequency separation=100GHz

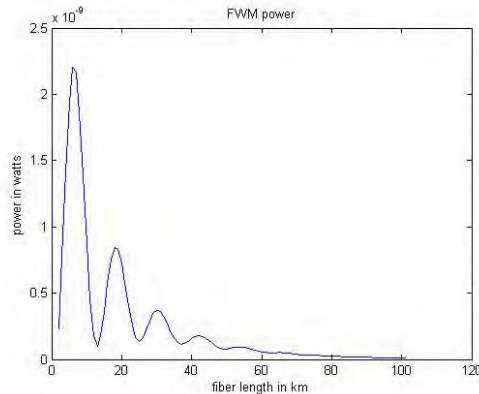


Fig 14. FWM power for two wave transmission for $D_c=16\text{ps/nm/km}$ and frequency separation=100GHz

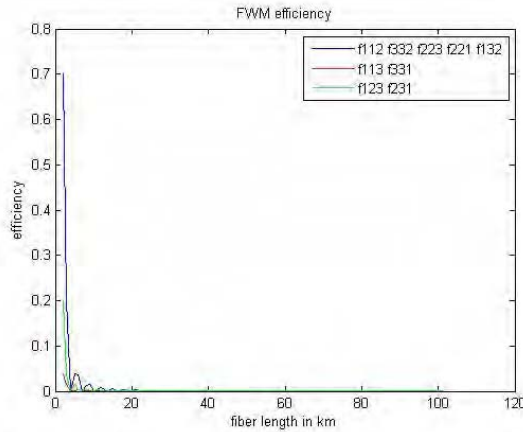


Fig 15. FWM efficiency for three wave transmission for $D_c=16\text{ps/nm/km}$ and frequency separation= 100GHz . Influence of Chromatic Dispersion

We see that as we decrease the chromatic dispersion, the FWM efficiency increases with increase in both frequency spacing and fiber length. FWM efficiency is analyzed for three values of chromatic dispersion(D_c)- 16, 1, 0.1 in this paper. The results are shown below

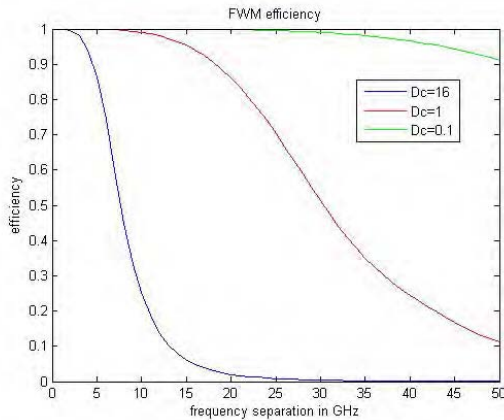


Fig 16. FWM efficiency vs frequency separation for two wave fiber transmission for various values of D_c

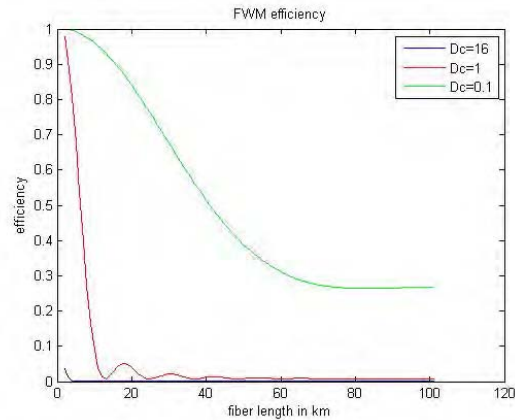


Fig 17. FWM efficiency vs fiber length for two wave fiber transmission for various values of D_c

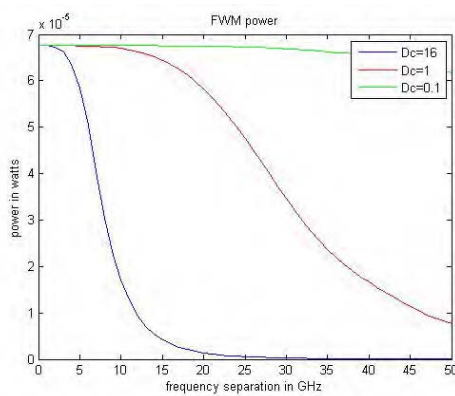


Fig 18. FWM power vs frequency separation for two wave fiber transmission for various values of D_c

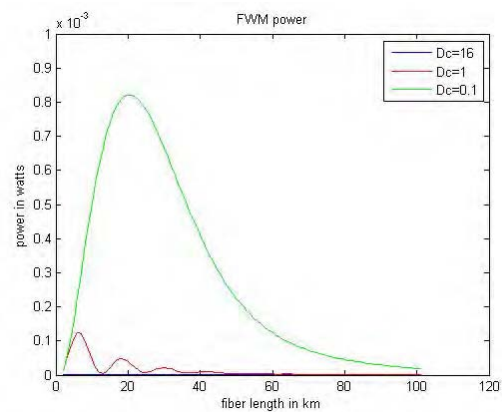


Fig 19. FWM power vs fiber length for two wave fiber transmission for various values of D_c

TABLE I
Parameters Used in Simulation

n	1.465
c	3×10^8 m/s
X_{1111}	$6e-14$ m ³ /J
A_{eff}	$80 \mu\text{m}^2$
α	0.2 dB/km
P	1 mW
D_c	16 ps/km/nm
$\frac{dD_c}{d\lambda}$	0.066 ps/km.nm ²
β	0.03
D	3
λ	1550 nm

IV. CONCLUSION

Numerical analysis of nonlinear Schrödinger equation using Split Step algorithm is done to analyze the effects of nonlinearity in fiber, which is coded and optimized in Matlab. The results show that the input signal get distorted and dispersed after transmission through the fiber. So, we can analyze the signal at the receiver and improve it by minimizing the fiber attenuation and dispersion. Also, FWM efficiency is calculated for two wave and three wave fiber transmissions. FWM efficiency decreases for both increase in frequency separation and fiber length. And, also chromatic dispersion should be high for FWM effect to be less. The results show that the FWM effect is increased when we decrease the chromatic dispersion. Thus, in order to reduce FWM effect and crosstalk between channels frequency separation should be high and chromatic dispersion also should be high.

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