Propagation of Rayleigh Waves due to the Presence of a Rigid Barrier in a Shallow Ocean

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***Abstract*- In the present paper the problem of propagation of Rayleigh waves due to a finite rigid barrier in a shallow ocean has been analyzed. The reflected, transmitted and scattered waves have been obtained by fourier transformation and Wiener-Hopf technique. Numerical computations for the amplitude of the scattered and the reflected waves have been made versus the wave number. It has been done by taking the barriers of different sizes. The scattered waves behave as a decaying cylindrical wave at distant points. The amplitude of the reflected and the scattered waves falls off rapidly as the wave number increases slowly.**

Keywords: **scattered waves, rigid barrier, fourier transforms, Wiener-Hopf technique**.

I. INTRODUCTION

Seismic waves appear on the surface of the earth during an earthquake and loose their energies around the inhomogeneities and irregularities on the surface of the earth. Rayleigh waves are responsible for the destruction of buildings and loss of human lives. This is due to the retrograde nature of motion of Rayleigh waves. Several studies have been carried out to analyze the effect of irregularities like rocks, mountains, grooves, trenches etc. on the incidence of Rayleigh waves. The scattering of Rayleigh waves due to irregularities in the surface leads to large amplification and variation in ground motion during earthquakes.

The propagation of Rayleigh waves in the presence of a rigid barrier in the surface of a shallow ocean lying over the solid half space has been discussed here using Wiener-Hopf technique [9] and Fourier transform [10]. The effect of a vertical barrier, fixed in an infinitely deep sea, on normally incident surface waves was first considered by Ursell [12] for a two dimensional case. The problem of attenuation of Rayleigh waves due to the presence of a surface impedence in the surface of a solid half space has been studied by Gregory [6]. The problem of diffraction of compressional waves due to a rigid barrier in the surface of a deep sea-water and in an ocean superimposed on a solid half space has been studied by Deshwal [3, 4] using the technique of Wiener and Hopf. Momoi [7, 8] has considered the scattering of Rayleigh waves by semicircular and rectangular discontinuities in the surface of a solid half space using the technique of Fourier transformation. The problem of reflection and transmission of a plane SH-wave at a corrugated interface between a dry sandy half space and an anisotropic elastic half space has been studied by Tomar and Kaur [11]. They have used the Rayleigh's method of approximation for studying the effect of sandiness, the anisotropy, the frequency and the angle of incidence on the reflection and transmission coefficients. The reflection of shear waves in visco-elastic medium at parabolic irregularity has been studied by Chattopadhyay et al. [1]. They found that amplitude of reflected wave decreases with increasing length of notch and increases with increasing depth of irregularity. Here we discuss the propagation of Rayleigh waves through irregularity in the form of a rigid barrier and the results have been discussed by taking different sizes of barrier.

II. FORMULATION OF THE PROBLEM

The scattering of incident Rayleigh waves at the rigid barrier in the shallow ocean has been discussed in this paper. The problem is two dimensional and is being analyzed in zx- plane. The z-axis has been taken vertically downward and x-axis along the free surface of the ocean. The rigid barrier of depth h is held fixed in the free surface of water of depth H along the z-axis. The water layer and the solid half space are given by $0 \le z \le H$ and $z \ge H$ respectively. The geometry of the problem is shown in fig. 1.

The wave equations in the liquid and solid media are

$$\left(\nabla^2 + k^2\right)\phi = 0,\ 0 \le z \le H \tag{1}$$

$$\left(\nabla^2 + k_1^2\right)\phi_1 = 0, \; z \geq H \tag{2}$$

$$\left(\nabla^2 + k_2^2\right)\Psi_1 = 0, \; z \geq H \tag{3}$$

where $k = k' + ik''$, $k_1 = k_1' + ik_1''$, $k_2 = k_2' + ik_2''$ (4)

The imaginary parts of k, k_1, k_2 are assumed to be small and positive. Let the incident potential distribution in the solid and liquid be

$$\phi_{1i} = \left[A\left(2\alpha_N^2 - k_2^2\right)\beta_N \cos\beta_N H \exp\left(-i\,\alpha_N x - \beta_{1N}\left(z - H\right)\right)\right] \Big/ \beta_{1N}\left(k_2^2\right), \; z \geq H \tag{5}$$

$$\Psi_{1i} = \left[2Ai\,\alpha_N \beta_N \cos\beta_N H \exp\left(-i\,\alpha_N x - \delta_{1N}\left(z - H\right)\right)\right] \Big/ \left(k_2^2\right), \; z \geq H \tag{6}$$

and

$$\phi_i = A \sin\beta_N z \exp\left(-i\,\alpha_N x\right), \; 0 \leq z \leq H \tag{7}$$

where α_N is a root of the equation

$$\tan\beta_N H = \rho_1 \beta_N \left[4\alpha_N^2 \beta_{1N}\delta_{1N} - \left(2\alpha_N^2 - k_2^2\right)^2 \right] \Big/ \rho\,\beta_{1N}\left(k_2\right)^4 \tag{8}$$

$$\beta_N^2 = k^2 - \alpha_N^2, \beta_{1N}^2 = \alpha_N^2 - k_1^2, \delta_{1N}^2 = \alpha_N^2 - k_2^2 \tag{9}$$

A is a constant and ρ_1 and ρ are densities of two media. Let the total potentials be

$$\phi_t = \phi + \phi_{i,} \; \phi_{1t} = \phi_1 + \phi_{1i}, \; \Psi_{1t} = \Psi_1 + \Psi_{1i} \tag{10}$$

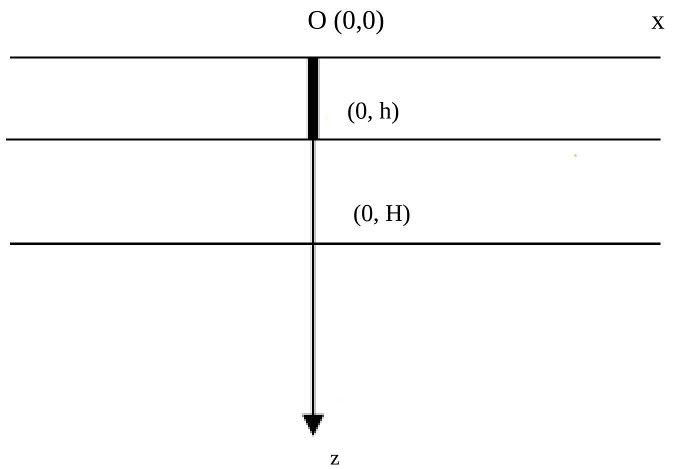

Fig. 1. Geometry of the Problem

The boundary conditions are

$$\tau_{zz} = 0, \; -\infty < x < \infty, \; z = 0 \tag{11}$$

$$u = 0, \; 0 \leq z \leq h, \; x = 0 \tag{12}$$

$$\tau_{zz} = \left(\tau_{zz}\right)_1, \; \left(\tau_{zx}\right)_1 = 0, \; w = w_1, \; -\infty < x < \infty, \; z = H \tag{13}$$

and ϕ_1 and Ψ_1 are bounded as z tends to ∞. The condition (12) shows that the barrier is rigid and there is no displacement across it.

III. SOLUTION OF THE PROBLEM

We take Fourier transform of (1) to obtain

$$\frac{\partial^2 \bar{\phi}\left(p, z\right)}{\partial z^2} - \beta^2 \bar{\phi}\left(p, z\right) = 0 \tag{14}$$

where $\beta = \pm\sqrt{p^2 - k^2}$ and $\bar{\phi}\left(p, z\right)$ represents the Fourier transform of $\phi\left(x, z\right)$ which can be defined as

$$\bar{\phi}(p,z) = \int_{-\infty}^{\infty} \phi(x,z) e^{ipx} dx$$

$$= \int_{-\infty}^{0} \phi(x,z) e^{ipx} dx + \int_{0}^{\infty} \phi(x,z) e^{ipx} dx = \bar{\phi}_-(p,z) + \bar{\phi}_+(p,z) \qquad (15)$$

If for given z, as $x \to \infty$ and M, d > 0, $|\phi(x,z)| \sim Me^{-d|x|}$, then $\bar{\phi}_+(p,z)$ is analytic in $\eta > -d$ and $\bar{\phi}_-(p,z)$ is analytic in $\eta < d$. By analytic continuation, $\bar{\phi}(p,z)$ and its derivatives are analytic in the strip $-d < \eta < d$ in the complex p-plane. It holds similarly for $\phi_1(x,z)$ and $\Psi_1(x,z)$.

Solving equation (14), we have

$$\bar{\phi}(p,z) = B(p)e^{\beta z} + C(p)e^{-\beta z} \qquad (16)$$

Similarly, we can obtain

$$\bar{\phi}_1(p,z) = B_1(p)e^{-\beta_1 z} \qquad (17)$$

$$\bar{\Psi}_1(p,z) = C_1(p)e^{-\delta_1 z} \qquad (18)$$

where $\beta_1 = \pm\sqrt{p^2 - k_1^2}$ and $\delta_1 = \pm\sqrt{p^2 - k_2^2}$ and we have used the condition that $\bar{\phi}_1$ and $\bar{\Psi}_1$ are bounded as z tends to infinity. The signs for β, β_1 and δ_1 are chosen so that their real parts are always positive for all p. Multiplying equation (1) by e^{ipx}, integrating from x=0 to x=∞ and using boundary condition (11), we get

$$\bar{\phi}_+(p,z) + \bar{\phi}_+(-p,z) = \left(2A_1 \sinh\beta z - 2iA\,\alpha_N \sin\beta_N z\right) \Big/ \left(p^2 - \alpha_N^2\right) \qquad (19)$$

Putting z = h in (19) and in its derivative and using the notations $\bar{\phi}_+(p), \bar{\phi}_-(p)$, $\bar{\phi}_+'(p)$ and $\bar{\phi}_-'(p)$ for $\bar{\phi}_+(p,h)$, $\bar{\phi}_-(p,h)$, $\bar{\phi}_+'(p,h)$ and $\bar{\phi}_-'(p,h)$ respectively, we have after eliminating A_1 from the resulting equations

$$\bar{\phi}_+(p) + \bar{\phi}_+(-p) = \frac{1}{\beta}\tanh\beta h\left[\bar{\phi}_+'(p) + \bar{\phi}_+'(-p)\right]$$

$$+ 2iA\,\alpha_N\beta_N \cos\beta_N h \tanh\beta h \Big/ \beta\left(p^2 - \alpha_N^2\right) - 2iA\,\alpha_N \sin\beta_N h \Big/ \left(p^2 - \alpha_N^2\right) \qquad (20)$$

Integrating equation (1) as x varies from $-\infty$ to 0, an equation in $\bar{\phi}_-(p)$ and $\bar{\phi}_-(-p)$ is obtained as in (20). Adding the resulting equation to (20) to find

$$\bar{\phi}_+(p) + \bar{\phi}_+(-p) + \bar{\phi}_-(p) + \bar{\phi}_-(-p) = \frac{1}{\beta}\tanh\beta h\left[\bar{\phi}_+'(p) + \bar{\phi}_+'(-p) + \bar{\phi}_-'(p) + \bar{\phi}_-'(-p)\right] \qquad (21)$$

We take the fourier transform of (13) and using equations from (16) to (18) to get

$$B(p)e^{\beta H} + C(p)e^{-\beta H} = (E_1 + E_2)B_1 e^{-\beta_1 H} \qquad (22)$$

$$B(p)e^{\beta H} - C(p)e^{-\beta H} = EB_1 e^{-\beta_1 H} \qquad (23)$$

where

$$E_1 = \left[(\lambda_1 + 2\mu_1)k_1^2 - 2\mu_1 p^2\right] \Big/ \lambda k^2 \qquad (24)$$

$$E_2 = 4\mu_1 p^2 \beta_1 \delta_1 \big/ \lambda k^2 \left(p^2 + \delta_1^2\right),\ E = \beta_1 k_2^2 \big/ \beta\left(p^2 + \delta_1^2\right) \qquad (25)$$

Solving equations (22) and (23) for $B(p)$ and $C(p)$ and using their values in (16), we get

$$\bar{\phi}(p,z) = \left[(E_1 + E_2)\cosh\beta(z - H) + E\sinh\beta(z - H)\right]B_1 e^{-\beta_1 H} \qquad (26)$$

We put z=h in (26) and its derivative w.r.t. z and find after eliminating B_1

$$\bar{\phi}_+(p)+\bar{\phi}_-(p)+\bar{\phi}_+(-p)+\bar{\phi}_-(-p)=\frac{1}{\beta}\left[\frac{(E_1+E_2)\cosh\beta(h-H)+E\sinh\beta(h-H)}{(E_1+E_2)\sinh\beta(h-H)+E\cosh\beta(h-H)}\right]\Big[\bar{\phi}_+'(p)+\bar{\phi}_-'(p)$$

$$+\bar{\phi}_+'(-p)+\bar{\phi}_-'(-p)\Big] \qquad (27)$$

We can find from (21) and (27) that

$$\bar{\phi}_+(p)+\bar{\phi}_-(-p)=-\bar{\phi}_+(-p)-\bar{\phi}_-(p)=T(p) \qquad (28)$$

$\bar{\phi}_+(p)+\bar{\phi}_-(-p)$ is analytic in the region $\eta>-d$ and $-\bar{\phi}_+(-p)-\bar{\phi}_-(p)$ is analytic in the region $\eta<d$. So by analytic continuation, they represent an entire function $T(p)$. Since each member of $T(p)$ tends to 0 as $|p|$ tends to ∞. Hence by Liouville's theorem, the entire function is identically zero. So from (28), we get

$$\bar{\phi}_+(p)=-\bar{\phi}_-(-p) \text{ and } \bar{\phi}_-(p)=-\bar{\phi}_+(-p) \qquad (29)$$

The results in equation (29) also hold for their derivatives.

IV. SOLUTION OF THE WIENER-HOPF EQUATION

We get a Wiener-Hopf type differential equation, when B_1 in (26) is eliminated between $\bar{\phi}(p)$ and $\bar{\phi}'(p)$ and (20) is used,

$$2\bar{\phi}_-(-p)=M(p)\bar{\phi}_+'(p)+G(p)\bar{\phi}_-'(p)+iA\,\alpha_N\left[M(p)-G(p)\right]\beta_N\cos\beta_N h+2\sin\beta_N h\Big/\left(p^2-\alpha_N^2\right) \qquad (30)$$

where

$$M(p)=M_+(p)M_-(p)=H(p)/N(p)L(p)$$

$$=\left[\frac{(E_1+E_2)\cosh\beta H-E\sinh\beta H}{(E_1+E_2)\sinh\beta(h-H)+E\cosh\beta(h-H)}\right].\frac{1}{\beta\cosh\beta h}=\frac{R_+(p)R_-(p)}{L_+(p)L_-(p)}\prod_{N=1}^{\infty}\left(\frac{p^2-\alpha_N^2}{p^2-\alpha_{2N}^2}\right) \qquad (31)$$

$\pm\alpha_N$ and $\pm\alpha_{2N}$ are the zeros of $H(p)$ and $N(p)$ respectively. $R_\pm(p)$ and $L_\pm(p)$ are obtained asymptotic to constants as $|p|\to\infty$. Similarly if

$$G(p)=G_+(p)G_-(p)=G_1(p)/N(p)L(p)$$

$$=\left[\frac{(E_1+E_2)\cosh\beta(2h-H)+E\sinh\beta(2h-H)}{(E_1+E_2)\sinh\beta(h-H)+E\cosh\beta(h-H)}\right].\frac{1}{\beta\cosh\beta h} \qquad (32)$$

$G_\pm(p)$ are asymptotic to constants as $|p|\to\infty$.

Now using equations (31) and (32) in (30), we have

$$\frac{2\bar{\phi}_-(p)}{G_+(p)M_-(p)}-\frac{G_-(p)\bar{\phi}_-'(p)}{M_-(p)}+\frac{iA\,\alpha_N\beta_N\cos\beta_N hG_-(p)}{M_-(p)\left(p^2-\alpha_N^2\right)}-\frac{2iA\,\alpha_N\sin\beta_N h}{M_-(p)G_+(p)\left(p^2-\alpha_N^2\right)}$$

$$=\frac{M_+(p)\bar{\phi}_+'(p)}{G_+(p)}+\frac{iA\,\alpha_N\beta_N\cos\beta_N hM_+(p)}{G_+(p)\left(p^2-\alpha_N^2\right)} \qquad (33)$$

We can decompose (33) as

$$\bar{\phi}_+'(p)=\frac{A_1'G_+(p)}{M_+(p)}-\frac{A_1'}{M_+(p)}-\frac{iA\,\beta_N\cos\beta_N hG_-(-\alpha_N)G_+(p)}{2(p+\alpha_N)M_-(-\alpha_N)M_+(p)}-\frac{iA\,\alpha_N\beta_N\sin\beta_N h}{\left(p^2-\alpha_N^2\right)}$$

$$+\frac{iA\,\beta_N\cos\beta_N hG_+(p)M_+(\alpha_N)}{2(p-\alpha_N)M_+(\alpha)G_+(\alpha_N)}-\frac{2iA\,\alpha_N\sin\beta_N h}{(p_i-\alpha_N)(p+\alpha_N)M_+(p)M_-(p_i)} \qquad (34)$$

where $p_i=-\alpha_N, p_t$ and p_t are the roots of $G_+(p)=0$. $A_1'=\dfrac{-2\bar{\phi}_-(p_t)}{M_-(p_t)}$, which is independent of p.

$\bar{\phi}_-'(p)$ can be obtained from $\bar{\phi}_+'(p)$ by changing p to –p and using $\bar{\phi}_-'(p) = -\bar{\phi}_+'(p)$. We take the inverse Fourier transform of (15) and using (26) to obtain

$$\phi(x,z) = \frac{1}{2\pi}\int_{-\infty+i\eta}^{\infty+i\eta}\left[(E_1+E_2)\cosh\beta(z-H) + E\sinh\beta(z-H)\right]B_1 e^{(-\beta_1 H - ipx)}dp \tag{35}$$

where $-d < \eta < d$ and B_1 can be obtained by differentiating (26) and using (34).

V. REFLECTED AND TRANSMITTED WAVES

For finding the reflected component we evaluate the integral in (35) along a closed contour in the p-plane. Inside the contour, $p = \pm\alpha_N, \pm\alpha_{2N}$ are the poles and $p = \pm k, \pm k_1, \pm k_2$ are the branch points. The branch cuts are obtained by taking the real parts of β, β_1 and δ_1 equal to zero. The contour of integration is shown in figure 2.

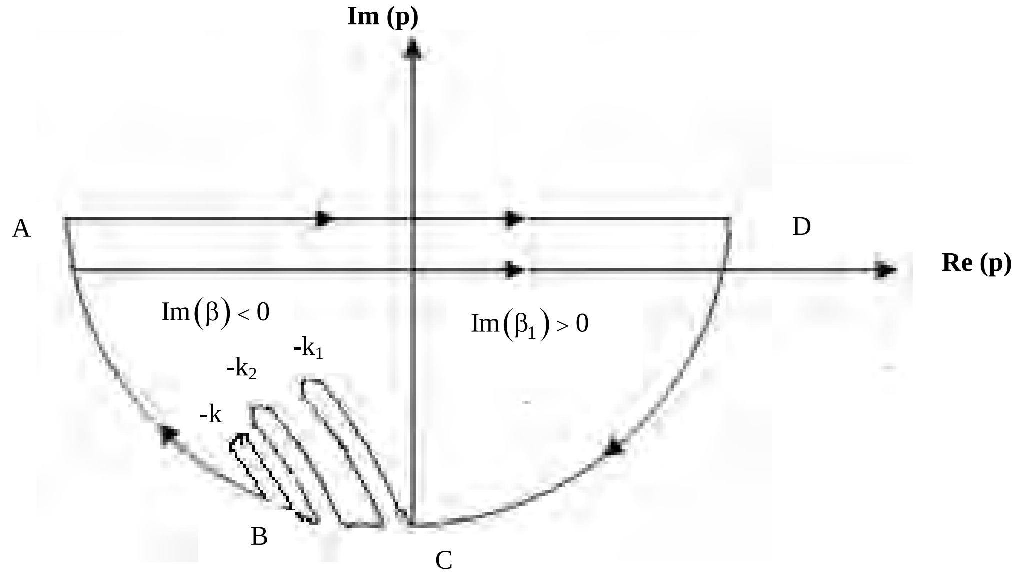

Fig. 2. The contour of integration in complex p- plane

By Cauchy Residue theorem, we have

$$\int_{-\infty+i\eta}^{\infty+i\eta} + \int_A^B + \int_C^D + \int_{M_k} + \int_{M_{k_1}} + \int_{M_{k_2}} = 2\pi i \sum \text{Res}. \tag{36}$$

where M_k, M_{k_1} and M_{k_2} are the branch cuts corresponding to the branch points $-k, -k_1, -k_2$ respectively. There is a pole at $p = \alpha_N$ and the corresponding wave is given by

$$-\frac{M_1}{2}\left[\left(E_1' + E_2' - E''\right)e^{i\beta_N(z-H)} + \left(E_1' + E_2' + E''\right)e^{-i\beta_N(z-H)}\right]e^{-i\alpha_N x} \tag{37}$$

where

$$M_1 = \left[A_1'\left(G_+(-\alpha_N) - 1\right) - \frac{iA\beta_N\cos\beta_N h M_+(\alpha_N)G_+(-\alpha_N)}{4\alpha_N G_+(\alpha_N)} + \frac{iA\beta_N H'(\pm\alpha_N)}{2M_-(-\alpha_N)}\right].\frac{\beta_N M_-(-\alpha_N)\cos\beta_N h}{H'(\pm\alpha_N)} \tag{38}$$

Equation (37) gives the transmitted waves in the region x < 0.

Again, there is a pole at $p = -\alpha_N$ and the corresponding wave is given by

$$\frac{M_1}{2}\left[\left(E_1' + E_2' - E''\right)e^{i\beta_N(z-H)} + \left(E_1' + E_2' + E''\right)e^{-i\beta_N(z-H)}\right]e^{i\alpha_N x} \tag{39}$$

Equation (39) gives the waves reflected from the barrier and from the free surface after its reflection from the barrier. We see that the reflected and transmitted waves have equal but opposite amplitudes.

VI. SCATTERED WAVES

The incident Rayleigh waves are scattered when these waves encounter with surface irregularities like rigid barrier in the surface of a shallow ocean. For finding the scattered component of the incident Rayleigh waves, we consider the contribution of the integral along the branch cuts. We find out that the contribution of the integral vanishes along the branch cut M_k. On M_{k_1}, the main contribution comes from the neighbourhood of the branch point $p=-k_1$. We put $p=-k_1-it$, t being small. Along the cut t varies from 0 to ∞.

Now, $\beta_1=\pm\sqrt{\left(k_1+it\right)^2-k_1^2}=\pm i\sqrt{2k_1''t}=\pm i\,\beta_1'$

Integrating (35) along the two sides of branch cut L_{k_1}, we get

$$S_1\left(x,z\right)=\frac{i}{2\pi}\int_0^{\infty}\Bigg[\Big[\left\{\left(E_1+E_2\right)\cosh\beta\left(z-H\right)+E\sinh\beta\left(z-H\right)\right\}B_1e^{-\beta_1 H}\Big]_{\beta_1=i\,\beta_1'}$$
$$-\Big[\left\{\left(E_1+E_2\right)\cosh\beta\left(z-H\right)+E\sinh\beta\left(z-H\right)\right\}B_1e^{-\beta_1 H}\Big]_{\beta_1=-i\,\beta_1'}\Bigg]e^{-tx}dt \tag{40}$$

where we have used the result of Ewing et al. [5] i.e.

$$\int_0^{\infty}t^{1/2}\Psi\left(t\right)e^{-tx}dt=\frac{\Psi\left(0\right)\boldsymbol{\Gamma}\left(3/2\right)}{x^{3/2}}+\frac{\Psi'\left(0\right)\boldsymbol{\Gamma}\left(5/2\right)}{x^{5/2}}+\frac{\Psi''\left(0\right)\boldsymbol{\Gamma}\left(7/2\right)}{x^{7/2}}+\ldots\ldots\ldots.. \tag{41}$$

Where $\boldsymbol{\Gamma}\left(x\right)$ is a Gamma function. By considering h to be small in comparison to H and for small kh, k_1h etc., we have

$$S_1\left(x,z\right)=\frac{\boldsymbol{\Gamma}\left(3/2\right)\Psi\left(0\right)}{x^{3/2}}e^{-k_1''x} \tag{42}$$

where

$$\Psi\left(0\right)=\frac{\lambda\left(k''k_2\right)^2}{\pi\ \beta'\ \lambda_1\left(ik_1''\right)^{3/2}\left(\left(k_2''\right)^2-\left(2k''\right)^2\right)}\left[\left[A_1'-\frac{iA\,\alpha_N\beta_N}{\alpha_N^2+\left(k_1''\right)^2}-\frac{A_1'}{M_-\left(-k_1\right)}-\frac{iA\,\beta_N h}{\left(\alpha_N-ik_1''\right)M_-\left(-\alpha_N\right)}\right].\right.$$
$$\left.\frac{\beta'\left(\cosh\beta' z-\beta' h\sinh\beta' z\right)}{\sinh^2\beta' H}+\left(A_1'-\frac{iA\,\beta_N h}{\left(\alpha_N-ik_1''\right)M_-\left(-\alpha_N\right)}\right)\frac{\left(\sinh\beta' z\right)M_-\left(-k_1\right)}{\cosh^2\beta' H}\right]$$

and $\beta'=\left(i\left(k_1''\right)^2k^2\right)^{1/2}$

Scattered waves are obtained in (42) which are of the form $\dfrac{e^{-k_1''x}}{x^{3/2}}$.

VII. NUMERICAL COMPUTATIONS AND DISCUSSION OF RESULTS

The incident Rayleigh waves are scattered as well as reflected due to the presence of rigid barrier. The scattered wave propagate with a velocity equal to the velocity of the compressional waves of the solid half space. The mathematical calculations have been done by taking $\lambda_1\approx\mu_1=0.8,\lambda=0.6$ for H= 0.5 km, z = 0.01km, $\alpha_N=0.99$, k = 1.0, $k_2=0.8$, h= 0.05. The graph of amplitude versus the wave number of the scattered waves has been plotted in figure 3. The graph indicates that the amplitude of the scattered waves depend on the wave number and hence on the wave length of the scattered waves. Also the scattered waves given in equation (42) are of the form $\dfrac{e^{-k_1''x}}{x^{3/2}}$ and they behave as cylindrical waves decaying exponentially for large x. The reflected and transmitted waves are respectively given by equations (39) and (37). The graph showing the variation of amplitude versus wave number of reflected Rayleigh waves is being plotted by taking the barriers of different sizes and is shown in figure 4.

For computational and graphical purpose, we have fixed H = 0.5 km and graph has been plotted by taking h = 0.01, 0.05, 0.10, 0.15, 0.20, 0.25 km. The graph shows that amplitude of the reflected Rayleigh waves falls of rapidly as the wave number increases.

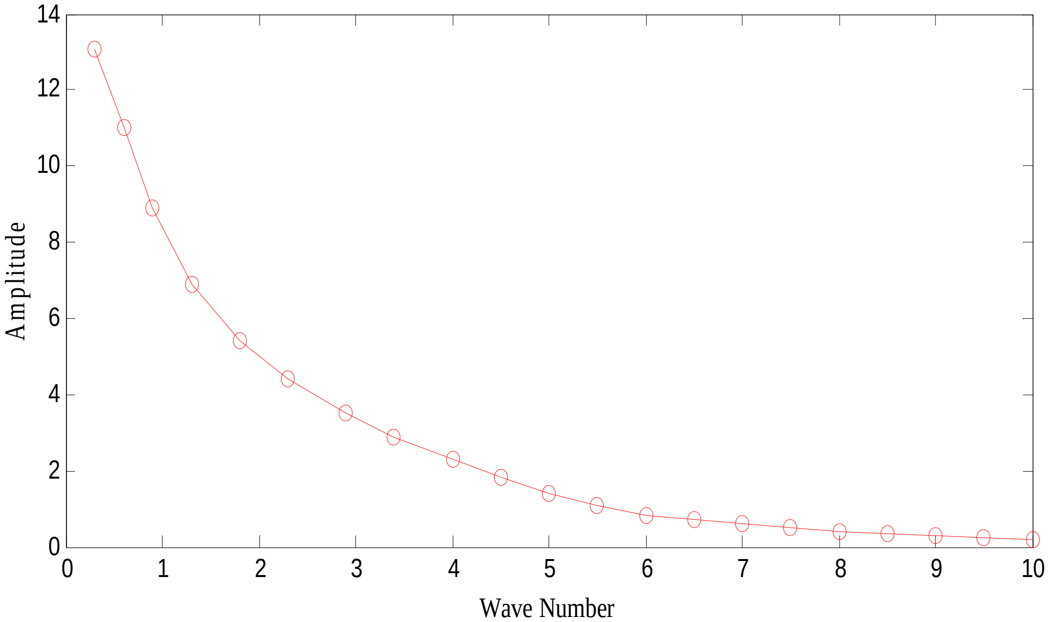

Fig. 3. Variation of amplitude versus wave number of scattered waves

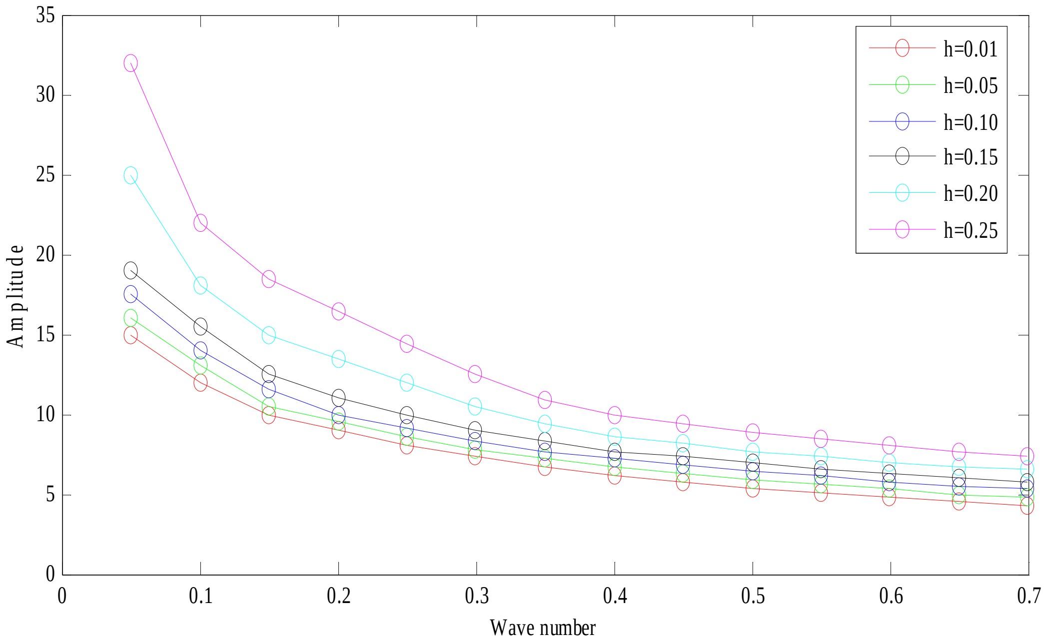

Fig. 4. Variation of amplitude versus wave number of reflected waves for different values of h

VIII. CONCLUSIONS

The scattered waves given in (42) are dominant near the scatterer and die out exponentially as they move away from scatterer. In the free surface (z = 0) the scatterer waves are of the form $\frac{e^{-k_1''x}}{x^{3/2}}$ both for near and far-off points. When h = 0, the scattered waves do not vanish as there is a discontinuity of surface at the point (0, 0).

The amplitude of the scattered waves decreases very rapidly with the slower increment in the value of wave number which signifies that as the wave number increases, the amplitude decreases at a faster rate but reduces to zero after a long time. The variation of amplitude with the wave number by taking barriers of different sizes shows that the behavior of reflected Rayleigh waves depend on the size of irregularity. In particular, this paper shows that larger the size of the barrier, larger is amplitude of the reflected Rayleigh waves resulting into more energetic reflected Rayleigh waves. This explains why the regions with more irregularities in earth surface face frequent earthquake with high intensity.

REFERENCES

[1] Chattopadhyay, A., Gupta, S., Sharma, V. K., Kumari, P., "Propagation of shear waves in visco-elastic medium at irregular boundaries", Acta Geophysica, 58, 2009, pp. 195-214.

[2] Copson, E. T., "Theory of functions of complex variables", Oxford University Press, 1935.

[3] Deshwal, P. S., "Diffraction of a compressional wave by a rigid barrier in a liquid half-space", Pure Appl. Geophys., 85, 1971, pp. 107-204.

[4] Deshwal, P. S., "Diffraction of a compressional wave by a rigid barrier in a liquid layer-Part II", Pure Appl. Geophys., 91, 1971, pp. 14-33.

[5] Ewing, W., Jardetzky, W.S. and Press, F., "Elastic Waves in Layered media", McGraw-Hill Book Co., Inc., New York, 1957.

[6] Gregory, R.D., "The attenuation of a Rayleigh wave in a half-space by surface impedance", Proc. Camb. Phil. Soc., 62, 1966, pp. 811-827.

[7] Momoi, T., "Scattering of the Rayleigh wave by a semi-circular rough space", J. Phys. Earth, 28, 1980, pp. 497-519.

[8] Momoi, T., "Scattering of Rayleigh waves by a rectangular rough space", J. Phys. Earth, 30, 1982, pp.295-319.

[9] Noble, B., "Methods based on the Wiener-Hopf technique", 1958.

[10] Oberhetttinger, F., Badii, L., "Tables of Laplace Transforms", Springer-Verlang, New York, 1973.

[11] Tomar, S. K., Kaur, J., "SH-waves at a corrugated interface between a dry sandy half-space and an anisotropic elastic half space", Acta Mechanica, 190, 2007, pp. 1-28.

[12] Ursell, F., "The effect of a fixed vertical barrier on surface waves in deep water", Proc. Camb. Phil. Soc., 43, 1947, pp. 374-382.