Dynamic Modeling and Control of Quad Rotor

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Abstract - This paper presents a nonlinear compensation technique to solve the trajectory tracking problem for a quadrotor. The kinematics and dynamic equations are obtained using Lagrange –Euler principle. The proposed control algorithm is robust to pay load variations even at high speed or low speeds. The controller can compensate the complex dynamics and also other external disturbances. The torque demanded for the particular motion and maximum payload is computed and it is very low compared with other control techniques. The simulation results demonstrate the proposed control algorithm can track the desired trajectory as well as control the attitude levels and also the thrust force is reduced.

Keywords – Quad rotor, Computed torque control, tracking and stablity

I. INTRODUCTION

The developments of Unmanned Aerial Vehicles (UAV's) have stimulated great significance in the automatic control area in the past years. In specific, Quadrotor platforms are promising rotorcraft in the area of UAV research, due to its simplicity in construction and maintenance, their ability to hover, and their vertical takeoff and landing (VTOL) capability. These autonomous vehicles are used in multiple vehicle teams, mobile sensor networks, collecting information (imaging, pursuit, searching, video acquisition and reconnaissance), security, surveillance, control (smuggling prevention), targeting, meteorological and agricultural applications, traffic management and steering, telemetry (remote sizing) and crisis management after natural disasters. Compared with existing helicopters, quad rotors are having simple in construction avoiding the complex mechanical linkage structures which lessens the weight of system. In addition, the individual four rotors are smaller in size compared with the main rotor on a helicopter which can also reduce the weight. The quad rotors can reach at different altitude levels in the closely clustered or dense environments. They have an Autopilot where the human pilot is not needed and maintaining the flight through an appropriate scheduling of aerodynamic forces either autonomously or by remote control.

The dynamics and control of quad rotor are highly nonlinear and time varying behaviors which are studied by the various researchers In Pounds *et al*., the control structure was based on internal linearization while in Tayebi and McGilvray, a quaternion based PD feedback control scheme is implemented for quad rotor. Koo *et al*., derived a complete dynamic model of quad rotor and control algorithm is developed. A sequential nonlinear control strategy is implemented for the derived 6-DOF model, constituted of translational and rotational subsystems. The controller for translational subsystem stabilizes the altitude and generates the desired roll and pitch angles to the rotational subsystem controller. The rotational controller stabilizes the quad rotor near hover. In Mokhtari et al., a robust feedback linearization with a linear H1 controller was applied to deal with the path following problem with parameter uncertainties and external disturbances. Although many control algorithms are available in the literature, they are complicated and most of the above control applications assume that the computed control actions will never reach the saturation limits of the actuators, although in practice it is possible. For instance, when the UAV is far away from its destination, the generated control signals are normally higher than the admissible values. Moreover, the vehicles are composed of mechanical and electrical parts, which are also subject to physical constraints. In this paper, the computed torque control presented in [6] is used and their merits over the PID control are also discussed. In the computed torque control scheme, the generated control torque signals are very low and which can be used to actuate the motors in real time which can be demonstrated with the help of simulation results.

II. KINEMATICS AND DYNAMICS OF QUAD ROTOR

The schematic of quad rotor studied in this paper is shown in Fig. 1. The quad rotor main hub is considered to be a rigid body connected by frames to it and it is actuated by varying the speed of four rotors. The following motions are achieved in the quad rotor: The longitudinal motions are accomplished with the help of changing the speed in turn the forces of front (F_1) and rear (F_3) rotors which is nothing but pitching of the rotor. The lateral motion is realized be means of varying the forces on right (F_2) and left (F_4) propellers.

Normally, the mobile robot has restriction along the sideways and this kind of Nonholonomic constraint is avoided in the quad rotor. The lateral motion can also be realized as a rolling motion in the quad rotor. The yaw motion is accomplished from the difference in speeds and corresponding forces of each pair of propellers namely (F_1 and F_3) and (F_2 and F_4) pairs respectively. i.e., accelerating the two clockwise turning rotors while decelerating the counter clockwise turning rotors, and vice versa. The final motion to lift the quad rotor namely thrust is achieved by increasing the speed of the four propellers. i.e accelerating and decelerating the pair of rotors running in clockwise and counterclockwise directions.

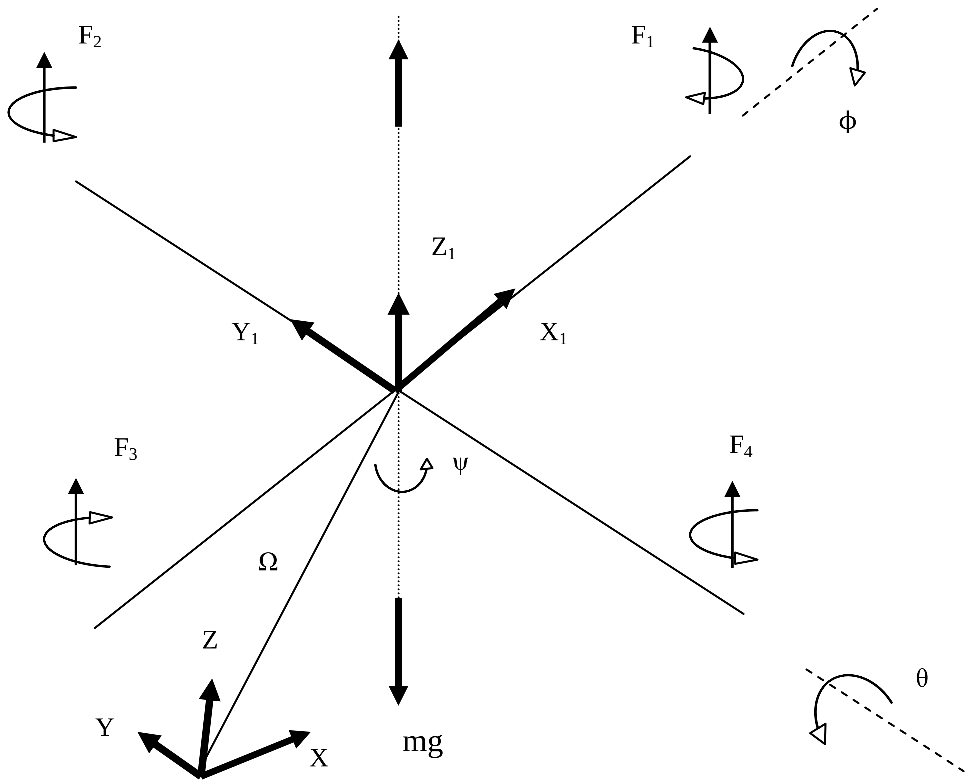

Fig. 1 Schematic of Quad rotor

A. **Kinematics**

The kinematic relation of quad rotor can be obtained by assigning co-ordinate frames as shown in Fig. 1. Let, the body coordinate frame is X_1, Y_1, Z_1, attached to the hub and inertial or fixed frame is denoted as X, Y, Z. The longitudinal motion of the vehicle is achieved along X_1 direction, lateral motion is attained in Y_1 direction and thrust is developed in Z_1 direction. The vector $\Omega = [x, y, z]'$ represents the position of the quad rotor mass center expressed in the inertial frame. Indeed, the most practical way to carry out the rotation relating the inertial frame and body frame is along the thrust direction [8]. In general, the rotation matrix is obtained by successive rotations around the axes of the body frame. In this paper, the XYZ fixed angle [13] is used to describe the rotation of the body with respect to fixed frame.

The structure of the matrix is [13]

$$R = \begin{bmatrix} C\psi C\theta & C\psi S\theta S\phi - S\psi C\phi & C\psi S\theta C\phi + S\psi S\phi \\ S\psi C\theta & S\psi S\theta S\phi + C\psi C\phi & S\psi S\theta C\phi - C\psi S\phi \\ -S\theta & C\theta S\phi & C\theta C\phi \end{bmatrix} \quad (1)$$

where ϕ, θ, ψ are the roll, pitch and yaw angles respectively. S and C represent sin (.) and cos (.) of the respective Euler angles in the above matrix.

The rotational kinematics can be obtained from the relationship between the rotation matrix and its derivative with a skew symmetric matrix

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & \sin\phi \tan\theta & \cos\phi \tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi \sec\theta & \cos\phi \sec\theta \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \quad (2)$$

The above can be written as,

$$\dot{\xi} = J^{-1}{}_n \omega \quad (3)$$

Where, $\xi = [\phi, \theta, \psi]'$ $\omega = [p \quad q \quad r]'$

are the angular velocities in the body fixed frame.

B. **Dynamics of the Quad rotor**

The dynamic equation of motion is formulated with the help of Euler- Lagrange equation which is given by

$$\begin{bmatrix} f \\ \tau \end{bmatrix} = \frac{d}{dt}\left(\frac{\partial L}{\partial qi}\right) - \frac{\partial L}{\partial qi} \quad (4)$$

Where L is the Lagrangian of the quad rotor model, $q = [\Omega \quad \xi]'$ is the state vector and "τ "represents the roll, pitch and yaw moments and "f " is the translational force applied to the quad rotor.

In order to simplify the system dynamics, the total dynamics is divided into translational and rotational dynamics by considering the respective state vectors.

The translational dynamics can be represented with the help of fixed angle is as follows [7]:

$$\ddot{X} = \frac{1}{m}(\cos\psi \sin\theta \cos\phi + \sin\psi \sin\phi) U_1$$
$$\ddot{Y} = \frac{1}{m}(\sin\psi \sin\theta \cos\phi + \cos\psi \sin\phi) U_1$$
$$\ddot{Z} = -g + \frac{1}{m}(\cos\theta \cos\phi) U_1 \quad (5)$$

where m is the quad rotor mass, g is the acceleration due to gravity and U_1 is the thrust load acting along the Z axis.

Let us define the following with the aid of Jacobian matrix from the kinematics (3)

$$\upsilon = J'_n I J_n \quad (6)$$

where I is the diagonal moment of inertia tensor. Then, rotational kinetic energy can be written as,

$$K.E = \frac{1}{2}\dot{\xi}' \vartheta \dot{\xi} \quad (7)$$

and potential energy can be written as

$$P.E = mgz \tag{8}$$

Lagrangian can be formulated as L = K.E – P.E

Then, applying Lagrange – Euler equations, the equations of motion (4), the rotational dynamics is yield into [7]

$$M(\xi)\ddot{\xi} + C(\xi,\dot{\xi})\dot{\xi} = \tau \tag{9}$$

Where

$$M(\xi) = \begin{bmatrix} Ixx & 0 & -IxxS\theta \\ 0 & IyyC^2\phi + IzzS^2\phi & (Iyy - Izz)C\phi S\phi C\theta \\ -IxxS\theta & (Iyy - Izz)C\phi S\phi C\theta & IxxS^2\theta + IyyS^2\phi C^2\theta + IzzC^2\phi C^2\theta \end{bmatrix}$$

$$C(\xi,\dot{\xi}) = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{12} & C_{22} & C_{23} \\ C_{13} & C_{23} & C_{33} \end{bmatrix}$$

$$C_{11} = 0$$

$$C_{12} = (Iyy - Izz)(\dot{\theta}C\phi S\phi + \dot{\psi}S^2\phi C\theta) + (Izz - Iyy)\dot{\psi}C^2\phi C\theta - Ixx\dot{\psi}C\theta$$

$$C_{13} = (Izz - Iyy)\dot{\psi}C\phi S\phi C^2\theta$$

$$C_{21} = (Izz - Iyy)(\dot{\theta}C\phi S\phi + \dot{\psi}S^2\phi C\theta + (Iyy - Izz)\dot{\psi}C^2\phi C\theta + Ixx\dot{\psi}C\theta$$

$$C_{22} = (Izz - Iyy)\dot{\phi}C\phi S\phi$$

$$C_{23} = -Ixx\dot{\psi}S\theta C\theta + Iyy\dot{\psi}S^2\phi C\theta S\theta + Izz\dot{\psi}C^2\phi S\theta C\theta$$

$$C_{31} = (Iyy - Izz)\dot{\psi}C^2\theta S\phi C\phi - Ixx\dot{\theta}C\theta$$

$$C_{32} = (Iyy - Izz)(\dot{\theta}C\phi S\phi S\theta + \dot{\phi}S^2\phi C\theta) + (Iyy - Izz)\dot{\phi}C^2\phi C\theta + Ixx\dot{\psi}S\theta C\theta - Izz\dot{\psi}C^2\phi C\theta$$

$$C_{33} = (Iyy - Izz)\dot{\phi}C\phi S\phi C^2\theta - Iyy\dot{\theta}S^2\phi C\theta S\theta - Izz\dot{\theta}C^2\phi C\theta S\theta + Ixx\dot{\theta}C\theta S\theta$$

III. CONTROL STRATEGY

Besides the speed and payload, the application for which the system is designed plays a vital role in evaluating the importance of the method of control. The controller should compensate the complete dynamics of the system with increasing in pay load and other uncertainties. The compute torque control has been widely used to compensate variety of dynamical systems. The proposed method can overcome PID control in many complex mechatronics systems. This paper utilizes similar control algorithm to control the quad rotor in the desired trajectory as well as to regulate the performance.

Computer torque control is a model based control method utilizes the dynamics of the system to compute the control torque signals that are input to the system. It is a form of Non-linearity cancellation technique for which the dynamic model of the system is known prior.

Computed torque control law is given by,

$$\tau = M\,V + C(\xi, \dot{\xi}) \tag{10}$$

where

$$V = \ddot{q}_d - 2\lambda\,\dot{\tilde{q}} - \lambda^2\,\tilde{q}$$

Substitution of control torque (10) into the rotational dynamics (9) yields

$$\dot{\tilde{q}} + 2\lambda\,\dot{\tilde{q}} + \lambda^2\,\tilde{q} = 0 \tag{11}$$

where

$$\tilde{q} = q - q_d$$

The above equation (11) conveys that the system exponentially decays to zero which necessitate the stability of the system. The proposed control law can be applied to any arbitrary systems which can compensate the non linear dynamics of the system considered.

IV. SIMULATION

The suggested computed torque control has been tested by the simulation to check its performance. The initial conditions of the quad rotor of state vectors are = [0.8; 0.9; 0.7; 0.1; 0.1; 0.1]; The following model parameters are considered for the simulation m=0.74kg, Ixx = 0.004 kg.m2, Iyy =0.004 Kg.m2, and Izz =0.008 Kg.m2. The reference trajectory is a circular path which is defined by Xd = sin(t) and Yd = cos(t), also to reach the desired altitude in Z direction is Zd = 0.3m.

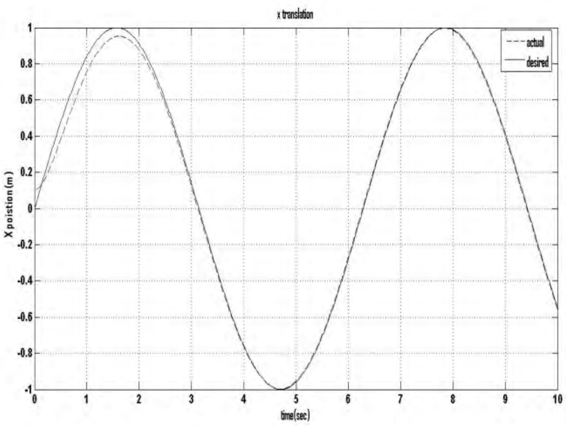

Fig. 2 X - motion tracking

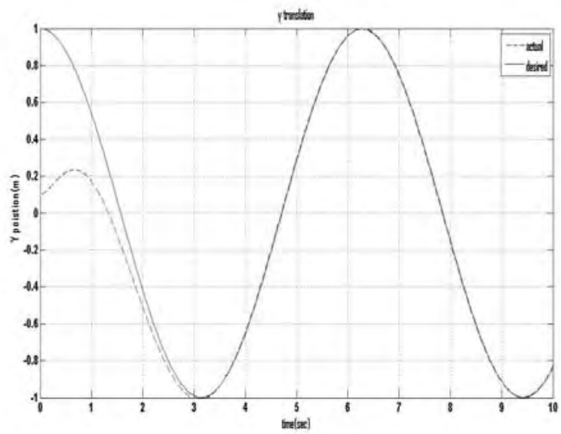

Fig. 3 Y - motion tracking

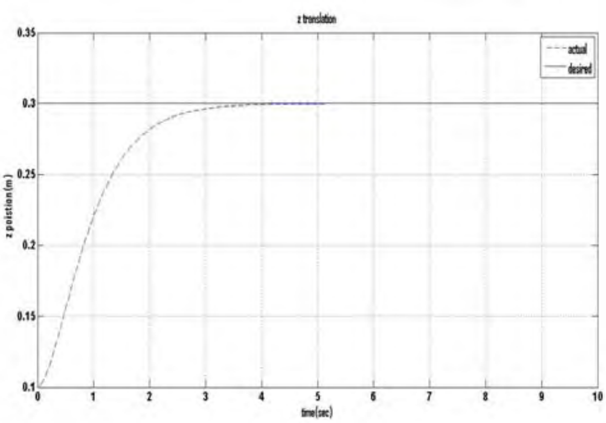

Fig. 4 Z - motion

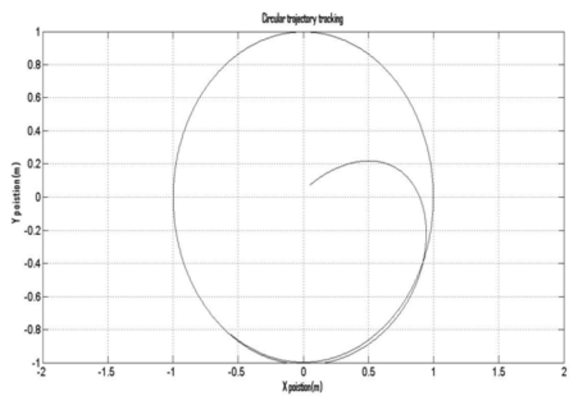

Fig. 5 Circular trajectory tracking

It can be observed from the Fig. 2 and Fig. 3 that the system can reach the desired trajectory approximately 2 secs and also achieve the desired altitude along z direction in 3 secs. The circular trajectory tracked by the quad rotor system is shown in Fig 5. It is also evident from the Fig. 6, Fig. 7 and Fig. 8 that the attitude of quad rotors namely roll, pitch and yaw angles is also obtained the desired level. The thrust force is calculated and the Fig. 9 shows that at the equilibrium it is against the gravity force. The simulation results shows that the proposed controller can track the desired trajectory and also achieve the desired attitude levels.

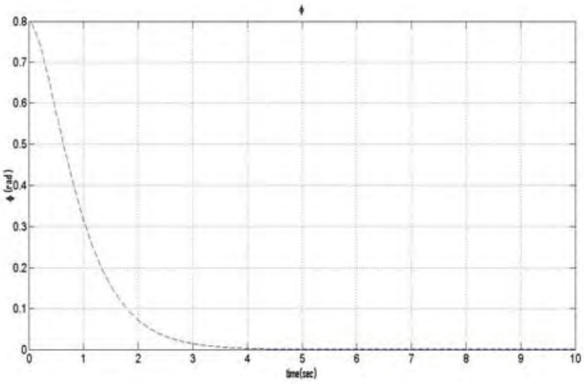

Fig. 6 Roll motion

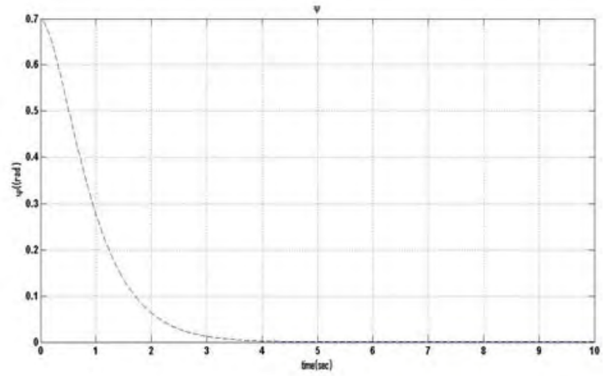

Fig. 7 Pitch motion

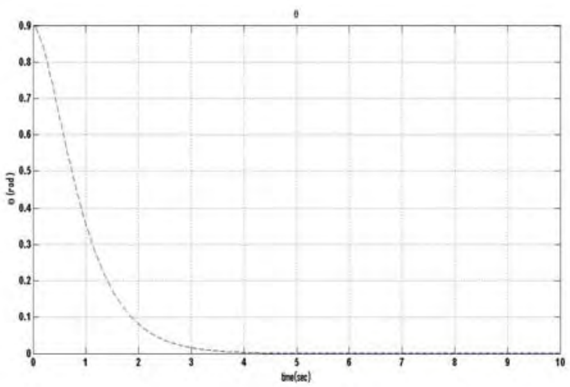

Fig. 8 Yaw motion

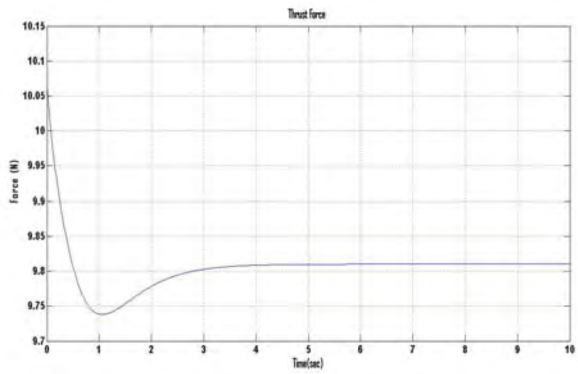

Fig. 9 Thrust Force

V. CONCLUSION

In this article, the quad rotor kinematics and dynamics has been studied. In order to control the quad rotor in the desired trajectory and also to achieve the required attitude, computed torque control is considered. This method is practically robust to external disturbances in the system dynamics and the exact torque is calculated for the desired motion trajectory and also for the particular payload. It is also evident from the simulation results that, this control algorithm is an efficient choice to implement it in real time applications.

VI. REFERENCES

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