BER PERFORMANCE IMPROVEMENT USING MIMO TECHNIQUE OVER RAYLEIGH WIRELESS CHANNEL with DIFFERENT EQUALIZERS

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Abstract

Wireless communication using Multiple-Input Multiple-Output (MIMO) links has emerged as one of the most significant breakthroughs in modern communications because of the huge capacity and reliability gains promised even in worst fading environment. This paper presents an overview of some important behaviors of MIMO systems under Rayleigh channel environments. This work describes the basic ideas of MIMO transmission systems and focused and investigated the BER performance. All analysis was performed under ideal identical independent fading conditions by the use of MATLAB. At the initial stage of the work we related the SNR and the error performance of MIMO systems with the diversity schemes, in the later part of the paper, implementations of different equalizers are also verified for the improvement of the BER performance. Each chapter is rounded by a number of simulations to deepen the understanding of the performance with the use of multiple antennas and equalizers in wireless communication over Rayleigh wireless radio channels. MIMO fading channels are correlated to observe mutual coupling between antenna elements. Receiver diversity is analyzed especially with the Maximal Ratio Combining(MRC) technique and fair comparison is done with Equal Gain Combining(EGC) and Selection- Combing(SC). Further study is done with integration of Maximum Likelihood (ML), Maximum Mean Square Equalization(MMSE) and Zero Forcing(ZF). All the results obtained are simulated by using the MATLAB, under Rayleigh channel conditions.

Keywords: MIMO,EGC,MRC techniques, ZF, MMSE and ML equalizers, SNR BER.

I. INTRODUCTION

The wireless communications is evolving rapidly for high data rates and better quality of services,. It can also be done by increasing transmit power but there is a limitation for the biological hazards, interference and also it is technically difficult and costly to create linear receivers with sensitivity more than 30-35 dB [1]. The information theoretic capacity of the MIMO wireless channel has been characterized under various assumptions since the prominent works of Foschini [2] and Telatar [3]. The performance improvement of wireless connection by the use of array of antenna elements is an old technique. It was used by Guglielmo Marconi in 1901 to get the better gain of the Atlantic transmissions of Morse codes [4] .This paper discuses the merits of the MIMO System and the BER Performance of the Rayleigh Wireless channel under BPSK modulation scheme. Different diversity techniques like EGC, MRC, SC has been analyzed here. Again various equalization techniques are also implemented to achieve even better link performance.

II. ANALYTICAL MODEL FOR MIMO

MIMO is a narrowband technology. For the H Channel matrix we have

$$\mathbf{y = Hx + n.}$$

The number of independent channels that a signal travels from the sender to the receiver is called as the diversity gain[5]. The proper operation of MIMO systems requires careful design, with the encoded signals received from each transmitting antenna and the multiple communication channels achieving specified orthogonality conditions.[6]. The better combination of number of transmitting and receiving antenna for MIMO systems in BPSK modulation technique that satisfy the good SNR is to be investigated primarily.

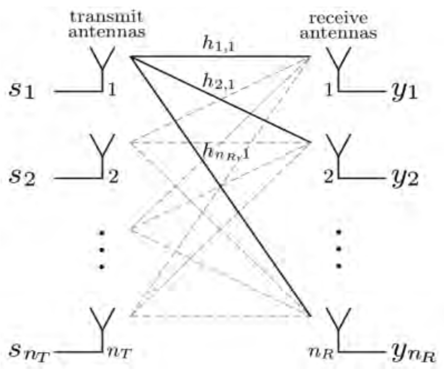

Fig.1.(A MIMO Channel with nT transmit and nR receive antennas)

An adequate description of a MIMO channel is a research area of itself [7], and many publications have investigated for the classification and description of MIMO transmission phenomena and their impact on MIMO performance parameters. Previous work has shown that the system capacity could be linearly increased with the number of antennas when the system is operating over flat fading channels [8,9]. The main arguments today, for using multiple antennas when transmitting over a wireless link are: Array gain ,Interference suppression, Spatial diversity, Transmitter localization, Bit rate, Reliability , Complexity[10]. The capacity of wireless communication systems can also be improved by the Diversity technique.[11] Multiple versions of the same signal may be transmitted and/or received and combined at the receiver end. The fundamental phenomenon which makes reliable wireless transmission difficult is time-varying multipath fading [6,11].

Multiplexing scheme has also introduced by transmitting multiple data streams to a single user with multiple transmit and receive antennas.[11,12]. Among Transmit diversity and receiver diversity, The receiver diversity technique for flat fading and BPSK modulation by taking N number of receiver antennas and noise n, the Gaussian probability density function on each receive antenna is [13],

$$P(n) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(n-\mu)^2}{2\sigma^2}} \quad \text{……………………………….}(1)$$

with $\mu = 0$ and $\sigma^2 = \frac{N_0}{2}$.

From the discussion on chi-square random variable, we know that, if **h_i** is a Rayleigh distributed random variable, then h_i^2 is a chi-squared random variable with two degrees of freedom. The probability density function of γ_i is

$$P(\gamma_i) = \frac{1}{(E_b/N_0)} e^{\frac{-\gamma_i}{(E_b/N_0)}} \text{…………………………...……………}(2)$$

III. OBSERVATIONS

Making different antenna arrangements for simplest conditions, from the study one can observe the BER performance with BPSK modulation scheme for SISO, SIMO, MISO, and MIMO in figure-2. In receiver diversity like Selection combine (SC), Equal Gain Combining(EGC), Maximal Ratio Combining(MRC) more number of antennas are used in receiver side i.e. For N number of antennas.

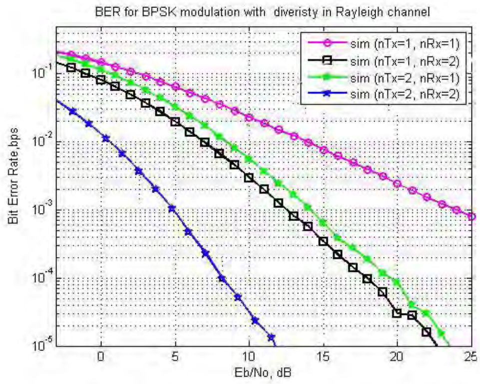

Fig.2. (BER for BPSK modulation scheme for SISO, SIMO, MISO, MIMO)

The outage probability on the ***i***th receiver antenna for selection combining technique is

$$P_{out,\gamma_i} = P[\gamma_i < \gamma_s]$$

$$= \int_0^{s} \frac{1}{E_b/N_0} e^{\frac{-\gamma_i}{E_b/N_0}} d\gamma_i$$

$$= 1 - e^{-\frac{\gamma_s}{E_b/N_0}}$$

the total joint probability is the product of individual probabilities.

$$P_{out} = P[\gamma_1 < \gamma_s] P[\gamma_2 < \gamma_s] \ldots\ldots P[\gamma_N < \gamma_s]$$

$$= \prod_{i=1}^{N} P[\gamma_i < \gamma_s] \left[1 - e^{\frac{-\gamma_s}{E_b/N_0}}\right]^N$$

The probability density function (PDF) is then the derivative of the Cumulative Distribution Function (CDF)

$$P(\gamma) = \frac{dP_{out}}{d\gamma}$$

$$= \frac{1}{E_b/N_0} e^{\frac{-\gamma_s}{E_b/N_0}} \left[1 - e^{\frac{-\gamma_s}{E_b/N_0}}\right] = \frac{E_b}{N_0} \sum_{i=1}^{N} \frac{1}{i}$$

the BER for BPSK in AWGN is derived as

$$P_b = \frac{1}{2} erfc\left(\sqrt{\frac{E_b}{N_0}}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

This equation reduces to

$$P_e = \frac{1}{2} \sum_{k=0}^{N} (-1)^k \binom{N}{k} \left(1 + \frac{k}{(E_b/N_0)}\right)^{-\frac{1}{2}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

Similarly for EGC and two receiving antenna BER is

$$P_e = \frac{1}{2}\left[1 - \{\sqrt{E_b/N_0(E_b/N_0 + 2)}\}/(E_b/N_0 + 1)\right] \ldots(5)$$

And for MRC we have

$$P_e = \int_0^{\infty} \frac{1}{2} erfc(\sqrt{\gamma}) p(\gamma) d\gamma$$

$$= \int_0^{\infty} \frac{1}{2} erfc(\sqrt{\gamma}) \frac{1}{(N-1)!\,(E_b/N_0)^N} \gamma^{N-1} e^{\frac{-\gamma}{(E_b/N_0)}} d\gamma$$

This equation can be reduced to

$$P_e = p^N \sum_{k=0}^{N-1} (N-1-k)\,(1-p)^k \quad \ldots\ldots\ldots\ldots \quad (6)$$

Where,

$$p = \frac{1}{2} - \frac{1}{2}\left(1 + \frac{1}{E_b/N_o}\right)^{1/2}$$

Using Equation 3, 5 and 6 in MATLAB we can see the SNR improvement taking number of receiver antennas in X axis and gain in dB in Y axis in figure-3.

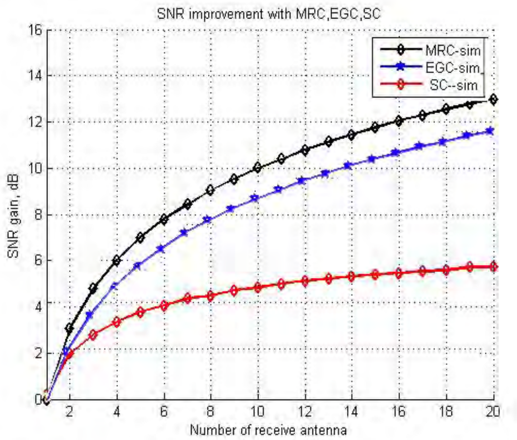

Fig.3:(Improvement in SNR for MRC,EGC,SC techniques)

We can observe that by increasing the number of receiving antenna 1 to 8 keeping the number of transmitting antenna one how the BER performance is increasing in figure-4

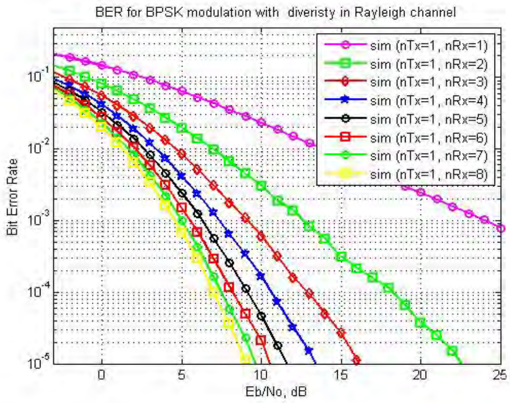

Fig.4.(BER for BPSK Modulation in Rayleigh Channel showing receiving diversity with Rx = 1 to 8)

A. ZERO FORCING (ZF) EQUALIZER:

Zero-forcing (ZF) is one of the common equalization techniques used for high data rates with complex modulation schemes, minimum mean-square-error (MMSE) adaptive (LMS, RLS) algorithms and the Decision Feed-back (DFE) algorithm. (The maximum likelihood(ML) equalizers employing Viterbi decoders usually are not employed for high-rate systems.) [14] .This paper introduces the concept of the ZF equalizer design using the residues of the estimated channel multi-path propagation transfer function. It is assumed that the channel could be sufficient accurately estimated, at the receiver and thus the issues related to channel estimation are not discussed in the paper[15]. It is recently shown that, by a combination of 2^{nd} and 4^{th} order statistics, the channel can be estimated with accuracy better than any estimation techniques reported in the literature [16]. The proposed architecture is useful in finite word-length embedded circuit implementations such as those designed using FPGA or ASIC technology [17]. The proposed residue ZF equalizer architecture could be easily extended for the implementation of adaptive or DFE type equalizers. [18].

In 2 x 2 MIMO system for first receiver antenna received signal

$$y_1 = h_{1,1x_1} + h_{1,2x_2} + n_1 = [h_{1,1x_1} h_{1,2}] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} n_1$$

the received signal on the second receive antenna is

$$y_2 = h_{2,1x_1} + h_{2,2x_2} + n_2 = [h_{2,1x_1} h_{2,2}] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} n_2$$

the above equation can be represented in matrix notation as follows:

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

$$or \quad y = Hx + n$$

To solve for x we know that we need to find a matrix **W** which satisfies

$$WH = I$$

The Zero Forcing (ZF) linear detector for meeting this constraint is given by,

$$W = (H^H H)^{-1} H^H$$

This matrix is also known as the pseudo inverse for a general **m x n** matrix.

$$H^H H = \begin{bmatrix} h_{1,1}^* & h_{2,1}^* \\ h_{1,2}^* & h_{2,2}^* \end{bmatrix} \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix}$$

$$= \begin{bmatrix} |h_{1,1}|^2 + |h_{2,1}|^2 & h_{1,1}^* h_{1,2} + h_{2,1}^* h_{2,2} \\ h_{1,2}^* h_{1,1} + h_{2,2}^* h_{2,1} & |h_{1,2}|^2 + |h_{2,2}|^2 \end{bmatrix}$$

BER with ZF equalizer for higher order MIMO for BPSK modulation scheme in Rayleigh fading channel is derived as

$$P_b = \frac{1}{2}\left(1 - \sqrt{\frac{E_b/N_0}{(E_b/N_0) + 1}}\right)$$

By Keeping the number of Transmission antenna fix if the receiver antenna will be increased then the BER Performance will be better. The matlab output of the program for the 2 Tx and M Rx antenna is in figure-5

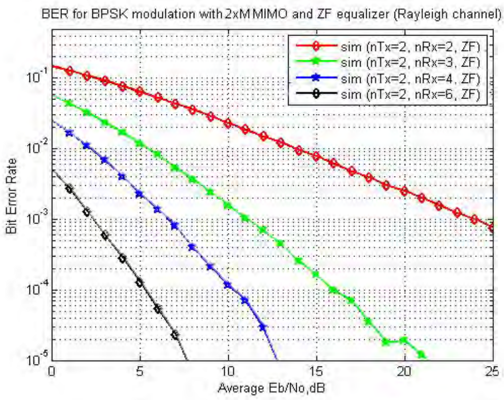

Fig.5.(BER for BPSK Modulation in Rayleigh Channel with 2Tx and M Rx antenna with ZF Equalizer, M = 2,3,4,6)

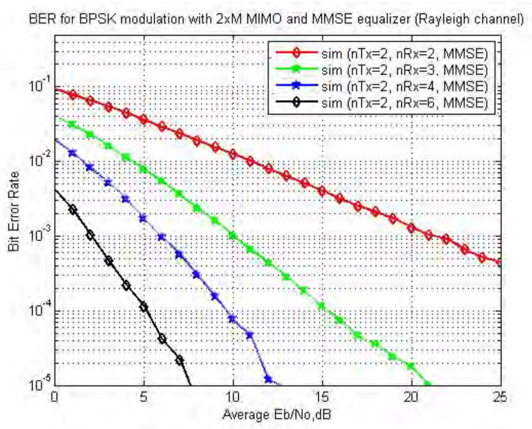

Fig.6.(BER for BPSK Modulation in Rayleigh Channel with 2Tx and M Rx antenna with MMSE Equalizer, M = 2,3,4,6)

B. MINIMUM MEAN SQUARE ERROR (MMSE) EQUALIZER:

The Minimum Mean Square Error (MMSE) approach tries to find a coefficient which minimizes the criterion,

$$E\{[Wy - x][Wy - x]^H\}$$

Solving, $W = [H^H H + N_0]^{-1} H^H$

If N_0 is zero in Zero Forcing equalizer then the MMSE equalizer reduces to Zero Forcing equalizer.

By Keeping the number of Transmission antenna fix if the receiver antenna will be increased then the BER Performance will be better. The mat lab output of the program for the 2 Tx and M Rx antenna is in figure-6

C. MAXIMUM LIKELIHOOD (ML) RECEIVER:

The Maximum Likelihood receiver tries to find $\hat{x}_1$ which minimizes, $J = |y - H\hat{x}|^2$.

$$J = \left\| \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} - \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix} \right\|^2$$

Since the modulation is BPSK, the possible values of x_1 is +1 or -1 similarly x_2 also take values +1 or -1. So, to find the Maximum Likelihood solution, we need to find the minimum from the all four combinations.

$$J_{+1,+1} = \left\| \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} - \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} +1 \\ +1 \end{bmatrix} \right\|^2$$

$$J_{+1,-1} = \left\| \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} - \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} +1 \\ 1 \end{bmatrix} \right\|^2$$

$$J_{-1,+1} = \left\| \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} - \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} -1 \\ +1 \end{bmatrix} \right\|^2$$

$$J_{-1,-1} = \left\| \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} - \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} -1 \\ -1 \end{bmatrix} \right\|^2$$

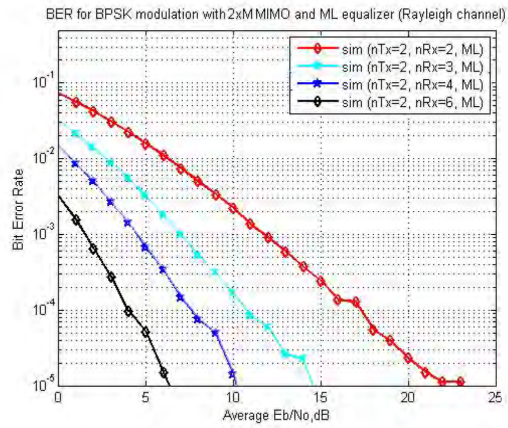

Fig.7(Plot for 2 X M Rayleigh channel with ML equalization for M=2,3,4,6)

IV. RESULT DISCUSSION

The MATLAB simulation results are shown in the figures from 2 to 7. In figure 2,BER performance for SISO, SIMO, MISO, MIMO system is studied. In figure 3, the SNR improvement using various combining techniques is observed. Figure 4 shows the BER performance with receiving diversity. Figures from 5 to 7 demonstrates the effect of equalizers. Observation is to trace the equalizer that performs better BER for Rayleigh wireless Channel environment. Citing single point observation for 2X6 MIMO system, at the SNR of 10 dB, the ZF equalizer achieves the BER of 10^{-4} for, where as with the same SNR, MMSE equalizer shows the BER in between 10^{-4} to 10^{-5} But the ML equalizer even performs better than the above two by providing a BER of exactly 10^{-5} .All the studies are done under Rayleigh channel environment for MIMO communication.

V. CONCLUSION

The BER Performance of the Rayleigh Wireless channel under BPSK modulation scheme is gradually changed by changing the number of antenna either in transmitter side or in receiver side .The study confirms that the better BER performance is achieved if receiver diversity is more than transmission diversity under the MIMO conditions (Fig 2,3 & 4). When MIMO system is integrated with ZF and MMSE equalizers MMSE-performs better(Fig.5 & Fig.6) But by adopting ML Equalizer for the MIMO technique, the system achieved best performance(Fig.6 & Fig.7)

VI. FUTURE SCOPE

The BER performance of MIMO system with various equalizers can be optimized with different bio-inspired optimization techniques.

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